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Venture Capital Contracting with Renegotiation

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Abstract

This paper examines contracting between a venture capitalist and an entrepreneur in a setting with unobservable effort when contracts are renegotiated each period. The contribution of our paper lies in the insights it provides on optimal contracts in this setting. The insights from our model prove to be significantly different in certain respects than those obtained under a multi-period contract without renegotiation or a single period setting. An example is worked out to illustrate the division of payoff between the venture capitalist and the entrepreneur each period.

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1. Introduction

> Venture capital firms take chances on financing new companies that traditional sources of money wouldn't and shouldn't touch. In return for taking that extraordinary risk, the theory goes, VCs stand to gain extraordinary returns by owning shares of companies that increase in value much faster than the average publicly held company. [Stewart Alsop *Fortune*, April 15, 2002, page76]

Venture capital funding for startups, and in turn, research on it, has grown in importance in the last two decades. The focus of this paper is the analysis of optimal contracts between the Venture Capitalist (VC), who provides funding, and the entrepreneur, who has the knowledge. The paper implements dynamic optimization techniques to analyze the nature of the relationship between the VC and entrepreneur in a startup company where the VC and the entrepreneur renegotiate the contract each period. As such, as far as we know, our setting is unique in the VC literature. We analyze the trajectory of optimal contracts over time and examine the entrepreneur's trajectory of optimal effort over time.

The contribution of this paper lies in the insights it provides on optimal contracts in this setting, which prove to be significantly different in certain respects than those obtained under a multi-period contract without renegotiation or a single period setting. Just how important is the issue of contracting between the VCs and entrepreneurs? A recent article in *Business Week*, has described contracting between VCs and entrepreneurs:

> To improve their chances of a payoff, they're putting the screws to entrepreneurs, their teams, and other VCs. Talks have become so long and tortuous that the legal costs for venture financing have doubled. ``There is a frenzy of extraordinarily draconian terms going on right now," says Craig Johnson, a lawyer at Venture Law Group who represents startups. [Linda Himelstein, *Business Week*, May 27, 2002, Iss. 3784, page 82]

Our analysis suggests that entrepreneurs who perceive the overall rewards for success to be high will work harder than in the case where rewards to success are low. It also implies that the VC should not reward failure at any stage and that the guiding principle should be "Pay For Performance", which we detail. Another such result is that the payment to the entrepreneur should be reduced when the probability of success, relative to the effect on entrepreneurs' effort of is high. In addition, our study shows that the payoff to the VC from current period and future periods should be such that the rate of improvement in chances of success, resulting from added effort from the entrepreneur, should equal the rate of increase in the VC's payoff.

2. Background and Literature Review

Entrepreneurs of firms typically come up with the idea and the product. The financing raised from VCs is typically used for continuing the development of the product and marketing it effectively. VCs have helped many well-known companies in their beginning. Examples of firms that VCs have financed in early stages include Microsoft, Intel, Lotus, Apple, Staples, TCBY, and Federal Express (for an excellent discussion on how the VC industry operates and its history please see Gompers and Lerner (1999) and Kaplan and Stromberg (2002)). VCs, such as Tommy Davis, Eugene Kleiner, Tom Perkins and Arthur Rock, have become legends in the high-tech industry for their part in the creation of the computer industry as we know it today.

Why do VC firms exist? Can't entrepreneurs just go and get a loan from the bank? Amit et. al [1998] argue that VCs exist because of their ability to reduce informational asymmetries. Specifically, banks and other institutional lenders, in contrast with VCs, are unable to distinguish between high and low quality entrepreneurs. As such, VCs are in essence financial intermediaries who thrive because of their superior ability to screen and monitor entrepreneurs.

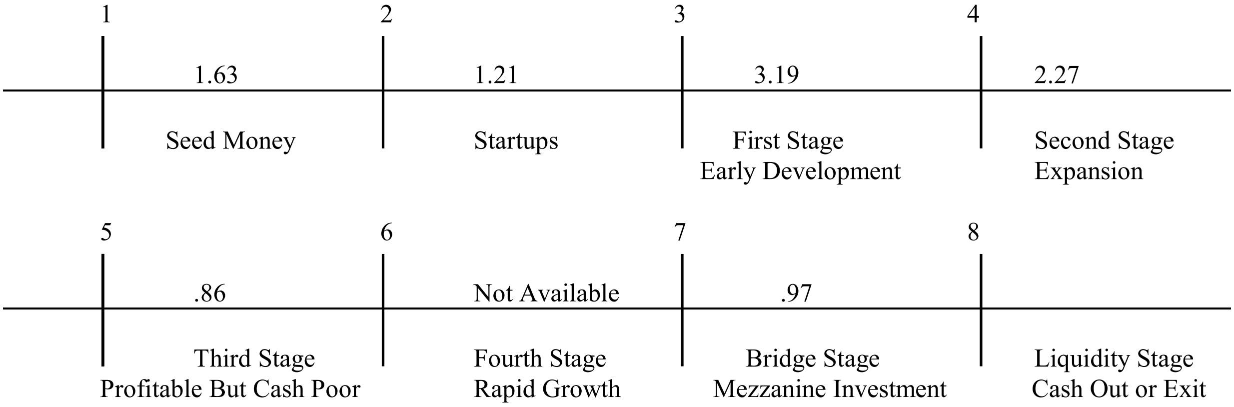
How do VCs make money?

> Venture firms make money in two ways. They get a share of a fund's profits (the "carry"), and they also earn a percentage of the capital committed in management fees, regardless of whether the capital is actually invested. [*The Economist* , April13, 2002, pg. 75]

Virtually all of the studies in the area view venture capital as a short-term source of funding. VCs aim to exit the firm once it reaches sufficient size and credibility so it can be sold to the public through an IPO or to another company (see, for example MacIntosh [1997] Tables 12 and 13, and Amit et. al [1998] figure 5).

Financing from VCs comes usually in several rounds. Amit et. al [1997] discuss the various funding rounds that companies go through with VCs prior to the cash out stage (see figure 3 in Amit et. al [1997]). Based on Gompers (1995) and Sahlman (1990) the various stages of VC investing can be characterized as follows:

Figure 1 - Stages of Investment by Venture Capitalists



Note: Numbers between stages denote time in years.

Several studies examine the asymmetric information aspects of the VC and entrepreneur relationship and their effect on the optimal form of financing (examples include Chan (1983), Admati and Pfleiderer (1994), Barry (1994), Bergemann and Hege (1998), Trester (1998), Neher (1999), Elitzur and Gavious (2003b), and Wang and Zhou(2004)). Some of these papers focus on the mechanism of staged financing (e.g., Neher (1999)). Others investigate what form the financing instrument should take and ask, for instance, whether it should it be provided in the form of debt, equity, or a hybrid instrument (examples of such studies include Bergemann and Hege (1998) and Elitzur and Gavious (2003b)).

Our study examines repeated contracts between the VC and the entrepreneur of the company where the effort of the entrepreneur is unobservable by the VC. Furthermore, this study is, as far as we know, the first one to investigate the VC and entrepreneur relationship with repeated contracts that are renegotiated each period between the parties rather than a single long-term contract or a single period contract.[1] The contribution of our paper lies in the insights it provides on optimal contracts between the two players when such contracts are renegotiated at the beginning of each period. The multi-period model with renegotiated contracts is consistent with reality because, as the above stages described in Sahlman (1990) and Gompers (1995) show, the relationship between VCs and entrepreneurs develops over several stages and several years and that VCs do not commit themselves to a single multi-period contract.

An aspect of the relationship between VCs and entrepreneurs that we do not investigate in this study is the adverse selection problem. The problem that we examine here deals mainly with the moral hazard aspect of the relationship that, in essence, starts after the adverse selection problem has been resolved. Sahlman (1990) provides a description of the mechanism that VCs employ to deal with the adverse selection problem. Another aspect of the relationship we do not investigate here is the syndication of investments by several VCs (Lerner (1994) provides an insightful discussion of this topic). We also do not investigate the role of 'angel' investors in financing of startups (Elitzur and Gavious (2003a) provide a formal analysis of this relationship).

3. The Model

Our setting involves a startup company with two risk neutral players: an entrepreneur and a VC.[2] It is assumed that the contract between the two players cannot be canceled in the middle of a period. The game has up to K stages where K is common knowledge. In every stage, k, of the game the entrepreneur chooses optimal effort level, $e_k \geq 0$. The VC cannot observe or infer e_k but only whether the stage was a success or a failure. e_k, in turn, affects the probability of success in this stage, $\alpha_k(e_k)$. The following assumptions are made with respect to α_k:

$$1 \geq \alpha_k(e_k) \geq 0, \quad \alpha_k'(e_k) > 0, \quad \alpha_k''(e_k) < 0, \quad k = 1,...,K$$

The above assumptions state that the probability of success (1) is between 0 and 1, (2) increases in effort, and (3) has diminishing marginal returns to effort.

Interest is held at zero to increase the mathematical tractability. The assumption of zero interest is made because incorporating a positive interest rate would make the model more cumbersome and, as we found out, does not really add any insights as we have here a finite horizon game.

If the outcome of stage k is a success the payoff of the VC changes by B_k and both players continue to the next period. B_k is, in essence, the expected change in the value of the VC's holdings in the firm. The VC cannot infer the entrepreneur's effort, e_k, since it is unobservable by him and he cannot infer the effort from the fact that the firm fail or succeed in the kth stage. B_k can assume in our model any value and, thus, will be negative or positive. In each stage the VC invests $I_k \geq 0$, I_k is common knowledge constant. The entrepreneur is awarded $S_k \geq 0$. Another way to look at this is that until period K the award to the

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[1] One could argue that the approach to use to describe this relationship is incomplete contracts (for an excellent discussion of the topic please see Aghion and Bolton (1992), Hart (1995) and Maskin (2002)) where the concept of renegotiated contracts is also used. One of the prerequisites of incomplete contracts is to have a parameter which is observable but non-verifiable. In the case of VCs and entrepreneurs, however, the parameter (entrepreneur's effort) is unobservable and thus the approach of incomplete contracts is ruled out in our case.

[2] The assumption of risk neutrality is quite common for papers on this topic (see, for example, Trester (1998) and Wang and Zhou (2004)).

entrepreneur, S_k, and the residual claim of the VC, $\mathrm{B_k} - S_k$, reflect the share in the company that the entrepreneur and the VC respectively get each period. Note that S_k is generic and, thus, we do not impose any structure on this sharing rule on purpose because we do not want to limit the contract to any specific form of financing instrument.[3] Furthermore, we are not focusing in this paper on the financing instrument that should be used as there are other studies who deal with this question (see, for example, Bergemann and Hege (1998) and Elitzur and Gavious (2003b)).

If, on the other hand, the outcome is a failure the company earns 0, the entrepreneur‘s reward is $F_k \geq 0$ and the game terminates.[4] We assume that the VC demands that his overall payoff, $\mathrm{B_k} + \mathrm{V_{k+1}}$, should exceed the payment to the entrepreneur, S_k, otherwise the VC will have no incentive to remain in this firm. B_k is assumed to be common knowledge. If the outcome in each stage is a success for the company the game ends at stage K with a cash-out to the VC who exits the firm.[5] The entrepreneur incurs a cost of effort $C(e_k) \geq 0$ at each stage k=1,..,K.[6] It is assumed that $C'(e_k) > 0$ and $C''(e_k) > 0$. We assume that both functions $\alpha_k(.)$ for k=1,...,K, and C(.) are common knowledge.[7] The time line each period is depicted in the Figure 2:

The expected payoff to the VC in each stage k, V_k, follows the recursive equation below:[8]

$$V_k = \alpha_k(e_k)\left[B_k - S_k + V_{k+1}\right] - \left[1 - \alpha_k(e_k)\right]F_k - I_k \qquad \mathrm{k} = 1,\ldots,\mathrm{K} \qquad \textbf{(1)}$$

The expected payoff to the entrepreneur in each stage k, U_k, follows the recursive equation below:[9]

$$U_k = \alpha_k(e_k)\left[S_k + U_{k+1}\right] + \left[1 - \alpha_k(e_k)\right]F_k - C(e_k) \qquad \mathrm{k} = 1,\ldots,\mathrm{K} \qquad \textbf{(2)}$$

Define $V_{K+1} = 0$ and $U_{K+1} = 0$.

We use backward induction to look for the sub game perfect Nash equilibrium in this model.

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[3] As opposed to this study, in most finance models, the entrepreneur has all the bargaining power (captures all surplus), not the VC.

[4] From our assumptions it follows that the entrepreneur cannot be dismissed until the end of the period and only if it is a failure.

[5] For characterization of this exit point see Elitzur and Gavious (2003b).

[6] Alternatively, we could redefine the notations in the model as follows:

$$c_k = C(e_k), \quad \mathrm{k} = 1,2,\ldots,\mathrm{K}, \quad \tilde{\alpha}_k(c_k) = \alpha_k(C^{-1}(c_k)), \quad \mathrm{k} = 1,2,\ldots,\mathrm{K}$$

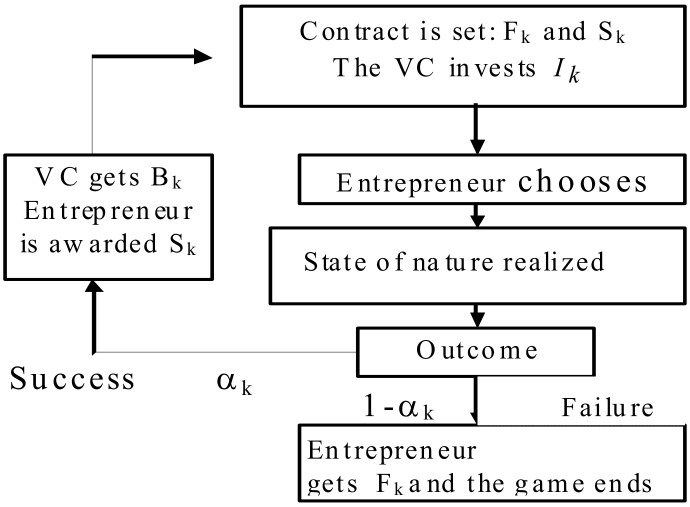
where C^{-1} is the inverse function of C. This would streamline the results but, unfortunately, some of the model's results would seem to be less intuitive.

[7] Note that we assume that the functions of α and C are known but not their realized values because they depend on the entrepreneur's effort, which cannot be observed by the VC.

[8] V_k is a function of S_k and F_k. Thus, for the sake of simplicity, we omit these variables and write V_k instead of $V_k(S_k, F_k)$.

[9] $U_k = \alpha_k(e_k)\left[S_k + U_{k+1}\right] + \left[1 - \alpha_k(e_k)\right]F_k - C(e_k)$ U_k is a function of e_k, thus, for the sake of simplicity, we omit this variable and write U_k instead of $U_k(e_k)$.

FIGURE 2 - THE TIME LINE EACH PERIOD



4. Results

In this section we analyze the model and obtain results. We start by examining the optimal effort of the entrepreneur over time. Then we analyze the characteristics of the trajectory of the optimal compensation scheme over time. Next, the optimal payment to the entrepreneur for failure is obtained. The last part of the section defines the optimal payment, S_k, over time.

The following Proposition examines the optimal effort of the entrepreneur over time:

Proposition 1: *The optimal effort of the entrepreneur in each period, e_k^*, k=1, ..., K, has the following structure:*

$$e_k^* = e_k^*(S_k - F_k + U_{k+1}), \quad \text{k} = 1,...,\text{K} \qquad \textbf{(3)}$$

Proof: Relegated to Appendix 1

In essence, Proposition 1 states that e_k^* is a function of the difference between the benefit from success, $S_k + U_{k+1}$, and the reward for failure, F_k. The Proposition is intuitively appealing because it shows that the entrepreneur is going to exert effort depending on the rewards for either success or failure and the future prospects. Consequently, entrepreneurs who perceive the overall rewards for success to be high will work harder than in the case where rewards to success are low. As such, one should expect entrepreneur to exert high levels of effort where the capital market is bullish on their industry, say, like during the Internet bubble. Furthermore, this may explain the fact that VCs invest in industries that are in high growth and not very competitive where rewards to success are substantial.

> One myth is that venture capitalists invest in good people and good ideas. The reality is that they invest in good industries- - that is, industries that are more competitively forgiving than the market as a whole. [Zider, 1998, page 133].

Proposition 1 may shed some light on why VCs moved from the energy industry in the early eighties to genetic engineering, specialty retailing and computer hardware, then to CD-ROMS, multimedia, telecommunications, and software, then they flocked to invest in Internet companies and, now they are moving to Biotech companies. The pattern that Zider [1998] describes is that industries that VCs invest are 'sexy' at the time of investment. 'Sexy' industries are those that provide substantial rewards to the entrepreneur for success, and, in turn, elicit a high effort level from the entrepreneur.

The VC chooses the optimal trajectory of compensation to the entrepreneur (in essence, the share of the company awarded to the entrepreneur), taking into account, the self-interested choice of effort by the entrepreneur. This scheme will be set according to the following Proposition:

Proposition 2: *The optimal trajectory of compensation to the entrepreneur over time satisfies the following condition:*

$$\boldsymbol{\alpha'_k (e_k)\left[S_k - F_k + U_{k+1}\right] = C'(e_k)} \tag{4}$$

Proof: Relegated to Appendix 1

Proposition (2) implies that the optimal compensation awarded each period by the VC to the entrepreneur, anticipating effort selection by the entrepreneur, is at the point where the marginal expected benefit from effort to the entrepreneur, $\alpha'_k (e_k)\left[S_k - F_k + U_{k+1}\right]$ is equal to the entrepreneur's marginal cost of effort, $C'(e_k)$. The following Proposition characterizes the payment to the entrepreneur for failure:

Proposition 3: *Not rewarding the entrepreneur for failure at each stage is an optimal strategy for the VC.*

Proof: See Appendix 1.

The intuition behind this Proposition is straightforward, a payment for failure for entrepreneurs is inconsistent with a 'pay for performance' compensation strategy because such a payment creates an incentive for the entrepreneur to exert little effort or none, as effort is costly for the entrepreneur but not rewarded.[10] Proposition 3 is consistent with, and complementary to, the work of Bergemann and Hege (1998) and Elitzur and Gavious (2003b) who argue that the optimal share contract should reward the entrepreneur only if he was successful.

Proposition 3 implies that we can ignore F_k and write the revised program as follows:

$$V_k = \alpha_k (e_k)\left[B_k - S_k + V_{k+1}\right] - I_k \qquad \mathrm{k} = 1,\ldots,\mathrm{K} \tag{1'}$$

$$U_k = \alpha_k (e_k)\left[S_k + U_{k+1}\right] - C(e_k) \qquad \mathrm{k} = 1,\ldots,\mathrm{K} \tag{2'}$$

Define $e_k{}^*$ as the optimal effort level for the entrepreneur and $e_k{}^{*'}$ as $\frac{de_k{}^*}{dX}$ where $X = B_k - S_k + V_{k+1}$. The following Proposition characterizes the optimal payment to the entrepreneur, S_k, over time:

Proposition 4: *The optimal payment scheme satisfies the following conditions:*

$$S_k = \frac{C'(e_k{}^*)}{\alpha_k{}'(e_k{}^*)} - U_{k+1} \tag{5}$$

$$S_k = V_{k+1} + B_k - \frac{\alpha_k (e_k{}^*)}{\alpha_k{}'(e_k{}^*) e_k{}^{*'}} \tag{6}$$

Proof: See Appendix 1

[10] While the Proposition may seem straightforward ex-post it still needs to be proved in order to show payment under all possibilities.

Condition (5) in Proposition 4 states that the optimal payment to the entrepreneur, S_k, is the ratio of the entrepreneur's marginal cost of effort to the marginal probability of success, $\frac{C'(e_k*)}{\alpha_k'(e_k*)}$, less the entrepreneur's expected future payoffs, U_{k+1}.

Condition (6) in Proposition 4 tells us that the entrepreneur is never paid the entire payoff of the VC, $V_{k+1} + B_k$, because it follows from (6) that $S_k < V_{k+1} + B_k$. Further, condition (6) states that the optimal payment to the entrepreneur, S_k, increases in the rewards to success, $V_{k+1} + B_k$. Another insight provided by equation (6) is that, ceteris paribus, the payment to the entrepreneur, S_k, is reduced when $\frac{\alpha_k(e_k*)}{\alpha_k'(e_k*)e_k*'}$ is high, i.e., when the probability of success, $\alpha_k(e_k*)$, relative to the effect on entrepreneurs' effort of S_k, $\alpha_k'(e_k*)e_k*'$, is high. In other words, payment to the entrepreneur will be high as long as he provides some added value in terms of his effort.

When we rearrange and combine conditions (5) and (6) the following obtains,

$$\frac{V_{k+1} + B_k - S_k}{U_{k+1} + S_k} = \frac{\alpha_k(e_k*)}{C'(e_k*)e_k*'} \qquad \textbf{(7)}$$

Condition (7) implies that if the probability of success, $\alpha_k(e_k*)$, is large relative to the effect on entrepreneurs' cost of effort, $C'(e_k*)e_k*'$, then the sharing rule would favor the VC and vice versa. Proposition 4 is different from the results in Elitzur and Gavious (2003b) where the optimal incentive scheme backloads all incentive payments to the entrepreneur as much as possible and thus the VC takes the entire payoff from the firm until a certain threshold point from which the entrepreneur takes it all. Consequently, Elitzur and Gavious (2003b) argue that a straight debt contract is optimal in their setting. The underlying cause for the different result between this study and Elitzur and Gavious (2003b) is the use of renegotiated contracts in this study rather than a single long-term multi-period contract (which Elitzur and Gavious (2003b) use). We argue that contracts are often renegotiated and, consequently, we need to understand the implication of renegotiation. Proposition 4 leads to the following sharing rule between the two players:

Proposition 5: *The payoff to the VC from current period and future periods satisfies the following condition:*

$$\frac{d(\alpha_k(e_k*))}{\alpha_k(e_k*)} = \frac{d(V_{k+1} + B_k - S_k)}{V_{k+1} + B_k - S_k} \qquad \textbf{(8)}$$

Proof: See Appendix.

Condition (8) states that the payoff to the VC from current period and future periods should be such that the rate of improvement in chances of success resulting from added effort from the entrepreneur, $\frac{d(\alpha_k(e_k*))}{\alpha_k(e_k*)}$, is equal to the rate of increase in the VC's payoff, $\frac{d(V_{k+1} + B_k - S_k)}{V_{k+1} + B_k - S_k}$.

5. Summary and Conclusions

This paper examines the relationship between a venture capitalist and an entrepreneur in a multi-period setting when contracts are renegotiated each period. As far as we know this is the first paper in the VC literature that uses renegotiated contracts in its setting than a single long-term contract or a single period setting. The contribution of our paper lies in the insights it provides on optimal contracts between venture capitalists and entrepreneurs. These insights prove to be significantly different in certain respects than those obtained under a multi-period contract without renegotiation or a single period setting.

An interesting aspect of the relationship between VC s and entrepreneurs that we do not investigate here is the adverse selection problem. The problem that we examine here deals mainly with the moral hazard aspect of the relationship that, in essence, starts after the adverse selection problem has been resolved. A possible direction to extend this study is how should VCs select entrepreneurs in a manner that alleviates the adverse selection problem.

REFERENCES

Admati, A., and P. Pfleiderer, 1994, Robust Financial Contracting and the Role for Venture Capitalists," *Journal of Finance,* 49, 371-402.

Aghion, P., and P. Bolton, 1992,"An Incomplete Contracts Approach to Financial Contracting," *Review of Economic Studies*, 473-494

Alsop, Stewart, 2002, "The Rules of Venture Capital," *Fortune*. New York: Apr 15, 2002. Vol. 145, Iss. 8, 76.

Amit, R., Brander, J. & Zott, C., 1997. "Venture Capital Financing Of Entrepreneurship In Canada,". In *Capital Markets Issues in Canada*, Paul Halpern (Ed.). Industry Canada, Ottawa, Canada.. 237-277.

Amit, R., Glosten, L., and E. Muller,1990, "Entrepreneurial Ability, Venture Investments, and Risk Sharing, " *Management Science* 36, 1232-1245.

Barry, C.,1994, "New Directions in Research on Venture Capital Firms," *Financial Management* 23, 3-15.

Bergemann, Dirk; Hege, Ulrich, 1998, "Venture Capital Financing, Moral Hazard, And Learning," *Journal of Banking & Finance*, Volume 22, 703-735.

Brav A., and P. Gompers, 1997, "Myth or Reality? The Long-Run Underperformance of Initial Public Offerings: Evidence from Venture and Nonventure Capital-Backed Companies," *Journal of Finance*, Vol. 52, No. 5. pp. 1791-1821.

Chan, Y., 1983, "On the Positive Role of Financial Intermediation in Allocation of Venture Capital in a Market with Imperfect Information," *Journal of Finance*, 38, 1543-1568.

The Economist, 2002, Business: Sharing the pain; Venture capital in America, Apr 13, 2002. Vol. 363, Iss. 8268; pg. 75

Elitzur, R., and A. Gavious, 2003a, "Contracting, Signaling and Free Riders: A Model of Entrepreneurs, 'Angels', and Venture Capitalists", *Journal of Business Venturing*, 18,709-725.

Elitzur, R., and A. Gavious, 2003b, "A Multi-Period Game Theoretic Model of Venture Capital Investment, *European Journal of Operational Research*. Vol 144, (2), 440-453

Gompers, Paul A., 1995, "Optimal Investment, Monitoring, and the Staging of Venture Capital," *Journal of Finance* 50: 1461-1489.

Gompers, P. and J. Lerner, 1999, "The Venture Capital Cycle," Cambridge Mas: MIT Press.

Hart, O., 1995, *Firms, Contracts, and Financial Structure*, Clarendon Press, Oxford,

Himelstein, Linda, 2002, "VCs TURN THE SCREWS: Today's venture capitalists are demanding onerous terms. That could hurt both innovation and investors," Business Week. May 27, 2002. , Iss. 3784, 82.
Kaplan, Steven,. N. and P. Stromberg, 2002, "Financial Contracting Theory Meets the Real World: Evidence From Venture Capital Contracts," *Review of Economic Studies*, 00, 1-35.
Lerner, J., 1994, "The Syndication of Venture Capital Investments," *Financial Management*. 23.16-27.
Maskin, Eric, 2002, "On Indescribable Contingencies and Incomplete Contracts, "*European Economic Review*," European Economic Review, Vol: 46, Issue: 4-5, 725-733.
MacIntosh., J., 1997, "Venture Capital Exits in Canada and the United States," in *Capital Markets Issues in Canada*,. Paul Halpern (Ed.). Industry Canada, Ottawa, Canada. 279-356
Neher, Darwin, 1999, "Staged Financing: An Agency Perspective," *Review of Economic Studies*, Vol. 66, 255-274.
Sahlman, W., 1990, "The Structure and Governance of Venture Capital Organization", *Journal of Financial Economics* 27, 473-524.
Trester, Jeffrey J., 1998, "Venture Capital Contracting Under Asymmetric Information," Journal of Banking & Finance Volume: 22, Issue: 6-8, 675-699.
Wang, S. and H. Zhou, 2004, "Staged Financing in Venture Capital: Moral Hazard and Risks," *Journal of Corporate Finance*, Volume: 10, Issue: 1, January, 131-155.
Zider, B., 1998, "How Venture Capital Works," *Harvard Business Review*, November-December 1998, 131-139.

APPENDIX 1 - MATHEMATICAL ADDENDUM

PROOF OF PROPOSITION 1

The entrepreneur's first-order condition with respect to effort is explicitly the following equation

$$\frac{dU_k}{de_k} = \alpha_k'(e_k)\left[S_k + U_{k+1}\right] - \alpha_k'(e_k)F_k - C'(e_k) = 0 \qquad \textbf{(A.1)}$$

Define e_k* as the optimal effort of the entrepreneur in k (the result of $\frac{dU_k}{de_k} = 0$). Also, define $X = \left[S_k + U_{k+1} - F_k\right]$. Consequently, (A.1) implies:

$$\alpha_k'(e_k*)X = C'(e_k^*) \qquad \textbf{(A.2)}$$

(A.2), thus, implies the following function

$$e_k* = e_k*(X) \qquad \textbf{(A.3)}$$

■

PROOF OF PROPOSITION 2

Rearranging (A.1) yields the result. ■

PROOF OF PROPOSITION 3

To get the optimal payment the VC solves the following problem:

$$\underset{F_k}{Max}\, V_k = \alpha_k(e_k)\left[B_k - S_k + V_{k+1}\right] - \left[1 - \alpha_k(e_k)\right]F_k - I_k \quad \mathrm{k} = 1,\ldots,\mathrm{K} \qquad \textbf{(A.4)}$$

s.t. $B_k + V_{k+1} \geq S_k$

Given the following problem for the entrepreneur:

$$\underset{e_k}{Max}\ U_k = \alpha_k(e_k)\left[S_k + U_{k+1}\right] + \left[1-\alpha_k(e_k)\right]F_k - C(e_k) \quad \text{k} = 1,\ldots,\text{K} \qquad \textbf{(A.5)}$$

Consequently, the VC solves:

$$\frac{\partial V_k}{\partial S_k} = 0 \ \text{ subject to } \frac{dU_k}{de_k} = 0 \qquad \textbf{(A.6)}$$

Define $e_k{*}'$ as $\frac{de_k{*}}{dF_k}$.

$$e_k^{*\prime} = \left.\frac{de_k{*}(X)}{dX}\right|_{X=\left[S_k+U_{k+1}-F_k\right]} \frac{dX}{dF_k} = \left.\frac{de_k{*}(X)}{dX}\right|_{X=\left[S_k+U_{k+1}-F_k\right]} \times(-1) < 0 \qquad \textbf{(A.7)}$$

$$\text{where } \left.\frac{de_k{*}(X)}{dX}\right|_{X=\left[S_k+U_{k+1}-F_k\right]} = -\frac{\frac{dU_k^2}{de_k dX}}{\frac{dU_k^2}{d^2 e_k}} = -\frac{\alpha_k{'}(e_k{*})}{\alpha_k{''}(e_k{*})X - C''(e_k)} > 0$$

The VC can use the function $e_k{*}$ in V_k to solve for $\frac{\partial V_k}{\partial F_k}=0$

$$\begin{aligned}\frac{\partial V_k}{dF_k} &= -\alpha_k'(e_k{*})e_k{*}'\left[B_k - S_k + V_{k+1}\right] - \alpha_k'(e_k{*})e_k{*}'F_k - \left[1-\alpha_k(e_k{*})\right] = \\ &-\alpha_k'(e_k{*})e_k{*}'\left[B_k - S_k + V_{k+1} + F_k\right] - \left[1-\alpha_k(e_k{*})\right]\end{aligned} \qquad \textbf{(A.8)}$$

$\left[B_k - S_k + V_{k+1}\right]$ should be positive otherwise the VC pays the entrepreneur more than his payoff. Since by the assumption $\alpha_k{'}(e_k{*})$ is positive. As in (A.7), $e_k{*}'$ is negative. Consequently, (A.8) is negative.

(A.8) implies that the payment for failure actually reduces the optimal effort of the entrepreneur and, hence, lowers the payoff of the VC. Accordingly, the best payment from the VC's view is the lowest possible one, which is zero. Further, V_k from (A.1) is monotonically increasing in e_k and monotonically decreasing in S_k. ■

PROOF OF PROPOSITION 4

In light of Proposition 1, in order to get the optimal payment, the VC solves the following problem:

$$\underset{A_k^f}{Max}\, V_k = \alpha_k(e_k)\left[B_k - S_k + V_{k+1}\right] \quad \text{k} = 1,\ldots,\text{K} \qquad \textbf{(A.9)}$$

Given the following problem for the entrepreneur:

$$\underset{e_k}{Max}\ U_k = \alpha_k(e_k)\left[S_k + U_{k+1}\right] - C(e_k) \quad \text{k} = 1,\ldots,\text{K} \qquad \textbf{(A.10)}$$

The entrepreneur's first-order condition with respect to effort is explicitly the following equation

$$\frac{\partial U_k}{\partial e_k} = \alpha_k{'}(e_k{*})\left[S_k + U_{k+1}\right] - C'(e_k{*}) = 0 \qquad \textbf{(A.11)}$$

Rearranging,

$$S_k = \frac{C'(e_k{*})}{\alpha_k{'}(e_k{*})} - U_{k+1} \qquad \textbf{(A.12)}$$

Define e_k*' as $\frac{de_k*}{dX}$ where $X = B_k - S_k + V_{k+1}$. The VC's first-order condition with respect to payment is the following equation

$$\frac{\partial V_k}{\partial S_k} = \alpha_k'(e_k*)e_k*'\left[B_k - S_k + V_{k+1}\right] - \alpha_k(e_k*) = 0 \text{ , k} = 1,\ldots,\text{K} \quad \textbf{(A.13)}$$

Rearranging (A.10)

$$S_k = V_{k+1} + B_k - \frac{\alpha_k(e_k*)}{\alpha_k'(e_k*)e_k*'} \quad \textbf{(A.14)}$$

■

PROOF OF PROPOSITION 5

Rearranging Equation (6) yields

$$V_{k+1} + B_k - S_k = \frac{\alpha_k(e_k*)}{\alpha_k'(e_k*)e_k*'} \quad \textbf{(A.15)}$$

Using the definitions of e_k*' as $\frac{de_k*}{dX}$ where $X = B_k - S_k + V_{k+1}$ and rearranging provides the following condition:

$$\frac{d(\alpha_k(e_k*))}{\alpha_k(e_k*)} = \frac{d(V_{k+1} + B_k - S_k)}{V_{k+1} + B_k - S_k} \quad \textbf{(A.16)}$$

■