A note on the neutrality of profit taxes and tax compliance with imperfect detection

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Abstract

In a tax-evasion model with profit tax, we reexamine and clarify the issues of neutrality and separability with imperfect detection of tax fraud. With this more realistic setting, we show that the profit tax is not necessarily neutral and the separability conclusion may not hold. Furthermore, the property of non-neutrality may coexist with that of separability or inseparability. However, in contrast to the traditional conclusion, raising the audit probability may reduce the tax compliance when the property of inseparability is present.
1. Introduction

The conventional view that the profit tax has no influence on the monopolist’s production decision is well known. However, much of the literature on profit taxation (e.g., Marrelli 1984, Kreutzer and Lee 1986 and 1988, Wang and Conant 1988, Wang 1990, Yaniv 1995 and 1996, Lee 1998) has incorporated tax evasion into the analysis of tax neutrality and, therefore, the issue of separability between production and evasion decisions has become a focus of research. In most models the neutrality of profit taxes and the separability of decisions are still robust under a fixed audit rate.

However, the possibility of an uncertain audit outcome has been neglected in these models. In the real world, taxpayers often complain about significant changes in the tax law, difficulties in interpreting the existing tax laws (Alm 1988, Beck and Jung 1989, Alm Cronshaw and McKee 1993) and the high cost of tax compliance (Scotchmer and Slemrod 1989). In practice, production costs or sales revenues are difficult to measure with a high degree of precision. In view of uncertain audit outcomes, taxpayers may overstate costs even though the tax-evading monopolist will be audited. The purpose of this note is to investigate non-neutrality and inseparability under a formulation of uncertain detection of tax fraud.

With this more realistic setting, the analysis shows that the profit tax is not necessarily neutral and the separability conclusion may not hold. However, non-neutrality can not imply inseparability. This is different from the view of Lee (1998) who did not distinguish inseparability from non-neutrality. Furthermore, when the evasion and output decisions are inseparable, there exist not only direct but also indirect effects of raising the audit probability on tax compliance. Thus, raising the audit probability may encourage tax evasion if the indirect effect outweighs the direct effect. To the best of our knowledge, there are few studies that are concerned with this possibility.

2. The Model

Following Wang and Conant, consider a monopolistic monopolist facing a proportional tax rate, \( t \) with \( 0 \leq t \leq 1 \). Denote the monopolist’s output level as \( q \) and true total costs as \( C(q) \) which is not known to the tax collector. Suppose that the monopolist can evade a profit tax liability by overstating its production costs by a positive fraction \( \delta \) (referred to as the declaration factor), which are either audited with probability \( p \), or remain unaudited with probability \( 1 - p \). He/she may either be audited with probability \( p \), or unaudited with

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1 Those studies mentioned above have implicitly assumed that the tax authority can perfectly discover the fraud reports when the tax returns are audited.

2 It is worth noting that the possibility that a very high penalty will be counterproductive in deterring crimes has been explored in several papers (Malik 1990, Andreoni 1991, Chang et al. 2000, Ueng and Yang 2006). However, the result that an increase in the audit probability will enhance compliance is rather robust.
probability $1 - p$. When audited, the tax authority does not detect evasion precisely, for the reasons mentioned above. Thus the monopolist, in addition to being audited or not being audited, faces another kind of uncertainty regarding the audit outcome. The authorized costs will become $(1 + x)C(q)$, where $x$ is the difference ratio between the authorized costs and the true production costs. The actual profit of a monopolist is $\pi(q) = R(q) C(q)$, where $R$ denotes total revenues.

Let the authorization factor $x$ be a random variable with distribution $F(x)$ that is continuously distributed throughout the population with $x^L \leq x \leq x^U$. The uncertain (certain) detection is defined by a non-degenerate (degenerate) $F(x)$. The penalty rate $s (> 1)$ is applied as the declared costs exceed the authorized costs. Thus, if tax evasion is not audited, the monopolist’s net profit will be

$$A = (1 - t)\pi - t\delta C. \quad (1)$$

However, if the monopolist is audited, then there exist three cases: $\delta < x^L$, $x^L \leq \delta < x^U$, and $\delta \geq x^U$. Case 1 can be ignored since it is irrational for the evading taxpayer to report a lower production cost. In case 2, the monopolist will pay the tax due plus a fine on the unreported profit if he is assessed as having lower production costs than those declared. However, if the monopolist is assessed as having higher production costs than those declared, he receives a rebate for the overpaid tax, but does not receive a rebate for the reward at the fine rate $s$. In case 3, the evader will always pay the tax plus a fine if he is audited. Therefore, the random audited profit becomes

$$B = A - st(\delta - x)C, \quad (2)$$

where $s > 1$ if $\delta > x$ and $s = 1$ if $\delta = x$. Since we focus on the issues of the neutrality and separability of the profit tax, and case 2 offers no further insight in this respect, we only discuss case 3 where $\delta \geq x^U$ for easy analysis.

Suppose that the monopolist’s preference function is given by a von Neumann-Morgenstern utility function $U(\pi)$ with $U'(\pi) > 0$ and $U''(\pi) < 0$, which implies that the monopolist is risk-averse. The monopolist’s problem is to choose $q$ and $\delta$ to maximize its expected utility,

$$\text{Max}_{\delta, q} E[U(\pi)] = (1 - p)U(A) + p\int_{x^L}^{x^U} U(B)dF(x) + (1 - p)U(A) - pE[U(B)]. \quad (3)$$

The first-order conditions for an interior maximum of expected utility are

$$(1 - p)A_q U'(A) + pE[B_q U'(B)] = 0 \quad (4)$$

and

$$(1 - p)U'(A) - p(s - 1)E[U'(B)] = 0, \quad (5)$$

where the subscript indicates a partial derivative. Equation (4) represents the optimal level of production. Another characterization of the optimal evading condition is obtained by

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3 Our model will reduce to Wang and Conant (1988) if $x^L = x^U \neq 0$. 
rewriting (5) as \( U'(A)/E[U'(B)] = p(s-1)/(1-p) \) and demonstrates that the optimal internal rate of overstated costs requires that the marginal rate of substitution (between unaudited profit and expected profit detected) be equal to the real price of evasion.

To more easily demonstrate the focus and the comparative statics of this paper in what follows, we substitute (5) into (4) to reveal that at the optimum

\[
(1-t)\pi'E[U'(B)] + tC'E[U'(B)x] = 0.
\]

Furthermore, to provide important insights into the issues of the neutrality of profit taxes and the separability of decisions in the tax-evasion model, we re-write Equation (6) as

\[
\pi' = \frac{tC'E[U'(B)x]}{(1-t)E[U'(B)]}.
\]

Obviously, the profit tax is non-neutral \( \pi' \neq 0 \) if \( E[U'(B)x] \neq 0 \), and the separability of decisions is not preserved if the RHS of (7) depends on \( \delta \). In general, these two conditions hold, and hence we have the following proposition:

**Proposition 1:** With uncertain detection the profit tax can affect the profit-maximizing output, and the monopolist’s output and tax evasion decisions are inseparable; that is, the neutrality and separability results can not be preserved in general.

As we argued in the Introduction, this is quite different from the conventional results indicated in the previous literature (e.g., Wang and Conant 1988, Yaniv 1995 and 1996). It is worthwhile mentioning that Lee (1998) obtained similar results to these by formulating an endogenous audit rate rather than an uncertain audit outcome. Note that if \( E[U'(B)x]/E[U'(B)] \) is a function of \( \delta \), then \( E[U'(B)x] \neq 0 \); that is, inseparability implies non-neutrality. However, the reverse is not true. To see this, we know that \( E[U'(B)x] \neq 0 \) (i.e., \( \pi' \neq 0 \)) may not imply that \( E[U'(B)x]/E[U'(B)] \) depends on \( \delta \). For example, if \( F(x) \) degenerates to a non-zero constant \( x \), then \( E[U'(B)x] = xE[U'(B)] \). Also, \( E[U'(B)x]/E[U'(B)] = x \), which is independent of \( \delta \). That is, non-neutrality may not imply inseparability. This result does not arise in Lee’s model (1998) and has been neglected in the literature.  

3. **Raising the Audit Probability may Reduce Compliance**

Now we turn to investigate the comparative statics of how raising the audit probability affects tax compliance. Totally differentiating equations (5) and (6) given the penalty and tax rate

\[\pi' = \frac{tC'E[U'(B)x]}{(1-t)E[U'(B)]}.\]
leads to the following equations:

\[ J^1 dq + J^1 d\delta = J' dp \] (8)

and

\[ J^2 d \neq J^2 d\delta = 0 \] (9)

where

\[
\begin{align*}
J^1_q &= (1 - p)A_q U^*(A)  \\
J^1_p &= tC(1 - p)U^*(A)  \\
J^2_q &= E \left[ (1 - p)\pi' \triangle C_{x}\right] B_q U^*(B)  \\
J^2_p &= E \left[ (1 - p)\pi' \triangle C_{x}\right] B_p U^*(B)  \\
J^1_p &= U'(A) (s + 1)E[U'(B)] \quad 0 \geq \\
\end{align*}
\]

By Cramer’s rule, we obtain the comparative statics

\[
\frac{\partial \delta}{\partial p} = \frac{-J^1_p J^2_q}{J^1_q J^2_p - J^1_p J^2_q}.
\] (11)

It is worth noticing that the monopolist’s output and evasion decision are separable with certain detection outcomes. In this case, \( J^2_q \) equals zero via condition (6), and we obtain

\[
\frac{\partial \delta}{\partial p} = \frac{J^1_p}{J^1_q} \quad 0
\]

which is consistent with the conventional result. However, if the monopolist’s output and evasion decisions are inseparable, then \( J^2_q \) may not equal zero. This may make the sign of \( \frac{\partial \delta}{\partial p} \) in equation (11) ambiguous. To demonstrate the underlying economic intuition, we substitute \( dq = (\ J^2_q / J^1_q \)d\delta \) from (9) into (8) to decompose these effects of raising the audit rate, which leads to

\[
[\ J^1_q / J^1_p - J^1_q J^2_q / (J^1_p J^2_q) \]d\delta = dp.
\] (12)

The first term in the brackets on the LHS of (12) represents the direct effect of changing the audit rate on tax evasion, while the second term denotes the indirect effect (via changing the level of output) due to inseparability. When these two effects have opposite impacts on \( \delta \), and the indirect effect outweighs the direct effect, increasing the audit probability may reduce tax compliance.

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5 The comparative statics of \( \frac{\partial \delta}{\partial p} \) with (4) and (5) is equivalent to that with (5) and (6). However, it is much easier to show the focus of this note using the latter. The fuller derivation can be obtained via an email (klueng@nccu.edu.tw) to Glen Ueng.
Proposition 2: Under uncertain detection with inseparability, raising the audit probability may encourage rather than discourage tax evasion.

4. Conclusions

It is conventionally believed that profit taxes are neutral, and the monopolist’s output and tax evasion decisions are separable under a fixed audit probability. However, re-examining neutrality and inseparability by the formulation of uncertain audit outcomes leads to different results. First, with uncertain detection, the neutrality and separability results cannot be preserved in general. Second, non-neutrality and inseparability are not equivalent since inseparability implies non-neutrality but not vice versa. This result cannot be obtained in Lee (1998) and has been neglected in the literature. Finally, under an uncertain detection outcome with inseparability, raising the audit probability may encourage rather than discourage tax evasion. Therefore, as for the policy implications, controlling assessment outcomes may be significantly relevant for tax compliance purposes.

References


