

## Volume 29, Issue 1

### On the Importance of Span of the Data in Univariate Estimation of the Persistence in Real Exchange Rates

Hyeongwoo Kim  
*Auburn University*

Young-Kyu Moh  
*Texas Tech University*

#### Abstract

This paper revisits the empirical evidence on real exchange rates' convergence to their purchasing power parity (PPP) levels. In their recent empirical study, Murray and Papell (2002) claim that the univariate approach provides no useful information on the size of the half-lives of real exchange rate deviations from PPP. However, we obtain finite confidence intervals for the half-life for a maximum of 8 out of 16 countries by applying the nonparametric grid bootstrap technique of Hansen (1999) to over a century of real exchange rates data for 16 developed countries relative to the US dollar. Our finding sharply contrasts to that of Murray and Papell (2002) with the post Bretton Woods real exchange rates. Our finding suggests that span of the data, not the estimation methods, matters more for obtaining useful information on long-run propositions such as PPP.

---

We thank Nelson Mark, Masao Ogaki, Peter Summers, and an anonymous referee for useful comments that helped to improve the paper and Brad Higginbotham for research assistance.

**Citation:** Hyeongwoo Kim and Young-Kyu Moh, (2009) "On the Importance of Span of the Data in Univariate Estimation of the Persistence in Real Exchange Rates", *Economics Bulletin*, Vol. 29 no.1 pp. 129-140.

**Submitted:** Dec 22 2008. **Published:** February 19, 2009.

# 1. Introduction

This paper revisits the empirical evidence on real exchange rates convergence to their purchasing power parity (PPP) levels. In his celebrated work, Rogoff (1996) suggests a 3- to 5-year consensus half-life of PPP deviations from studies using long-horizon data. A great deal of work has been devoted to exploring evidence on PPP, particularly evidence of the consensus half-life but a satisfactory understanding of the real exchange rates dynamics still remains elusive.

It is well known that the least squares estimate for the autoregressive process suffers from significant downward bias and half-life estimation based on that may not yield correct inference on the persistence of the PPP deviations. To correct this problem, Murray and Papell (2002) employ methods proposed by Andrews (1993) and Andrews and Chen (2004) and find that the confidence intervals for half-lives are infinite for most of the post Bretton Woods real exchange rates. Based on their finding, they conclude that univariate estimation methods provide virtually no information on the size of the half-lives.

One major difference between our approach and theirs is the span of the data. Since PPP is a long-run proposition, it would be natural to use long-horizon data, if any, to evaluate its validity. Unlike Murray and Papell (2002) and many others, we use century-long real exchange rate data of Taylor (2002) for 16 developed countries relative to the US dollar.<sup>1</sup> However, bias correction methods due to Andrews (1993) and Andrews and Chen (1994) require normality assumption for error terms. This may be a very strong assumption, since error terms often follow non-normal (possibly unknown) distribution. In this vein, we explore a different road than Murray and Papell (2002) take and apply Hansen's (1999) grid bootstrap methods to correct for the downward bias.<sup>2</sup> One clear advantage of Hansen's method is that it does not require any distributional assumption, such as normality. Furthermore, it can control the type I error globally in the entire parameter space.<sup>3</sup>

By providing bias-corrected point estimates and the confidence intervals for the half-lives of the PPP deviations, we attempt to deliver additional information on the true stochastic process of real exchange rates. It should be noted that one cannot obtain such information from hypothesis tests in the unit root context.

We find non-negligible downward bias in both the least squares estimates for the half-lives and the corresponding confidence intervals. Unlike the results of Murray and Papell (2002), however, we are able to obtain finite confidence intervals for 6 out of 16 countries. When we allow time trend in the regression, we obtain finite confidence intervals for two additional countries.<sup>4</sup> We find substantial information about the size of half-lives using a univariate

---

<sup>1</sup>Taylor (2002) constructs real exchange rate data for a group of 20 countries over 100 years and uses the data to find strong evidence in favor of PPP via linear unit root tests. Lothian and Taylor (1996) also found evidence in favor of mean-reversion for two-century long franc/sterling and dollar/sterling real exchange rates.

<sup>2</sup>Rossi (2005) used Hansen's method for the 17 current float real exchange rates relative to the US dollar and reported infinite confidence intervals for all countries. Therefore, it seems that our gains mainly come from using a long span of data set. We thank an anonymous referee for pointing this out.

<sup>3</sup>Putting it differently, his method has correct first order asymptotic coverage for not only stationary but also local-to-unity autoregressive models.

<sup>4</sup>This may be consistent with the Balassa-Samuelson type PPP. See Taylor (2002) for a detailed explanation.

estimation method. In contrast to the Murray and Papell’s (2002) claim, our finding implies that the span of the data may matter more than estimation methods. The rest of the paper is organized as follows. Section 2 describes Hansen’s (1999) grid bootstrap methods. In Section 3, we report our empirical results. Section 4 concludes.

## 2. The Econometric Model

Consider the following augmented Dickey-Fuller type regression for the natural logarithm of the real exchange rate,  $q_t$ .

$$q_t = c + \alpha q_{t-1} + \sum_{i=1}^k \beta_i \Delta q_{t-i} + \varepsilon_t \tag{1}$$

It is well-known that the least squares estimator for  $\alpha$  is significantly downward-biased when an intercept,  $c$ , is included in the regression equation.<sup>5</sup> It should be also noted that the distribution of  $\varepsilon_t$  is unknown in general and can often be far from being normal. This implies that the conventional *normal* approximation in constructing confidence intervals perform very poorly. When coupled with the downward-bias problem, the normal approximation performs even worse.

These issues have been *partially* resolved by Andrews (1993) for the first order autoregressive process. Utilizing empirical distributions of  $\alpha$  over a grid of values in its parameter space, he showed how to obtain the exactly median-unbiased estimates and bias-corrected confidence intervals. Andrews and Chen (1994) extend this approach to a higher order autoregressive process such as (1) and propose the approximately median-unbiased estimator and corresponding confidence intervals.

It should be noted, however, that their techniques require Gaussianity for  $\varepsilon_t$ , which may be a very strong assumption in many cases. Hansen (1999) proposes a nonparametric bootstrap-based estimator and bias-corrected confidence intervals. His method does not require any distributional assumption but provides excellent coverage properties even when  $\alpha$  is slightly greater than unity. Hansen’s grid bootstrap method asymptotically controls type I error not only for the stationary autoregressive process but also for the mildly explosive autoregressive process.

The method can be implemented as follows. We first define the grid- $t$  statistic at each of  $M$  grid points  $\alpha_j \in [\alpha_1, \alpha_2, \dots, \alpha_M]$  around the neighborhood of the least square point estimate  $\hat{\alpha}$ ,<sup>6</sup>

$$t_N(\alpha_j) = \frac{\hat{\alpha} - \alpha_j}{se(\hat{\alpha})}, \tag{2}$$

---

<sup>5</sup>Note that running the ADF regression (1) is equivalent to implementing a regression without an intercept for demeaned observations. Then, the regression error is correlated with the independent variable, because the current and future values of the dependent variable are embedded in the sample mean. It can be analytically shown that it creates a downward bias (see, among others, Kendall 1954). The same problem arises when an intercept and time trend are included.

<sup>6</sup>The parameter space is not limited to  $(-1, 1]$  as in Andrews (1993). Hansen’s method is valid even for the local-to-unity framework. In other words, it can control the type I error globally in the parameter space.

where  $se(\hat{\alpha})$  is the least square standard error of  $\hat{\alpha}$ .

For each grid point  $\alpha_j$ , we first run least squares estimations for  $\beta$ s by regressing  $q_t - \alpha_j q_{t-1}$  on  $\Delta q_{t-1}, \Delta q_{t-2}, \dots, \Delta q_{t-k}$ . The resulting  $\hat{\beta}$  estimates are treated as nuisance parameters that are functions of  $\alpha_j$ . Then, generating  $B$  pseudo samples for each grid point, we implement least squares estimations for each of the  $B$  bootstrap samples at each of  $M$  grid points. From the bootstrap simulations, we obtain the ( $p$  quantile) grid- $t$  bootstrap quantile function estimates,  $\hat{\psi}_{N,p}^*(\alpha_j) = \hat{\psi}_{N,p}^*(\alpha_j, \beta(\alpha_j))$ , where  $N$  is the number of observations.<sup>7</sup> It is important to realize that each function is evaluated at each grid point  $\alpha_j$  rather than at the point estimate.<sup>8</sup>

Next, we smooth the quantile function estimates by the kernel regression.<sup>9</sup> Finally, we obtain the median unbiased estimate by the following.

$$\tilde{\alpha} = \alpha_j \in R, \text{ s.t. } t_N(\alpha_j) = \tilde{\psi}_{N,50\%}^*(\alpha_j), \quad (3)$$

where  $\tilde{\psi}_{N,p}^*(\alpha_j)$  denotes the smoothed quantile function estimates. The corresponding 95% grid- $t$  confidence interval  $[\tilde{\alpha}_L, \tilde{\alpha}_U]$  is obtained as follows.

$$\begin{aligned} \tilde{\alpha}_L &= \alpha_j \in R, \text{ s.t. } t_N(\alpha_j) = \tilde{\psi}_{N,97.5\%}^*(\alpha_j) \\ \tilde{\alpha}_U &= \alpha_j \in R, \text{ s.t. } t_N(\alpha_j) = \tilde{\psi}_{N,2.5\%}^*(\alpha_j) \end{aligned} \quad (4)$$

The grid- $\alpha$  median unbiased estimate and the confidence intervals can be similarly obtained by defining the grid- $\alpha$  statistic as follows.

$$b(\alpha_j) = \hat{\alpha} - \alpha_j, \quad (5)$$

where  $\alpha_j \in [\alpha_1, \alpha_2, \dots, \alpha_M]$ . Note that the grid- $\alpha$  median unbiased estimate coincides with the Andrew's (1993) estimator when the error terms are normally distributed.

### 3. Empirical Results

Taylor (2002) constructed a century long annual real exchange rate data through 1996 for 19 countries relative to the US dollar.<sup>10</sup> We focus on 16 developed countries by dropping Argentina, Brazil, and Mexico from the original data set. We extended Taylor's (2002) data through 1998 for Eurozone countries and through 2004 for non-Eurozone countries using the IFS CD-ROM.

<sup>7</sup>The quantile function is random, since it is evaluated at  $\beta(\alpha_j)$ . This randomness leads to the approximately median-unbiased estimator in what follows. When  $k = 0$ , there is no such nuisance parameter, and one can obtain the exactly median-unbiased estimator.

<sup>8</sup>If we construct confidence intervals from the quantile function estimates that are evaluated only at the point estimate, the resulting confidence interval coincides with the conventional bootstrap- $t$  confidence interval (Efron and Tibshirani, 1993).

<sup>9</sup>Following Hansen (1999), we used the Epanechnikov kernel  $K(u) = \frac{3(1-u^2)}{4}I(|u| \leq 1)$ , where  $I(\cdot)$  is an indicator function. The bandwidth parameter was chosen by least squares leave-one-out cross-validation.

<sup>10</sup>All real exchange rates for developed countries are CPI-based rates with the exception of Portugal.

As a pretest, we implement the Jarque-Bera tests for the real exchange rates to see whether the normality assumption is not an issue (Table I). The tests reject the null of normal distribution for 13 out of 16 real exchange rates at the 5% significance level. This result leads us to use Hansen’s (1999) method to correct for downward bias rather than using the methods by Andrews (1993) and Andrews and Chen (1994) that require normally distributed errors.

**Table I. Normality Tests by Jarque-Bera Test Statistics**

Country	Sample	JB
Australia	1870-2004	3.641
Belgium	1880-1998	121.3***
Canada	1870-2004	30.17***
Denmark	1880-2004	6.967**
Finland	1881-1998	1473***
France	1880-1998	6.967**
Germany	1880-1998	6.071**
Italy	1880-1998	140.7***
Japan	1885-2004	8.684**
Netherlands	1870-1998	6.946**
Norway	1870-2004	33.94***
Portugal	1890-1998	3.236
Spain	1880-1998	0.043
Sweden	1880-2004	9.401***
Switzerland	1880-2004	11.79***
UK	1870-2004	7.171**

Notes: i) JB refers to the normality test statistics by Jarque and Bera (1980). The null hypothesis is that each exchange rate deviation is normally distributed. The test statistic has an asymptotic chi-square distribution with two degrees of freedom. ii) Superscripts \*, \*\*, and \*\*\* refer the rejections of the null hypothesis of normality at the 10%, 5%, and 1% significance level, respectively.

We begin with the conventional augmented Dickey-Fuller regressions with an intercept and with an intercept and time trend. The 95% confidence intervals are obtained by the conventional nonparametric bootstrap from the empirical distribution (Efron and Tibshirani, 1993). The number of lags ( $k$ ) was chosen by the general-to-specific rule (Hall, 1994) as recommended by Ng and Perron (2001). The results are reported in Tables II and III.

The point estimates of the half-lives without bias correction are slightly longer than Rogoff’s (1996) 3- to 5-year consensus half-life. When time trend is included, the point estimates become closer to the consensus half-life. Most lowerbound half-life estimates are shorter than 3 years, and all upperbound estimates are still finite.

In order to control for bias, we implement a nonparametric grid bootstrap by running 10,000 bootstrap replications from the empirical distribution on 50 grid points in the neighborhood of the least squares point estimates totalling 500,000 bootstrap simulations for each

real exchange rate. We obtained both the grid- $t$  and the grid- $\alpha$  median unbiased estimates and the corresponding 95% confidence intervals, but report the results by the grid- $t$  method in Tables IV and V since we obtained virtually identical results.

**Table II. Least Squares Estimates without Time Trends and Bootstrap Confidence Intervals**

$$q_t = c + \alpha q_{t-1} + \sum_{i=1}^k \beta_i \Delta q_{t-i} + \varepsilon_t$$

Country	$\hat{\alpha}_{LS}$	95% CI	HL $_{LS}$	95% CI
Australia	0.890	[0.763,0.934]	5.931	[2.559,10.15]
Belgium	0.779	[0.629,0.850]	2.777	[1.497,4.523]
Canada	0.895	[0.764,0.941]	6.271	[2.569,11.39]
Denmark	0.886	[0.742,0.934]	5.726	[2.326,10.11]
Finland	0.584	[0.421,0.689]	1.291	[0.802,1.862]
France	0.863	[0.715,0.919]	4.705	[2.070,8.202]
Germany	0.910	[0.801,0.947]	7.350	[3.129,12.84]
Italy	0.753	[0.585,0.837]	2.438	[1.293,3.903]
Japan	0.987	[0.839,0.997]	52.21	[3.939,220.7]
Netherlands	0.905	[0.783,0.945]	6.914	[2.834,12.24]
Norway	0.871	[0.752,0.918]	5.003	[2.427,8.108]
Portugal	0.897	[0.674,0.949]	6.408	[1.758,13.12]
Spain	0.875	[0.758,0.923]	5.193	[2.500,8.613]
Sweden	0.845	[0.678,0.908]	4.124	[1.782,7.185]
Switzerland	0.962	[0.815,0.983]	17.92	[3.388,41.26]
UK	0.856	[0.663,0.919]	4.447	[1.689,8.167]

Notes: i) The number of lags ( $k$ ) was chosen by the general-to-specific rule (Hall, 1994). ii) For each real exchange rate, the 95% nonparametric bootstrap confidence interval was obtained from 2.5% and 97.5% percentile estimates from 10,000 bootstrap replications from the empirical distribution at the least squares point estimates (Efron and Tibshirani, 1993).

We find a non-negligible downward bias for all least squares point estimates. For visual inspection of the bias, we plot the grid- $t$  statistics (solid line), 2.5%, 50%, and 97.5% quantile function estimates (dashed line), and theoretical values for quantiles by normal approximation (dotted line: -1.96, 0.00, 1.96) in Figure 1. For virtually every real exchange rate, one can see significant downward bias. For some countries, the bias was big enough to change statistical inference about the stochastic processes. That is, for Japan and Switzerland, the bias-corrected point estimates<sup>11</sup> were slightly over unity, so their stochastic processes are consistent with the nonstationary process after correcting for bias.<sup>12 13</sup>

<sup>11</sup>As explained in the previous section, one can obtain bias-corrected estimates by (3). Bias-corrected confidence intervals are similarly obtained by (4).

<sup>12</sup>Note that Hansen's method is valid for entire parameter space, so the AR coefficient is not limited to one.

<sup>13</sup>Note that our bias adjusted half-life point estimates are longer than the so-called 3 to 5-year consensus

**Table III. Least Squares Estimates with Time Trends and Bootstrap Confidence Intervals**

$$q_t = c + \gamma t + \alpha q_{t-1} + \sum_{i=1}^k \beta_i \Delta q_{t-i} + \varepsilon_t$$

Country	$\hat{\alpha}_{LS}$	95% CI	$HL_{LS}$	95% CI
Australia	0.816	[0.661,0.870]	3.399	[1.674,4.973]
Belgium	0.689	[0.501,0.768]	1.857	[1.004,2.626]
Canada	0.806	[0.659,0.864]	3.216	[1.660,4.760]
Denmark	0.827	[0.645,0.881]	3.644	[1.582,5.458]
Finland	0.559	[0.382,0.658]	1.193	[0.721,1.654]
France	0.778	[0.641,0.839]	2.764	[1.557,3.942]
Germany	0.888	[0.757,0.925]	5.850	[2.490,8.863]
Italy	0.751	[0.562,0.825]	2.425	[1.202,3.600]
Japan	0.916	[0.548,0.938]	7.942	[1.154,10.76]
Netherlands	0.883	[0.738,0.921]	5.594	[2.277,8.414]
Norway	0.845	[0.702,0.891]	4.126	[1.961,6.026]
Portugal	0.899	[0.616,0.932]	6.501	[1.432,9.859]
Spain	0.871	[0.734,0.911]	5.010	[2.241,7.463]
Sweden	0.752	[0.576,0.822]	2.434	[1.256,3.533]
Switzerland	0.868	[0.660,0.913]	4.886	[1.669,7.573]
UK	0.843	[0.613,0.895]	4.054	[1.418,6.264]

Notes: i) The number of lags (k) was chosen by the general-to-specific rule (Hall, 1994). ii) For each real exchange rate, the 95% nonparametric bootstrap confidence interval was obtained from 2.5% and 97.5% percentile estimates from 10,000 bootstrap replications from the empirical distribution at the least squares point estimates (Efron and Tibshirani, 1993).

Regarding the 95% bias-corrected confidence intervals for the half-life, we find finite confidence intervals for 6 out of 16 countries. For a robustness check, we correct for the bias using a method proposed by Rossi (2005) and report the bias-corrected half-life estimates in Table IV ( $HL_R$ ), which are similar to our median unbiased estimates.<sup>14</sup> When we adopt Samuelson-Balassa type PPP instead, we are able to obtain finite confidence intervals for two more countries, Australia and France.<sup>15</sup> <sup>16</sup> These results sharply contrast with those of

half-life (Rogoff, 1996), which are based upon point estimates with no bias correction. One related issue is the time aggregation bias (Taylor, 2001) meaning that averaging out observations tends to produce upward bias in estimating the persistence parameter  $\alpha$ . We do not attempt to correct for such bias as our main results, finite confidence intervals, will be largely unaffected by such correction.

<sup>14</sup>Following Rossi (2005), we used the modified Akaike Information Criteria (MAIC) to select lag length. The confidence intervals using her method are qualitatively very similar and are not reported.

<sup>15</sup>We employ a two-step procedure. First, we run the regression with an intercept only. If we fail to obtain a finite confidence interval, which provides evidence against PPP, we extend the concept of PPP to Samuelson-Balassa type PPP including time trend as well as an intercept. In this sense, Australia and France serve as additional evidence of PPP. Taylor (2002) also adopted this strategy in the context of unit root tests.

<sup>16</sup>Rossi's (2005) method is not applicable to models with time trends. So we do not report  $HL_R$  in Table V.

Murray and Papell (2002) who found infinite confidence intervals for most countries during the post Bretton Woods system. Based on their results, they claim that the univariate approach provides virtually no information about the size of the half-life. However, our results show that the claim is not valid as we obtain finite confidence intervals for a maximum of 8 out of 16 countries. Our finding suggests that what matters is the span of the data and not the univariate method.

**Table IV. Bias Corrected Estimates without Time Trends and Grid-t Confidence Intervals**

$$q_t = c + \tilde{\alpha}q_{t-1} + \sum_{i=1}^k \beta_i \Delta q_{t-i} + \varepsilon_t$$

Country	$\tilde{\alpha}_{MU}$	95% CI	HL <sub>MU</sub>	HL <sub>R</sub>	95% CI
Australia	0.910	[0.833,1.009]	7.336	7.084	[3.785, $\infty$ )
Belgium	0.797	[0.698,0.901]	3.052	5.404	[1.926,6.678]
Canada	0.919	[0.827,1.022]	8.176	7.642	[3.646, $\infty$ )
Denmark	0.910	[0.823,1.016]	7.348	7.142	[3.554, $\infty$ )
Finland	0.600	[0.467,0.734]	1.358	1.884	[0.910,2.241]
France	0.886	[0.788,1.010]	5.729	6.080	[2.913, $\infty$ )
Germany	0.927	[0.863,1.008]	9.153	18.97	[4.716, $\infty$ )
Italy	0.771	[0.657,0.891]	2.663	3.585	[1.648,5.991]
Japan	1.009	[0.978,1.022]	$\infty$	$\infty$	[30.83, $\infty$ )
Netherlands	0.924	[0.853,1.013]	8.785	8.939	[4.348, $\infty$ )
Norway	0.886	[0.818,0.959]	5.716	7.093	[3.453,16.51]
Portugal	0.942	[0.828,1.046]	11.61	5.357	[3.669, $\infty$ )
Spain	0.893	[0.814,0.986]	6.131	6.291	[3.361,49.54]
Sweden	0.868	[0.770,0.989]	4.903	4.575	[2.654,64.82]
Switzerland	1.004	[0.932,1.033]	$\infty$	$\infty$	[9.791, $\infty$ )
UK	0.887	[0.770,1.026]	5.792	4.240	[2.653, $\infty$ )

Notes: i) The number of lags ( $k$ ) was chosen by the general-to-specific rule (Hall, 1994). ii) For each real exchange rate, the 95% grid- $t$  confidence interval as well as the median unbiased estimate were obtained by 10,000 nonparametric bootstrap replications from the empirical distribution on 50 grid points in the neighborhood of the least squares point estimates (Hansen, 1999). iii) HL<sub>R</sub> denotes the bias-corrected half-life estimates by Rossi's (2005) method. We use the modified Akaike Information Criterion (Ng and Perron, 2001) for these estimates following Rossi (2005).

#### 4. Concluding Remarks

This paper revisits the work of Murray and Papell (2002) who claim that the univariate methods provide no useful information on the half-lives of real exchange rates deviations from



PPP. Finding strong evidence against the normality assumption, we implement nonparametric grid bootstrap techniques proposed by Hansen (1999) to correct for a downward bias in least squares estimate for half-lives. We report finite confidence intervals for a maximum of 8 countries out of 16 developed countries using over a century-long data. Our results sharply contrast with those of Murray and Papell (2002) and Rossi (2005) who used the current float data. We conclude, therefore, that the span of the data, not the univariate approach, matter for obtaining reasonable information with regards to the long-run propositions such as PPP.

**Table V. Bias Corrected Estimates with Time Trends and Grid-t Confidence Intervals**

$$q_t = c + \gamma t + \tilde{\alpha}q_{t-1} + \sum_{i=1}^k \beta_i \Delta q_{t-i} + \varepsilon_t$$

Country	$\tilde{\alpha}_{MU}$	95% CI	$HL_{MU}$	95% CI
Australia	0.848	[0.748,0.965]	4.198	[2.391,19.42]
Belgium	0.718	[0.605,0.844]	2.094	[1.380,4.075]
Canada	0.838	[0.723,1.001]	3.913	[2.133, $\infty$ )
Denmark	0.865	[0.764,1.013]	4.769	[2.574, $\infty$ )
Finland	0.587	[0.449,0.727]	1.301	[0.865,2.176]
France	0.808	[0.696,0.932]	3.253	[1.916,9.841]
Germany	0.918	[0.845,1.014]	8.066	[4.117, $\infty$ )
Italy	0.786	[0.666,0.916]	2.880	[1.708,7.892]
Japan	0.974	[0.902,1.021]	25.94	[6.687, $\infty$ )
Netherlands	0.917	[0.838,1.017]	8.019	[3.927, $\infty$ )
Norway	0.872	[0.796,0.957]	5.045	[3.043,15.91]
Portugal	1.026	[0.847,1.067]	$\infty$	[4.187, $\infty$ )
Spain	0.904	[0.816,1.018]	6.893	[3.419, $\infty$ )
Sweden	0.784	[0.677,0.898]	2.846	[1.777,6.410]
Switzerland	0.916	[0.813,1.027]	7.855	[3.357, $\infty$ )
UK	0.907	[0.770,1.041]	7.097	[2.655, $\infty$ )

Notes: i) The number of lags ( $k$ ) was chosen by the general-to-specific rule (Hall, 1994).  
ii) For each real exchange rate, the 95% grid- $t$  confidence interval as well as the median unbiased estimate were obtained by 10,000 nonparametric bootstrap replications from the empirical distribution on 50 grid points in the neighborhood of the least squares point estimates (Hansen, 1999).

**Figure 1. Grid-t Statistics and Quantile Function Estimates (Intercept Only)**

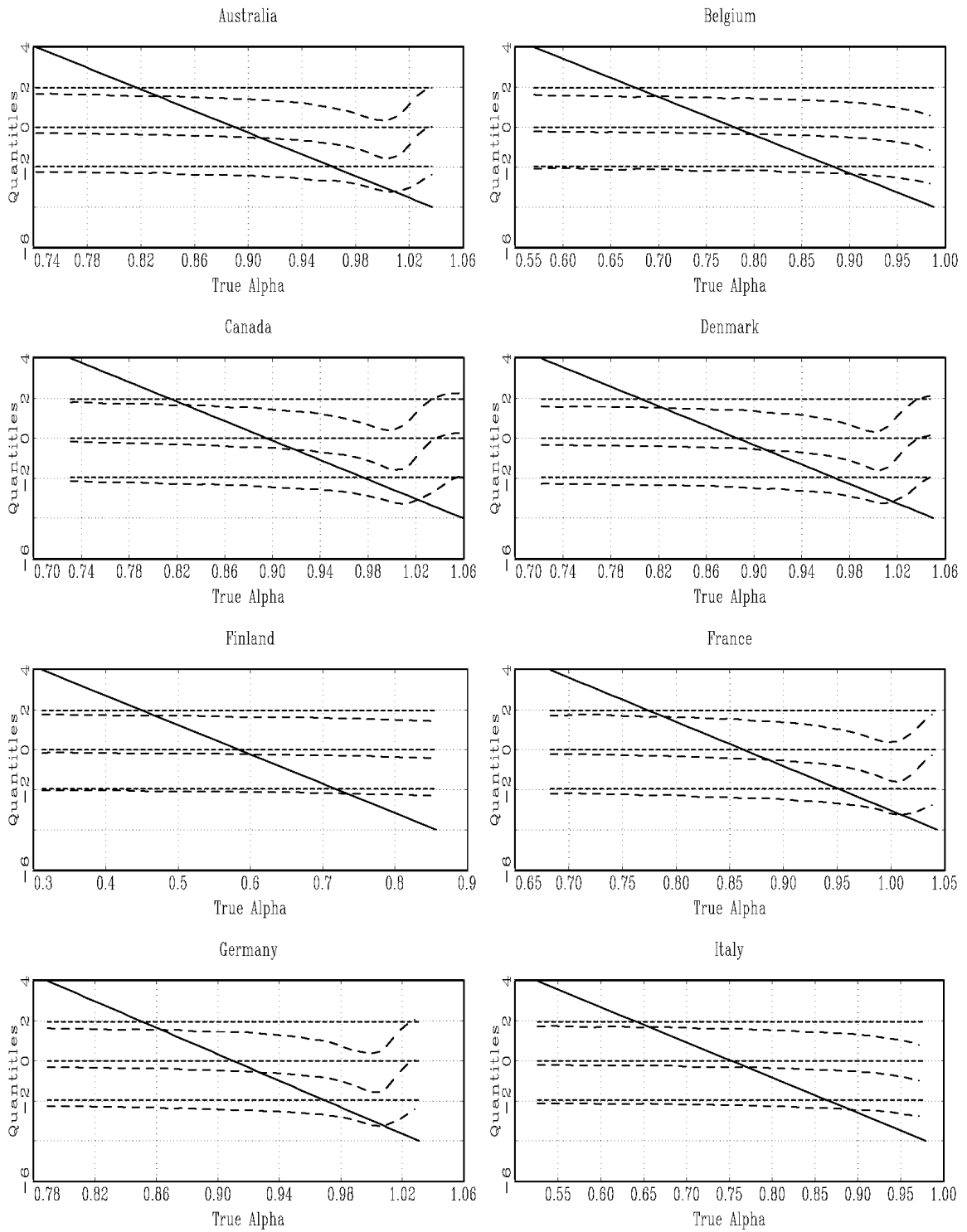
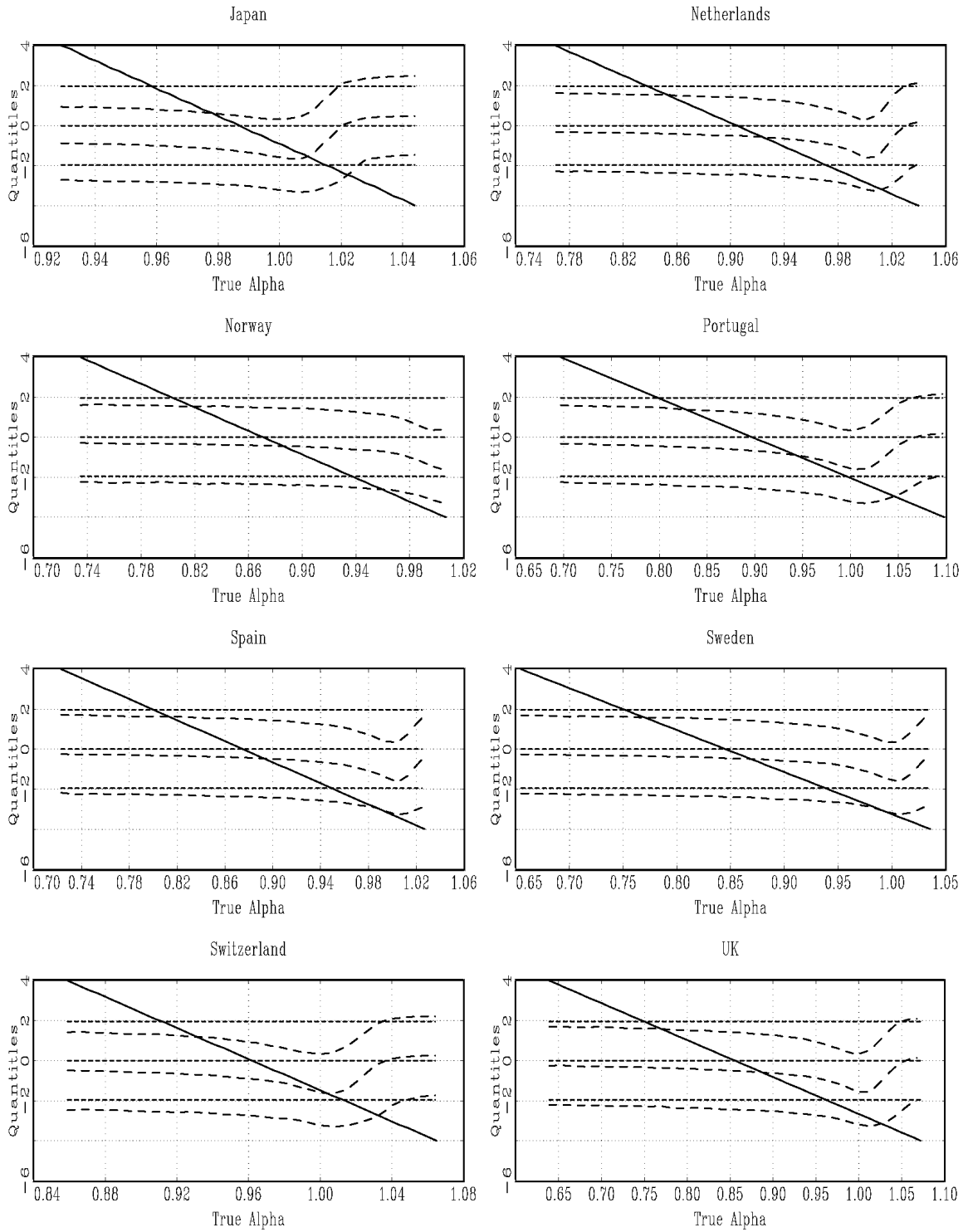


Figure 1. Continued



## References

- Andrews, D. W. K. (1993) "Exactly Median-Unbiased Estimation of First Order Autoregressive/Unit Root Models" *Econometrica* **61**, 139-165.
- Andrews, D. W. K. and H.-Y. Chen (1994) "Approximately Median-Unbiased Estimation of Autoregressive Models" *Journal of Business and Economic Statistics* **12**, 187-204.
- Efron, B. and R. J. Tibshirani (1993) *An Introduction to the Bootstrap*, Chapman and Hall/CRC, London, UK.
- Hall, A. (1994) "Testing for a Unit Root in Time Series with Pretest Data-based Model Selection" *Journal of Business and Economic Statistics* **12**, 461-470.
- Hansen, B. E. (1999) "The Grid Bootstrap and the Autoregressive Model" *Review of Economics and Statistics* **81**, 594-607.
- Kendall, M. G. (1954) "Note on Bias in the Estimation of Autocorrelation" *Biometrika* **41**, 403-404.
- Lothian, J. R. and M. P. Taylor (1996) "Real exchange Rate Behavior: The Recent Float from the Perspective of the Past Two Centuries" *Journal of Political Economy* **104**, 488-509.
- Murray, C. J. and D. H. Papell (2002) "The Purchasing Power Parity Persistence Paradigm" *Journal of International Economics* **56**, 1-19.
- \_\_\_\_\_ (2005) "The Purchasing Power Parity Puzzle is Worse than You Think" *Empirical Economics* **30**, 783-790.
- Ng, S. and P. Perron (2001) "Lag Length Selection and the Construction of Unit Root Tests with Good Size and Power" *Econometrica* **69**, 1519-1554.
- Rogoff, K. (1996) "The Purchasing Power Parity Puzzle" *Journal of Economic Literature* **34**, 647-668.
- Rossi, B. (2005) "Confidence Intervals for Half-Life Deviations from Purchasing Power Parity" *Journal of Business and Economic Statistics* **23**, 432-442.
- Taylor, A. M. (2001) "Potential Pitfalls for the Purchasing-Power-Parity Puzzle? Sampling and Specification Biases in Mean-Reversion Tests of the Law of One Price" *Econometrica* **69**, 473-498.
- \_\_\_\_\_ (2002) "A Century of Purchasing Power Parity" *Review of Economics and Statistics* **84**, 139-150.