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A Note on Strategic Delegation: The Role of Decreasing Returns to Scale

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Abstract

We build a model of optimal design of managerial incentive schemes when the production technology exhibits decreasing returns to scale and firms compete à la Cournot. We borrow Fershtman and Judd (1987) and Kräkel (2005) framework. We show how there is a dominant strategy for entrepreneurs to delegate output decisions. Results depend on the degree of diseconomies of scale. We demostrate how for a class of parameters, managers may increase profits through delegation, a result that with constant returns does not hold.

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1 Introduction

The analysis of the strategic use of managerial contracts in oligopolistic games started with Vickers (1985) and further developed in Fershtman (1985), Fershtman and Judd (1987), and Sklivas (1987) (hereafter referred to as VFJS). The design of managerial contracts on the part of firms' owners will affect managers' decisions. Attention is restricted to a class of linear combination contracts between profits and sales, that owners offer to managers. For example, if managers' compensation schemes are made to depend on sales rather than profits, managers will be less concerned about production costs. This will have the effect of encouraging them to expand output, which indeed will return above oligopolistic output levels. On the contrary if managers are encouraged to minimize costs, then they will be less concerned on sales. Therefore, market outcomes, and ultimately consumer welfare, are related to manager's compensation schemes. In order to make the problem meaningful, there must be a source of uncertainty (demand, costs etc.). Otherwise, if there was complete information owners could enforce a quantity type of contract.

Under quantity competition with linear cost of production, Fershtman and Judd (1987) show how a managerial contract that rewards managers on the basis of revenue has the effect of committing the firm to high levels of output, whereas under price competition owners put a negative weight on sales. Profits for owners are lower than in the standard Cournot result. However, some studies analyze the possibility of profit increase through delegation. Basu (1995) shows how when an additional decision to hire a manager stage is explictly included, then Stackelberg solution may be obtained if differences in cost are sufficiently large. Jansen et al. (2007) consider instead a (linear) combination of profits and market share. They show how this is indeed a dominant strategy in which owners get higher profits than with a VFJS type of contract, but lower than Cournot profits. These authors stress that the delegation of control to managers in Cournot settings can be advantageous in that it may give rise to Stackelberg leadership even though firms move simultaneously. The equilibrium arising in delegation games where firms are Cournot players indeed involves both firms delegating control in order to try to achieve a dominant position. All firms would prefer the rivals not to delegate, and the equilibrium is affected by a prisoner's dilemma.

Other studies have focused in different versions of VFJS framework. Ishibashi (2001) considers quality as well as price competition. Zhang and Zhang (1997) consider R&D in a strategic delegation game. These studies share a common feature. They include a third stage to the original two-stage game in order to allow for a third endogenous variable. Lambertini (2000) considers a four-stage game in which delegation and strategic variables are also endogenized. (Bhardwaj (2001) considers price competition and the choice of effort.in a five-stage model. Results depend on the intensity of price competition. Prices are not necessarily strategic complements as in Bulow at al. (1985). Miller and Pagal (2001) in a differentiated products oligopoly model, show the equivalence of price and quantity competition when owners are able to design compensation schemes sucha that also depend upon rival's profits.

Kräkel (2005) studies strategic delegation in duopolistic tournaments with increasing cost function. Despite the ex-ante symmetry of the problem, he shows the existence of an asymmetric equilibrium where one owner puts a positive weight on sales (more aggresive behavior) whereas the other puts a negative one.

In this note, we borrow VJFS framework to analyze the role of diseconomies of scale when entreprenuers are profit-maximizers. The convexity of the cost function can be motivated as an approach to the existence of capacity constraints and scarcity of resources. In this case, owners may be more concerned on cost minimization. We would expect that convexity of the cost function plays a significant role, as it does in Kräkel (2005). Optimal rewarding schemes take into account how expanding output also increases cost at a higher rate than under constant returns to scale. We assume demand is uncertain at the first stage of the game for the owners.[1]

The paper is organized as follows. Section 1 builds the model. Section 2 provides the optimal rewarding scheme under quantity competition with random demand. We show how the existence of a convex cost function may increase profits from delegation, as compared to entrepreneurial management. Section 3 concludes.

2 The Model

We consider a two-stage symmetric duopoly model with homogeneous product. Demand is linear and given by $Q(p) = \widetilde{\theta} - p$, $\widetilde{\theta}$ is distributed with mean $E(\theta)$ in the interval $\left[\underline{\theta}, \overline{\theta}\right]$.[2] The cost function is increasing: $C_i(q_i) = \frac{1}{2}cq_i^2$, where $c > 0$. Thus, marginal cost is given by $mc_i(q_i) = cq_i$ Each firm is characterized by an owner and a manager. On the first stage (compensation stage), each owner decides about an incentive scheme for the (single) manager. Following VFJS, the optimal rewarding structure for the manager is a linear combination of profits (Π_i) and sales (S_i) of the form,

$$O_i(Q) = \alpha_i \Pi_i(Q) + (1 - \alpha_i) S_i(Q) \text{ for } i = 1, 2$$

or equivalently,

$$O_i = S_i(Q) - \alpha_i C_i(q_i)$$

where $S_i(Q)$ is the sales function, and $\alpha_i \in \mathbb{R}$, that is no restrictions are imposed on the incentive parameter. The manager's total compensation is given by $M_i = A_i + B_i O_i$ $(B_i > 0)$. The model assumes incentive schemes are observable by both parties and no further renegotiations are allowed. Then, managers compete à la Cournot.

The timing of the game is the following. In stage 1, owners simultaneously choose the optimal rewarding structure (α_1^*, α_2^*). Then, in stage 2, managers simultaneously choose the optimal production levels (q_1^*, q_2^*). The model is solved by backward induction. We obtain the equilibrium in the second stage as a function of the incentive parameters, and without uncertainty on demand. Then, owners maximize expected profits choosing the optimal rewarding scheme and knowing the probability distribution of demand.

[1] Cost uncertainty turns out to be untractable under increasing costs. Since we are interested in the role strategic delegation under increasing costs, we do not consider this source of uncertainty as in Fershtman and Judd(1987).

[2] We have normalized the slope of the demand function to be one because the qualitative results are the same.

3 Incentive equilibrium

In stage two, the manager of each firm observes c and the realization of $\widetilde{\theta}$, θ. Each firm's quantity is function of the incentive parameters; $q_i(\alpha_i, \alpha_j)$ for $i, j = 1, 2$ and $i \neq j$. Manager of firm i maximizes O_i, as defined before, by choosing q_i as a function not only of the rival's production level, but also on the incentive scheme designed by the owner:

$$\max_{q_i} \quad (\theta - Q)\, q_i - \tfrac{1}{2}\alpha_i c q_i^2.$$

Given α_i, with $i = 1, 2$, the first order condition yields the best response function given by

$$q_i(q_j) = \frac{\theta - q_j}{2 + \alpha_i c}$$

and symmetrically for the other firm. Note how even under quantity competition, a priori it may be the case that quantities are strategic complements, in Bulow et Al (1985) sense as long as $\alpha_i < 0$ and $|\alpha_i| > \frac{2}{c}$. This possibility does not arise in constant linear marginal cost models. It holds that output is decreasing with α_i, that is as the manager is forced to be more concerned on costs, production shrinks. We find the equilibrium quantities as a function of (α_i, α_j), for $i, j = 1, 2$ and $i \neq j$,

$$q_i^*(\alpha_i, \alpha_j) = \frac{(1 + c\alpha_j)\,\theta}{2c(\alpha_i + \alpha_j) + c^2\alpha_i\alpha_j + 3}.$$

In stage 1, managers choose $\left(\alpha_i^*, \alpha_j^*\right)$ to maximize expected profits, given $q_i^*(\alpha_i, \alpha_j)$.

$$\max_{\alpha_i} \quad E\left[(\theta - Q^*(\alpha_i, \alpha_j))\, q_i^*(\alpha_i, \alpha_j) - \tfrac{1}{2}c\,(q_i^*(\alpha_i, \alpha_j))^2\right]$$

once the equilibrium results from stage 1 are included, the expected profit function can be written as

$$\max_{\alpha_i} \quad \frac{\left(2c\alpha_i - c + 2c\alpha_j - c^2\alpha_j + 2c^2\alpha_i\alpha_j + 4\right)}{\left(2c(\alpha_i+\alpha_j) + c^2\alpha_i\alpha_j + 3\right)^2} \tfrac{1}{2}\,(c\alpha_j + 1)\, E\left(\theta^2\right)$$

Proposition 1 summarizes the properties of the optimal rewarding scheme. Proof can be found in Appendix 1.

Proposition 1 *The optimal rewarding schedule in the symmetric equilibrium depends only on the curvature of the cost function, c, and is given by,*

$$\alpha^*(c) = \frac{1}{2} + \frac{1}{c}\left(\frac{1}{2}\sqrt{(c+4)\,c} - 1\right).$$

Therefore, in the unique symmetric equilibrium the output level and the corresponding profit *for each firm are,*

$$\begin{aligned}
q^*(c) &= \frac{2E(\theta)}{4+c+\sqrt{4c+c^2}} \\
\pi^*(c) &= \frac{\left(c+\sqrt{(c+4)\,c}-c^2\right)E\left(\theta^2\right)}{\left(2+c+c\sqrt{(c+4)\,c}\right)(4+c)}
\end{aligned}$$

which are positive for every $c \in (0, 2.5943)$. *It holds that* $\alpha^*(c) < 1$ *and* $\lim_{c\longrightarrow 0}\alpha^*(c) = -\infty$.

Note how profit is linearly increasing with θ. However, given a realization of the demand it can be shown profits are concave in $c > 0$. As a result, it is feasible for firms to increase profits increasing production. Therefore, designing incentive schemes that encourage output expansion is profit-enhancing. Corollary 1 discusses the threshold of c at which $\alpha^*(c) = 0$.[3]

Corollary 1 *If* $c = 0.5$ *then* $\alpha^*(0.5) = 0$. *Thus, if* $c \in (0, 0.5)$ *then* $\alpha^*(c) < 0$.

This means that the delegation contract puts a negative weight on profits. It actually encourages managers to be more aggresive in the product market, since as c tends to zero the cost function becomes flatter and the marginal cost tends to zero. Proposition 2 considers the range of the parameter c such that incentive scheme is either positive, but less than one, or negative

Proposition 2 *Assume* θ *is distributed with mean* $E(\theta)$, *then there is a unique subgame perfect Nash equilibrium of the two-stage game. If* $c\,(0, 0.5]$ *then* $\alpha^* \leq 0$, *whereas if* $c\,(0.5, 2.5943)$ *then* $\alpha^* \in (0, 1)$.

Putting a negative weight on profits is optimal for sufficiently flat marginal cost. Relatively high marginal cost is required to design contracts in which managers are encouraged to be more concern on cost. Note how as an additional result from Proposition 2, in equilibrium best response functions of managers cannot be upward sloping. Therefore, best response strategies can not be strategic complements. We also compare the predictions with those under no delegation. Call q^N the Nash equilibrium quantity when there is no delegation, that is when owners take production decisions.

$$\begin{aligned}
q^N &= \frac{E(\theta)}{c+3} \\
\pi^N &= \frac{(c+2)\,E\left(\theta^2\right)}{2\,(c+3)^2}.
\end{aligned}$$

Our main result is summarized in Proposition 3. It compares output and profits when owners offer incentive schemes to managers and when they do not.

[3] I would like to thank an anonimous referee for suggesting this Corollary. As the referee explains, it is interesting not only for technical reasons but also to understand the economics behind the model.

Proposition 3 *If* $c \in (0.1993, 0.8677)$ *then profits from delegation are larger than profits without delegation. Besides, total firm output, and as a result total output in the incentive equilibrium is always larger than in the absence of delegation.*

In Figure 1 we depict the difference in profits and difference in firm-ouput level under delegation versus absence of delegation, as a function of c for a realization of $\widetilde{\theta}$.

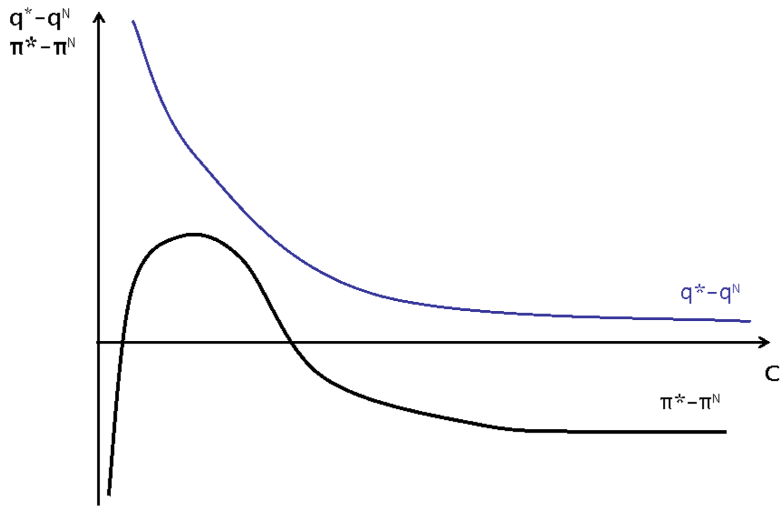

Figure 1.

If $c = 0.4461$ then the positive difference reaches its maximum. This result is in contrast with Fershtman and Judd (1987) for the case of linear costs, since delegation always reduces total profits for the owners when demand is random, for every non-random c.[4]

4 Concluding remarks

In this note, we show how under decreasing returns to scale, there exist a symmetric equilibrium in which delegation of production from owner to manager can be profitable. The result depends on the curvature of the cost function. This is in contrast with Fershtman and Judd (1987) where constant returns to scale are assumed. Therefore, the existence of diseconomies of scale play a significant role for owners when demand is uncertain.

5 References

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[4] They assume cost asymmetry, but when identical cost is assumed results do not change significantly.

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6 Appendix

Proof of Proposition 1: In stage 1, owners choose (α_1, α_2) to maximize expected profits, given $q_1^*(\alpha_1, \alpha_2)$ and $q_2^*(\alpha_1, \alpha_2)$. The first order condition for profit maximization is,

$$\frac{\partial \Pi_1(\alpha_1, \alpha_2)}{\partial \alpha_1} = \frac{c(c\alpha_2 + 1)(2c + c^2\alpha_2 - (2c\alpha_1 + c^2\alpha_1\alpha_2 + 1))}{(2c(\alpha_1 + \alpha_2) + c^2\alpha_1\alpha_2 + 3)^3} = 0$$

The best response function for firm 1 is:

$$\alpha_1(\alpha_2) = \frac{(2c + c^2\alpha_2 - 1)}{(c\alpha_2 + 2)c}$$

where

$$\frac{d\alpha_1}{\partial \alpha_2} = \frac{1}{(c\alpha_2 + 2)^2}$$

We look for symmetric equilibrium, that is $\alpha_1^* = \alpha_2^* = \alpha^*$. In equilibrium, since we have a cubic first order condition, there are three possible solutions:

$$\left\{\alpha^I, \alpha^{II}, \alpha^{III}\right\} = \left\{-\frac{1}{c}, \frac{1}{c^2}\left(-c + \frac{1}{2}c^2 - \frac{1}{2}\sqrt{c^4 + 4c^3}\right), \frac{1}{c^2}\left(-c + \frac{1}{2}c^2 + \frac{1}{2}\sqrt{c^4 + 4c^3}\right)\right\}$$

Properties of the solutions:

1. $\alpha^I = -\frac{1}{c} < 0$.
2. $\alpha^{II} = \frac{1}{c^2}\left(-c + \frac{1}{2}c^2 - \frac{1}{2}\sqrt{c^4 + 4c^3}\right) > 0$ if $c < \frac{1}{2}$.
3. $\alpha^{III} = \frac{1}{c^2}\left(-c + \frac{1}{2}c^2 + \frac{1}{2}\sqrt{c^4 + 4c^3}\right) < 0$ for every $c > 0$.

Since the first order condition is non-linear, we hace to check second order conditions are satisfied for each possible solution.

$$\frac{\partial}{\partial \alpha_1}\frac{\partial \Pi_1(\alpha_1, \alpha_2)}{\partial \alpha_1} = \frac{(4\alpha_1 - 2\alpha_2 - 3c\alpha_2 + 2c\alpha_1\alpha_2 - 6)(c\alpha_2 + 2)(c\alpha_2 + 1)c^3}{(2c\alpha_1 + 2c\alpha_2 + c^2\alpha_1\alpha_2 + 3)^4}$$

Then, we find profits for each of the possible solutions. We analyze the three possible symmetric equilibria:

$\left(\alpha^I, \alpha^I\right)$ The equilibrium output is zero for each firm, then profits are zero as well. The second order condition is not-determined. Therefore, it cannot be a solution.

$\left(\alpha^{II}, \alpha^{II}\right)$ The second order condition has three multiplying terms: (1) $\left(\sqrt{c^3(c+4)} - c^2 - 2c\right) < 0$ for every $c > 0$; (2) $\left(2\sqrt{c^3(c+4)} - c^3 - 4c^2 + c\sqrt{c^3(c+4)}\right)^{-4} > 0$ for every $c > 0$, and (3) $c^3(c+4) - 2c^4 - 8c^3 + 2c\sqrt{c^3(c+4)} + c^2\sqrt{c^3(c+4)} > 0$ for every $c > 0$. Therefore, the second order condition holds. Thus, the equilibrium level of output is,

$$q_1^*\left(\alpha_1^I, \alpha_2^I\right) = q_2^*\left(\alpha_1^I, \alpha_2^I\right) = \frac{2E(\theta)}{\left(4 + c - \sqrt{(4+c)c}\right)}$$

which is positive for every $c \in \mathbb{R}^+$. Thus, the equilibrium payoff-profits for each firm are,

$$\pi_1^*\left(\alpha_1^I, \alpha_2^I\right) = \pi_2^*\left(\alpha_1^I, \alpha_2^I\right) = \frac{\left(\sqrt{(4+c)c} + (c-1)c\right)E\left(\theta^2\right)}{\left(\sqrt{(4+c)c} - (2+c)\right)(4+c)}$$

which are negative since the denominator is always negative for every $c \in \mathbb{R}^+$). Therefore it cannot be a solution.

$\left(\alpha^{III}, \alpha^{III}\right)$ The second order condition has three multiplying terms: (1) $2\left(4c^2 + c^3 + 2\sqrt{c^3(c+4)} + c\sqrt{c^3\right.$ 0 for every $c > 0$, (2) $\left(c^3(c+4) - 2c^4 - 8c^3 - 2c\sqrt{c^3(c+4)} - c^2\sqrt{c^3(c+4)}\right) < 0$ for every $c > 0$, and $\left(2c + c^2 + \sqrt{c^3 (c+4)}\right)\left(c^2 + \sqrt{c^3 (c+4)}\right) c^2 > 0$ for every $c > 0$. Therefore, the second order condition holds. Thus, the equilibrium level of output is,

$$q_1^* \left(\alpha_1^{III}, \alpha_2^{III}\right) = q_2^* \left(\alpha_1^{III}, \alpha_2^{III}\right) = \frac{2E(\theta)}{4 + c + \sqrt{4c + c^2}}$$

and the corresponding level of profits are, for each firm are,

$$\pi_1^* \left(\alpha_1^{III}, \alpha_2^{III}\right) = \pi_2^* \left(\alpha_1^{III}, \alpha_2^{III}\right) = \frac{\left(c + \sqrt{(c+4)\,c} - c^2\right) E\left(\theta^2\right)}{\left(2 + c + c\sqrt{c^2 + 4c}\right)(4 + c)}$$

which is positive for every $c \in (0, 2.5943)$. We call $\alpha_1^{III} = \alpha_2^{III} = \alpha^*$.■

Proof of Proposition 2: The proof follows directly from Proposition 1. We show the magnitude of $\alpha^* = \frac{1}{2} + \frac{1}{c}\left(\frac{1}{2}\sqrt{(c+4)\,c} - 1\right)$. It is holds that if $c < \frac{1}{2}$ then $\alpha^* < 0$ and that $0 \leq \alpha^* < 1$ if $c \geq \frac{1}{2}$. On the one hand, let us find the limit when $c \to \infty$:

$$\lim_{c \to \infty} \frac{1}{2} + \frac{1}{c}\left(\frac{1}{2}\sqrt{(c+4)\,c} - 1\right)$$

this is nothing but $\frac{1}{2} + \lim_{c \to \infty} \frac{1}{c}\left(\frac{1}{2}\sqrt{(c+4)\,c} - 1\right)$, which is one. Thus, there is an upper bound on delegation, it cannot go beyond one. On the other hand, let us find the limit when $c \to 0$:

$$\lim_{c \to 0} \frac{1}{2} + \frac{1}{c}\left(\frac{1}{2}\sqrt{(c+4)\,c} - 1\right)$$

this is nothing but $\frac{1}{2} + \lim_{c \to 0} \frac{1}{c}\left(\frac{1}{2}\sqrt{(c+4)\,c} - 1\right)$, which is not defined and tends to minus infinite.■

Proof of Proposition 3: We substract profits and output with and without delegation to obtain the expression,

$$\begin{aligned}
\pi^*(c) - \pi^N(c) &= \frac{\left(c + \sqrt{(c+4)\,c} - c^2\right)\theta^2}{\left(2 + c + c\sqrt{(c+4)\,c}\right)(4 + c)} - \frac{(c+2)\,\theta^2}{2\,(c+3)^2} \\
q^*(c) - q^N(c) &= \frac{2\theta}{4 + c + \sqrt{4c + c^2}} - \frac{\theta}{c+3}
\end{aligned}$$

Expressions cannot be easily simplified. We simulate results for some feasible realizations of the random shock, $\widetilde{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$ such that $\underline{\theta} > 0$.■