Economics Bulletin

Volume 29, Issue 1

On the empirical relevance of st. petersburg lotteries

James C. Cox
Georgia State University

Vjollca Sadiraj
Georgia State University

Bodo Vogt
University of Magdeburg

Abstract

Expected value theory has been known for centuries to be subject to critique by St. Petersburg paradox arguments. And there is a traditional rebuttal of the critique that denies the empirical relevance of the paradox because of its apparent dependence on existence of credible offers to pay unbounded sums of money. Neither critique nor rebuttal focus on the question with empirical relevance: Do people make choices in bounded St. Petersburg games that are consistent with expected value theory? This paper reports an experiment that addresses that question.

The National Science Foundation provided research support (grant number IIS-0630805).
Citation: James C. Cox and Vjollca Sadiraj and Bodo Vogt, (2009) "On the empirical relevance of st. petersburg lotteries", *Economics Bulletin*, Vol. 29 no.1 pp. 214-220.
Submitted: Jan 28 2009. **Published:** February 27, 2009.

1. Introduction

The first theory of decision under risk, expected value maximization, was challenged long ago by the St. Petersburg Paradox (Bernoulli 1738).[1] The original St. Petersburg lottery pays 2^n when a fair coin comes up heads for the first time on flip n, an event with probability $1/2^n$. The expected value of this lottery "is infinite" (that is, larger than any finite number) because $\sum_{n=1}^{\infty} 2^n \times (1/2)^n = 1+1+\cdots+1+\cdots$. But Bernoulli famously reported that most people stated they would be unwilling to pay more than a small amount to play this lottery. He concluded that such reported preferences called into question the validity of expected value theory, and offered a theory with decreasing marginal utility of money as a replacement.

A traditional rebuttal of the alleged paradox is based on the observation that no agent could credibly offer the St. Petersburg lottery for another to play – because it could result in a payout obligation exceeding any agent's wealth – and therefore that this challenge to expected value theory has no bite. Suppose, for example, that the maximum prize offered is equal to $\$2^{35}$ (= \$34.36 billion), an amount that Exxon Mobil could have credibly offered to pay in 2008 since its reported profit for 2007 was \$40 billion. Consider a St. Petersburg lottery that pays $\$2^n$ if the first head occurs on flip *n*, for $n \le 35$, and pays $\$2^{35}$ in the event that there is a run of 35 tails in the first 35 coin flips. This lottery has expected value of \$36 $= \sum_{n=1}^{35} [\$2^n \times (1/2)^n] + \$2^{35} \times (1/2)^{35}$, so it would not be paradoxical if individuals stated they would be unwilling to pay large amounts to play the lottery. An alternative version of the lottery pays $\$2^n$ if the first head occurs on flip *n*, for $n \le 35$, and nothing if there is a run of 35 tails in the first 35 coin flips. The expected value of this lottery is \$35; an expected value maximizing agent would not pay more than \$35 to play it.

Arguments about the St. Petersburg Paradox are great sport, especially now that a generalized form of the paradox has been shown (by Cox and Sadiraj 2008, and Rieger and Wang 2006) to apply to cumulative prospect theory (Tversky and Kahneman 1992), dual theory of expected utility theory (Yaari 1987), rank dependent utility theory (Quiggin 1993), and expected utility theory as well as expected value theory. But neither side of such arguments provides an answer to the question that is relevant to positive economics: Do real offers to play St. Petersburg lotteries elicit real responses that are inconsistent with expected value theory? Surprisingly, there appears to have been no previous experiment that addresses this question.[2]

2. A Real Experiment with Finite St. Petersburg Lotteries

The experiment was designed as follows. Subjects were offered the opportunity to decide whether to pay their own money to play nine finite St. Petersburg lotteries. One of each subject's decisions was randomly selected for real money payoff. Lottery *N* had a maximum of *N* coin tosses and paid 2^n euros if the first head occurred on toss number n,

---

[1] Analysis of the paradox was published by Daniel Bernoulli (1738). It was, reportedly, previously described in a letter dated September 9, 1713 from Nicolas Bernoulli to Pierre Raymond de Montmort.

[2] Bernoulli (1738), Bottom, *et al*. (1989), and Rivero, *et al*. (1990) report experiments with hypothetical responses. After our experiment was completed (in February 2007) we became aware of research on St. Petersburg lotteries by Tibor Neugebauer from his presentation at the North American Regional Conference of the Economic Science Association, November 13-15, 2008. As of the date of this writing, Professor Neugebauer's working paper is not yet available.

for $n = 1, 2, \ldots N$, and paid nothing if no head occurred. Lotteries were offered for N = 1,2, …,9. Of course, the expected payoff from playing lottery N was N euros. The price offered to a subject for playing lottery N was 25 euro cents lower than N euros. Expected value theory predicts preference for the St. Petersburg lottery N for every one of these lotteries, i.e. for all N = 1,…,9.

The experiment was run in the Maxlab at the University of Magdeburg in February 2007. At the beginning of the experiment, subjects were seated at well-separated cubicles and were given printed subject instructions. After reading the instructions, each subject made nine decisions. Subsequently, the decision sheets were collected. Next, one ball was drawn, for each individual subject, from a bingo cage containing nine balls. The number on the ball drawn for a subject selected the decision that paid money for that subject. If a subject had chosen the lottery in the randomly selected decision, money payoff in the lottery was determined by repeatedly flipping a coin until either the first head appeared or until the specified maximum number of tosses in the selected lottery had been attained. Bingo balls were drawn and the coin was flipped in the presence of the subjects. Each subject was paid the amount determined by this procedure in cash, in Euros, immediately at the end of the experiment. An English version of the subject instructions is included in the appendix.

3. Do People Make Risk Neutral Choices with Finite St. Petersburg Lotteries?

Thirty subjects participated in the experiment: (a) 26 out of 30 (or 87%) of the individual subjects refused at least one opportunity to play a St. Petersburg lottery for less than its expected value; and (b) over all subjects, 127 out of 270 (or 47%) of their choices were inconsistent with expected value theory. We next ask whether the observed failure of expected value theory is statistically significant.

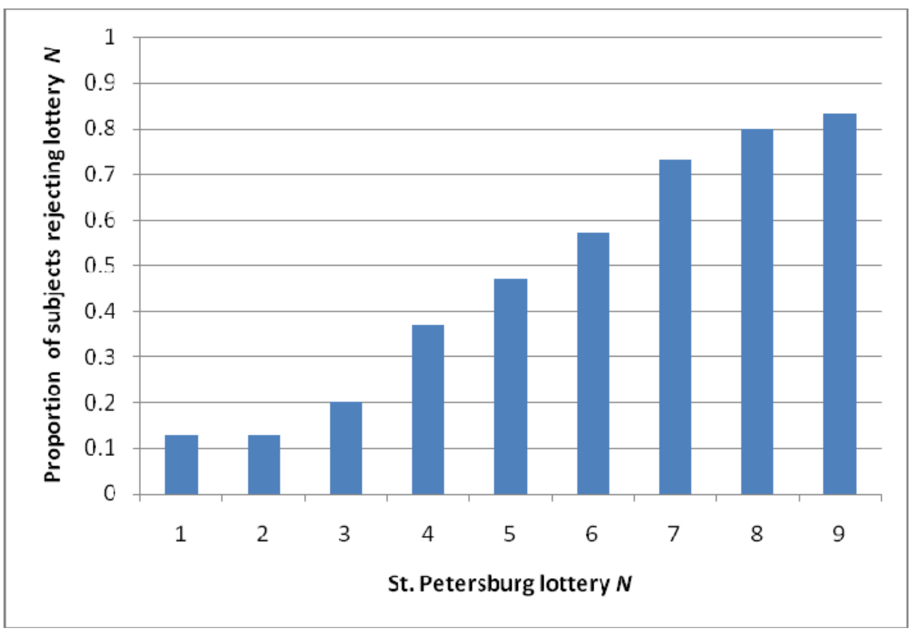

Figure 1. Proportions of Subjects Who Rejected St. Petersburg Lotteries

One way to pose the question is to ask which characterization of risk references is more consistent with the data: (a) risk neutrality; or (b) risk aversion sufficient to imply rejection of all offers to play the St. Petersburg games in the experiment. First, the observed proportions of subjects who reject St. Petersburg lotteries for $N = 1,2, \ldots$ or 9 are: [0.13, 0.13, 0.20, 0. 37, 0.47, 0.57, 0.73, 0.80, 0.83] , as shown in Figure 1. In the first 5 tasks more than half the subjects made choices that are consistent with expected value (EV) theory. However, as the stakes of the sure amount of money required for playing the St. Petersburg lotteries increase, and the variance of payoffs of the lotteries increase, subjects' risk-averse choices start to dominate. From lotteries for N = 6 to 9, the fraction of choices violating EV theory increases from 57% to 83%.

Next, we apply a linear mixture model (Harless and Camerer 1994) with stochastic preference specification for error rate ε: (a) if option Z is stochastically preferred then Prob(choose Z) $= 1 - \varepsilon$; and (b) if option Z is not preferred then Prob(choose Z) $= \varepsilon$. Let the letter a denote a subject's response that she accepts the offer to play a specific St. Petersburg lottery in the experiment. Let r denote rejection of the offer to play the game. The linear mixture model is used to address the specific question whether, for the nine St. Petersburg lotteries offered to the subjects, the response pattern (a,a,a,a,a,a,a,a,a) or the response pattern (r,r,r,r,r,r,r,r,r) is more consistent with the data. With this specification, the log-likelihood is -182 and the estimate of the error rate is 0.29 . The point estimate of the proportion of subjects in the experiment that are not risk neutral (or very slightly risk averse) is 0.48, and the Wald 90% confidence interval of this estimate is (0.30, 0.67). Allowing for a specification with two error rates (one error rate for lotteries 1-4 and another error rate for lotteries 5-9), the estimates of the error rates are 0.50 for the first four lotteries and 0.14 for the last five lotteries. The point estimate of the proportion of subjects in the experiment that are not risk neutral (or very slightly risk averse) is 0.77, with Wald 90% confidence interval of (0.63, 0.91) . The log-likelihood is -164. Using data only for lotteries 4 - 9, that require payments in excess of 3 euros to accept, the error rate is 0.18 and the point estimate of the proportion of subjects in the experiment that are not risk neutral (or very slightly risk averse) is 0.71 with 90% confidence interval (0.56, 0.87). The log-likelihood is -103. Table I summarizes these results.

Table I. Estimates for Error-rate Analysis

	Model 1	Model 2	Model 3 (lotteries 4-9)
nobs	270	270	180
Estimated fraction of risk averse subjects	0.48 (0.30, 0.67)	0.77 (0.63,0.91)	0.71 (0.56,0.87)
Estimated error rate 1	0.29	0.50	0.18
Estimated error rate 2		0.14	
Log-likelihood	-182	-164	-103

Figures in parentheses show Wald 90% confidence intervals.

4. Concluding Remarks

The St. Petersburg Paradox has been known for almost 300 years. Arguments about its relevance to judging the plausibility of expected value theory are almost as old. Although arguments about the paradox are great sport, they don't address the original question that motivated its introduction: Are decision makers risk neutral? We report what appears to be the first experiment with real money payoffs for finite St. Petersburg lotteries. Data from our experiment support the conclusion that a majority of subjects in the experiment are risk averse, not risk neutral.

References

Bernoulli, D. (1738) "Specimen Theoriae Novae de Mensura Sortis" *Commentarii Academiae Scientiarum Imperialis Petropolitanae*, **5**, 175-192. English translation: "Exposition of a New Theory on the Measurement of Risk" *Econometrica* **22**, 1954, 23-36.

Bottom, W.P., R.N. Bontempo and D.R. Holtgrave (1989) "Experts, Novices, and the St. Petersburg Paradox: Is One Solution Enough?" *Journal of Behavioral Decision Making* **2**, 139-147.

Cox, J.C. and V. Sadiraj (2008) "Risky Decisions in the Large and in the Small: Theory and Experiment" in *Risk Aversion in Experiments* by J.C. Cox and G.W. Harrison, Eds., Emerald: Bingley, UK, Research in Experimental Economics, Volume 12, 9-40.

Harless, D. and C. F. Camerer (1994) "The Predictive Utility of Generalized Expected Utility Theories" *Econometrica* **62**, 251-1290.

Quiggin, J. (1993) *Generalized Expected Utility Theory. The Rank-Dependent Model*, Kluwer: Boston.

Rieger, M.O. and M. Wang (2006) "Cumulative Prospect Theory and the St. Petersburg Paradox" *Economic Theory* **28**, 665–679.

Rivero, J.C., D.R. Holtgrave, R.N. Bontempo and W.P. Bottom (1990) "The St. Petersburg Paradox: Data, At Last" *Commentary* **8**, 46-51.

Tversky, A. and D. Kahneman (1992) "Advances in Prospect Theory: Cumulative Representation of Uncertainty" *Journal of Risk and Uncertainty* **5**, 297-323.

Yaari, M.E. (1987) "The Dual Theory of Choice under Risk" *Econometrica* **55**, 95-115.

Appendix: Subject Instructions

Please write your identification code here:
A coin is tossed no more than 9 times. Your payoff depends on the number of tosses until Head appears for the first time. If Head appears for the first time on flip number N then you are paid 2^N Euros. Table A.I shows the possible outcomes:

Table A.I

Head appears for the first time on coin toss	Probability this will occur	Your payoff in Euros
1	0.5	2
2	0.25	4
3	0.125	8
4	0.0625	16
5	0.03125	32
6	0.015625	64
7	0.0078125	128
8	0.0039062	256
9	0.0019531	512
Never		0

There are a variety of different lotteries offered to you that differ in the maximum possible number of coin tosses and the amount you have to pay if you want to participate in the lottery. Table A.II shows this.

Table A.II

Maximum number of tosses	Participation fee in Euros	I choose to pay to participate: Yes/No
1	0.75	
2	1.75	
3	2.75	
4	3.75	
5	4.75	
6	5.75	
7	6.75	
8	7.75	
9	8.75	

For example, if you decide to pay 3.75 Euros to participate in the lottery with a maximum of 4 tosses, the coin will be flipped 4 times. Your payoff is determined according to Table A.I.

If Head appears on the first toss then you will receive 2 Euros, regardless of the results of the further tosses. If Tail appears on the first toss and Head on the second, you will receive 4 Euros, regardless the results of the further tosses. If Tails appear on the first two tosses and then Head on the third toss, you will receive 8 Euros, regardless of the further tosses. If Tails appear on the first three tosses, followed by Head on the fourth toss, you receive 16 Euros. If Head never appears your payoff is 0 Euro.

Payoffs

After you make your decisions, one of the rows will be selected by chance and your Yes or No decision in that row will become binding. The selection of the row is carried out by drawing a ball from a bingo cage containing balls with numbers 1,2, …, 9. The number on the drawn ball determines the row of the table that is selected.

If, for example, row 6 is selected for payoff then: (a) if your decision in row 6 is “No” then no money changes hands; (b) if your decision in row 6 is “Yes” then you will pay the experimenter 5.75 Euros to play the coin toss lottery with a maximum of 6 tosses and possible outcomes in the first 6 rows of Table A.I.