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A note on quantity precommitment, cournot outcome and asymmetric capacity costs

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Abstract

This paper extends Kreps and Scheinkman's 1983 result, which shows that a production capacity choice stage followed by price competition yields the same outcome as a Cournot game, to a setting where capacity costs are asymmetric.

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1. Introduction

Bertrand and Cournot-type models are standard tools in industrial organization, and usually yield very different results and policy recommendations. In an important paper, Kreps and Scheinkman (1983) (hereafter referred to as KS) show that the two models are related: for symmetric firms, a game with a first stage in which firms choose production capacities, followed by a second Bertrand competition stage with an efficient rationing rule, yields the same outcome as the corresponding Cournot game.

However, this result is quite sensitive to the assumptions of the model. For example, as shown by Davidson and Deneckere (1986) and Madden (1998), it can only be extended to more general rationing rules if capacity costs are high enough.

More problematic, Deneckere and Kovenock (1996) show that the KS result no longer holds if costs are sufficiently asymmetric in the price competition stage (from now on, we will call distribution costs - as distinct from capacity costs - those costs that only depend on the quantity sold). In particular, in a KS setting with a suitably efficient firm, Cournot models tend to underestimate the efficient firm's incentives to choose a capacity above its Cournot level (in order to price its competitor out of the market in the following price competition subgame).

One could then think that Cournot models should be dropped whenever cost asymmetry is an important issue. Deneckere and Kovenock (1996) and Allen et al. (2000) present a way to deal with this problem and indeed find results which differ from models using Cournot-type subgames when the distribution costs are sufficiently asymmetric. However, their method is quite sophisticated and could not be incorporated in most models without making them intractable. Moreover, the KS result seems to hold when the distribution costs are not too asymmetric, or when the capacity costs are high enough. Consequently, it is important to understand under which conditions the KS conclusions are valid.

In this note, we extend KS's main result by showing that the unique perfect equilibrium of a KS game with asymmetric capacity costs (and null distribution costs) yields the corresponding Cournot outcome. KS show their result for any equilibrium, not just perfect equilibria, but they provide a proof which can not be directly extended to asymmetric capacity costs (in particular Steps 2 and 3 of their proof, page 336). Restricting our attention to perfect equilibria allows us to use their description of the price competition subgame's equilibrium to develop a specific proof holding in this case.[1]

2. The model

In this paper, we use the same assumptions (and notation) as KS, but allow for capacity cost asymmetry. Two firms $E1$ and $E2$ simultaneously select their production capacity x_1 and x_2 in a first stage, and then compete in price in a market for a homogenous product, in a second stage. The consumers' demand function is D and the inverse demand function is P. This function P is strictly positive, twice continuously differentiable, strictly decreasing and concave on its support, which is taken to be a bounded interval $[0, X)$.

At the price competition stage, firms sell quantities according to the efficient rationing rule. This amounts to assuming that the consumers with the highest willingness to pay will first buy from the cheaper firm. Then, if this firm cannot produce up to the quantity consumers are willing to buy at its price, the residual demand goes to the other firm. Moreover, we assume that capacity is costly. Firm i incurs a cost $b_i(x_i)$ to set up a capacity x_i. Each cost function b_i is assumed to be convex, twice-continuously differentiable, and such that $P(0) > b_i(0) > 0$.

The assumptions on P and b_i ensure that a Cournot game with inverse demand P and costs b_i (resp. null costs) has a unique equilibrium that we will denote (x_1^*, x_2^*) (resp. (x^*, x^*)). Let $r_{b_i}(.)$ (resp. $r(.)$) denote the Cournot best-response function for a firm with cost b_i (resp. null costs). The function R, defined by $R(q) \equiv r(q)P(q + r(q))$, is the profit a firm with null costs obtains in the Cournot game if it reacts optimally to the other firm's choice of quantity q.

3. The result

The only difference with KS's model is that the capacity cost functions need not be the same for both firms. Consequently, we can use KS's description of the subgame equilibrium of the price-setting stage as a function of the production capacities chosen in the first stage:

Lemma 1 *(Kreps-Scheinkman) There is a unique Nash equilibrium (possibly in mixed strategies) of the price competition subgame, and:*

[1] Note that in the particular case of constant marginal capacity costs, and under slightly different assumptions, our result could also be deduced from Allen et al (2000).

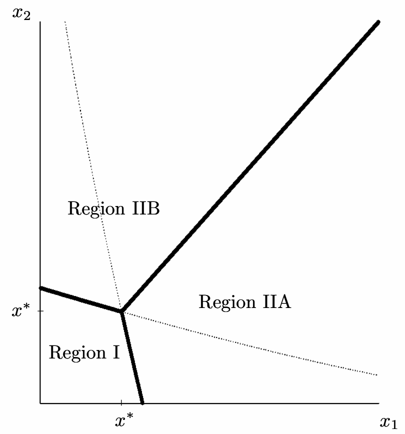

Figure 1:

- *In Region I (where $x_1 \leq r(x_2)$ and $x_2 \leq r(x_1)$), both firms choose the same price $P(x_1 + x_2)$ at the equilibrium, and firm i's profit equals*

$$\Pi_i^p(x_1, x_2) = x_i P(x_1 + x_2) - b_i(x_i).$$

- *In Region IIA (where $x_1 > r(x_2)$ and $x_2 \leq x_1$), the profits of the firms equal*

$$\Pi_1^p(x_1, x_2) = R(x_2) - b_1(x_1) \text{ and } \Pi_2^p(x_1, x_2) = \underline{p} x_2 - b_2(x_2),$$

where $\underline{p}$ is the smallest solution of $\underline{p} = \frac{R(x_2)}{\min\left[x_1, D(\underline{p})\right]}$.

- *In Region IIB (where $x_2 > r(x_1)$ and $x_1 \leq x_2$), the profits of the firms equal*

$$\Pi_2^p(x_1, x_2) = R(x_1) - b_2(x_2) \text{ and } \Pi_1^p(x_1, x_2) = \underline{p} x_1 - b_1(x_1),$$

where $\underline{p}$ is the smallest solution of $\underline{p} = \frac{R(x_1)}{\min\left[x_2, D(\underline{p})\right]}$.

First, note that (x_1^*, x_2^*), the equilibrium of the Cournot game with costs b_1 and b_2, belongs to Region I. Indeed, it is clear that the Cournot best-response functions with costs b_1 and b_2, $r_{b_1}(.)$ and $r_{b_2}(.)$, are such that $r(x) \geq r_{b_i}(x)$, for any x.

Interestingly, if the firms' capacities (x_1, x_2) belong to I, the firms' profits are the same as Cournot profits (considering capacities instead of quantities), and firms thus have incentives to behave locally as in a Cournot model. Thus, if there is a perfect equilibrium with capacities in Region I, then these capacities must equal (x_1^*, x_2^*).[2]

If the firms' capacities (x_1, x_2) belong to Region IIB, the firm with the highest capacity, firm 2, receives exactly the same expected revenue $R(x_1)$ whatever its capacity x_2. Thus, the interior of the Region IIB can be thought of as an area where firm 2 overinvests in capacity that does not increase its revenue. As capacity is costly, it is clear that firm 2 has no incentive to choose a capacity inside Region IIB. The same is true for firm 1 in Region IIA.

This proves that if we have a perfect equilibrium, it must either involve capacities (x_1^*, x_2^*) , or capacities on the frontier between Region IIA and IIB (included in the line $x_2 = x_1$). In fact, it is easy to rule out the second option:

Lemma 2 *Any perfect equilibrium of the game involves capacities* (x_1^*, x_2^*) .
Proof. *Consider that firm 1 has chosen a given capacity* $\widetilde{x_1} > x^*$. *If firm 2 chooses a capacity* $x_2 = \widetilde{x_1}$, *then its profit* $R(\widetilde{x_1}) - b_2(x_2)$ *is equal to* $\frac{x_2}{\widetilde{x_1}} R(x_2) - b_2(x_2)$. *Now, if firm 2 decides to choose a slightly lower capacity* $x_2' = \widetilde{x_1} - \varepsilon$, *so that* $(\widetilde{x_1}, x_2')$ *belongs to Region IIA but still verifies* $x_2' > x^*$ *(which is the case at least locally for small* ε*), then firm 2 receives a profit equal to* $\underline{p} x_2' - b_2(x_2')$, *which is higher than* $\frac{x_2'}{\widetilde{x_1}} R(x_2') - b_2(x_2')$.[3] *KS have shown (page 332) that the function* $xR(x)$ *is strictly decreasing whenever* $r(x) < x$, *which ensures that* $\frac{x_2'}{\widetilde{x_1}} R(x_2') - b_2(x_2') > \frac{x_2}{\widetilde{x_1}} R(x_2) - b_2(x_2)$.[4]
Thus, there can no perfect equilibrium with capacities on the frontier between Region IIA and IIB. ■

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[2] Indeed, under our assumptions the function $x_i P(x_1 + x_2) - b_i(x_i)$ is concave in x_i. This implies that for a given capacity of firm j, and as long as (x_1, x_2) belongs to Region I, firm i's profit is increasing when x_i gets closer to $r_{b_i}(x_j)$. So, if $(x_1, x_2) \neq (x_1^*, x_2^*)$, at least one of the firms can improve its profit by choosing another capacity in Region I.

[3] Note that $\underline{p} = \frac{R(x_2)}{\min[x_1, D(\underline{p})]}$ implies $x_1 \geq \min[x_1, D(\underline{p})] = \frac{R(x_2)}{\underline{p}}$, or equivalently $\underline{p} \geq \frac{R(x_2)}{x_1}$.

[4] Since r is strictly decreasing, $x_2 > x^*$ implies that $r(x_2) < r(x^*) = x^* < x_2$.

When Cournot capacities (x_1^*, x_2^*) belong to the third quadrant relative to (x^*, x^*), as is the case when capacity costs are symmetric, the existence of a unique perfect equilibrium is obvious. Considering firm 2's situation for example, as $x_1^* \leq x^*$, possible deviations from (x_1^*, x_2^*) either remain in Region I, where the firms have incentives to behave as Cournot competitors, or lead to Region IIB, in which we have seen above that firm 2 had no interest to choose its capacity.

The reason why our result is not obvious for asymmetric capacity costs is that, when $x_1^* > x^*$, if firm 1 chooses capacity x_1^*, then according to firm 2's choice of x_2, capacities (x_1^*, x_2) may lie in Regions I or IIB as before, but also in Region IIA (see Figure 2 below). In this region, it is not clear, a priori, whether firm 2's profit is lower than with $x_2 = x_2^*$ or not. Proposition 3 shows that this is the case and thus proves our result.

Proposition 3 *There is a unique perfect equilibrium in this game. In the equilibrium path, firms choose production capacities equal to standard Cournot quantities* (x_1^*, x_2^*) *,then post a price equal to* $P(x_1^* + x_2^*)$*, and receive the Cournot profits.*

Proof. *Suppose that* $x_1^* > x^*$ *and that firm* 1 *chooses a capacity* x_1^**.*

As $x_1^* \leq r(x_2^*) \leq r(0)$*, whatever capacity* x_2 *firm* 2 *chooses in Region IIA, firm* 2*'s profit will be equal to* $\Pi_2^p(x_1^*, x_2) = \underline{p}x_2 - b_2(x_2) = \frac{x_2 R(x_2)}{x_1^*} - b_2(x_2)$*. Indeed, there exists a positive number L such that the function* $f(p) \equiv p \min[x_1^*, D(p)]$ *increases on* $[0, L]$*, decreases on* $[L, P(0)]$ *and equals* px_1^* *on* $[0, L]$*, so that the smallest solution of* $\underline{p} \min\left[x_1^*, D(\underline{p})\right] = R(x_2)$ *is the solution of* $\underline{p}x_1^* = R(x_2)$*.*[5]

Deriving firm 2*'s profit on Region IIA with respect to* x_2 *gives*

$$\begin{aligned} \frac{\partial \Pi_2^p}{\partial x_2}(x_1^*, x_2) &= \frac{R(x_2) + x_2 R'(x_2) - x_1^* b_2'(x_2)}{x_1^*} \\ &\leq \frac{R(x_2) + x_2 R'(x_2) - r(x_2) b_2'(x_2)}{x_1^*} \quad \text{as } x_1^* \geq r(x_2). \end{aligned}$$

Since $R'(x_2) = r(x_2)P'(x_2 + r(x_2)) + r'(x_2) * \underbrace{[P(x_2 + r(x_2)) + r(x_2)P'(x_2 + r(x_2))]}_{=0}$*,*

[5]The demand for the price $P(r(0))$, which maximizes $pD(p)$, is equal to $r(0)$, and this demand $r(0)$ is higher than x_1^*. Consequently, $f(p)$ is first increasing and equal to px_1^*, then decreasing and equal to $pD(p)$.

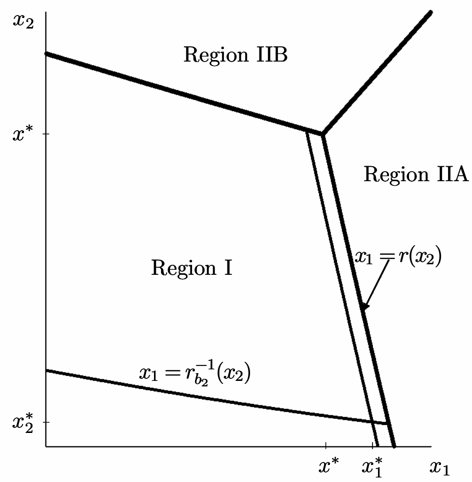

Figure 2:

we then have

$$\begin{aligned}\frac{\partial \Pi_2^p}{\partial x_2}(x_1^*, x_2) &\leq \frac{R(x_2) + x_2 r(x_2) P'(x_2 + r(x_2)) - r(x_2) b_2'(x_2)}{x_1^*} \\ &\leq \frac{r(x_2)(P(x_2 + r(x_2)) + x_2 P'(x_2 + r(x_2)) - b_2'(x_2))}{x_1^*}. \quad (1)\end{aligned}$$

As the derivative of firm 2's Cournot profit with respect to x_2, $P(x_2 + x_1) + x_2 P'(x_2 + x_1) - b_2'(x_2)$, *is non-positive when* $x_2 \geq r_{b_2}(x_1)$, *and as* $x_2 \geq r_{b_2}(r(x_2))$, *then inequality (1) implies that*[6]

$$\frac{\partial \Pi_2^p}{\partial x_2}(x_1^*, x_2) \leq 0.$$

Consequently, as firm 2's profit function is continuous on its capacity x_2, *its best response to* x_1^* *is in Region I and is therefore* x_2^*.

[6]Indeed, if $x_2 \geq x_2^*$, then $r_{b_2}^{-1}(x_2) \leq r(x_2)$, as the corresponding part of the "long-run" Cournot curve $x_1 = r_{b_2}^{-1}(x_2)$ lies in Region I. This proves that $x_2 \geq r_{b_2}(r(x_2))$, since r_{b_2} is decreasing.

A symmetric argument for firm 1 concludes the proof. ■

4. Concluding Remarks

This proposition extends the number of situations in which a Cournot model may be seen as appropriate. As shown by Deneckere and Kovenock (1996), KS's result does not hold when distribution costs are asymmetric enough, since when an inefficient firm chooses its Cournot capacity, the efficient firm may have an incentive to choose a capacity above its Cournot capacity and then flood the market in the price competition stage. This is no longer a problem when distribution costs are symmetric at the price competition stage and capacity costs are asymmetric.

One of the main features reported to support the use of Cournot for industries with symmetric firms is that the prices are easy to adjust as compared to the production capacities. Very-easily-adjusted prices usually means that most of the costs are sunk at the price-setting stage, so that the remaining costs are fairly symmetric at this point. Consequently, this note conveys a similar message for industries with asymmetric firms.

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