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Performance of short-term trend predictors for current economic analysis

Olivier Darne
LEMNA, University of Nantes

Estelle Bee Dagum
Department of Statistics, University of Bologna

Abstract

We study the performance of several short-term trend estimators for current economic analysis. These estimators are available in X11-ARIMA, X12-ARIMA, TRAMO-SEATS and STAMP. We also include two other trend-cycle estimators obtained by post-processing seasonally adjusted data with X11ARIMA, namely, a modified Henderson nonlinear filter by Dagum (1996) DMH, and a new modified version of it, DMH-D. The estimators are applied to a number of simulated non-seasonal data of various levels of variability.

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1. Introduction

Major financial and economic changes of global nature have introduced high levels of variability in time series data, particularly, in socioeconomic indicators often used for the analysis of current economic conditions, known to as recession and recovery analysis. Traditionally, these indicators were seasonally adjusted to determine the direction of the short-term trend for an early detection of a turning point. However, due to the presence of high levels of variability, there is a need for further smoothing seasonally adjusted data to facilitate the analysis of current economic conditions.

Dagum (1996) developed a nonparametric nonlinear filter to improve on the classical Henderson linear filters, which are used in the X11-ARIMA and X12-ARIMA methods for short-term trend-cycle estimation. Dagum's modified 13-term Henderson [DMH] nonlinear filter has the advantage of reducing the number of unwanted ripples (10-month cycles that can be falsely interpreted as true turning points) in the final trend curve without increasing the time lag to detect a true turning point. Furthermore, it also reduces the size of the revisions to the most recent trend-cycle estimates with respect to those of the Henderson filter. A study by Chaab et al. (1999) showed the superior performance of DMH relative to structural trend-cycle parametric models when applied to seasonally adjusted series with varying degrees of signal to noise ratios[1].

The purpose of this paper is twofold: first, we propose a modification to the Dagum (1996) nonlinear nonparametric filter, called here DMH-D, to improve on the size of the revisions to the last data point trend-cycle estimate; and, second, we perform a comparative analysis of the new estimator with that of Dagum (1996) and others incorporated in seasonal adjustment methods, namely, X11-ARIMA, X12-ARIMA, TRAMO-SEATS and STAMP. Differently to previous studies, the comparison is done on the basis of non-seasonal simulated data. Furthermore, the performance of the trend-cycle estimators discussed is evaluated on the basis of four instead of the three main criteria: (1) time delay of true turning point detection, (2) number of unwanted ripples (false turning points) produced, and (3) size of revisions to the last data point trend-cycle estimate. We added an extra criterion, that of the identification of true turning points for we observed that the parametric trend estimators not always detected a true turning point.

Section 2 briefly introduces the nonparametric short-term trend estimation procedures. Section 3 gives the criteria required for a short-term trend to be appropriate for current economic analyses. Section 4 shows the different data sets used for the comparative analysis. Section 5 presents the empirical results

[1]Similarly, Dagum and Capitano (1998) and Dagum and Luati (2000) showed that DMH gave superior results when compared to other non-parametric linear and non linear smoothers, namely the local weighted regression (loess) of degree two, Gaussian kernel, supersmoother and cubic spline.

and, finally, Section 6 concludes.

2. Trend-cycle Estimators

2.1. Henderson Trend-cycle Estimator

The trend-cycle Henderson estimator is a linear filter available in the Census X-11 seasonal adjustment method (Shiskin et al., 1967) and all its variants, particularly, the X11-ARIMA (Dagum, 1988) and X12-ARIMA (Findley et al., 1998) used in our study.
Henderson (1916) showed that the smoothness of the output from a given linear filter depends on the smoothness of its weight diagram, and he developed a formula which makes the sum of squares of the third differences of the smoothed series a minimum for any numbers of terms. In others words, the $\sum(\Delta^3 y_t)^2$ is minimized, where Δ is the difference operator and y_t is the output or smoothed series, if and only if $\sum(\Delta^3 h_k)^2$ is minimizes, where h_k' are the weights, subject to the constraints that $\sum h_k = 1$, $\sum kh_k = 0$ and $\sum k^2 h_k = 0$ (Dagum, 1978 and 1985).

The Henderson symmetric weight system of length $2n+1$, where $m = n+2$, is given by

$$h_k = \frac{315[(m-1)^2 - k^2][m^2 - k^2][(m+1)^2 - k^2][3m^2 - 16 - 11k^2]}{8m(m^2-1)(4m^2-1)(4m^2-9)(4m^2-25)} \tag{1}$$

To derive a set of 13 weights from (1), 8 is substituted for m and the values are obtained for each k from -6 to 6. The Henderson 9-term and 23-term trend-cycle filters are obtained by substituting m with 6 and 13, respectively. The Henderson 13-term trend-cycle [H13] filter is thus given by,

$$\begin{aligned} H_{13}(B) \;=\; & -0.019B^{-6} - 0.028B^{-5} + 0.00B^{-4} + 0.065B^{-3} + 0.147B^{-2} \\ & +0.214B^{-1} + 0.24B^{-0} + 0.214B^{1} + 0.147B^{2} + 0.065B^{3} \\ & +0.00B^{4} - 0.028B^{5} - 0.019B^{6} \end{aligned} \tag{2}$$

where B is the backshift operator defined by $B^m y_t = y_{t-m}$ and $B^0 = 1$[2].

The standard Henderson trend estimation consists of applying the automatically selected Henderson filter to a robust seasonally adjusted series. The robustification is done by the default replacement of extreme values where irregulars falling between $\pm 1.5\sigma$ and $\pm 2.5\sigma$ are scaled down linearly and beyond $\pm 1.5\sigma$ are replaced by their mean. The seasonally adjusted series is usually obtained from

[2]For a discussion on the asymmetric Henderson filters for the observations at the beginning and the end of the series, see Musgrave (1964), Ladiray and Quenneville (2001), Doherty (2001), Gray and Thomson (2002) and Quenneville et al. (2003).

the X11-ARIMA or the X12-ARIMA extending the original data with one year of ARIMA extrapolations.

2.2. Dagum's Modified 13-term Henderson Filter (DMH)

The modified 13-term Henderson filter developed by Dagum (1996) is nonlinear and basically, consists of: (a) extending the series (which is already seasonally adjusted if seasonality is present as often occurs with monthly or quarterly economic data) using ARIMA extrapolated values and (b) applying the 13-term Henderson filter to the extended series with modified extreme values identified using very strict sigma limits.

To facilitate the identification and fitting of simple ARIMA models, Dagum (1996) recommends, at step (a), to modify the input series for the presence of extreme values using the standard $\pm 2.5\sigma$ limits of X11ARIMA (versions 1988 or 2000) and X12ARIMA (Findley et al. 1998). These computer programs are those that can be used to implement DMH. In this way, a simple and very parsimonious ARIMA model, the $(0,1,1)\,(0,0,1)_s$, is often found to fit a large number of series. Concerning step (b), it is recommended to use very strict sigma limits, such as $\pm 0.7\sigma$ and $\pm 1.0\sigma$.

The extension of the series is performed with the purpose of reducing the size of the revisions of the most recent estimates. On the other hand, the use of stricter sigma limits for the identification and replacement of the extreme values has the purpose of reducing the number of unwanted ripples (false turning points) created by the classical 13-term Henderson filter.

To extrapolate the input series, Dagum (1996) advises to employ the $(0,1,1)$ $(0,0,1)$ ARIMA model. However, we observed from many empirical studies that the value of seasonal moving average parameter was often very small. Therefore, we propose to modify this method [thereafter DMH-D] by removing this parameter and using the $(0,1,1)\,(0,0,0)$ ARIMA model, this model change improves the revision size without altering the delay of turning point detection and the number of unwanted ripples[3].

3. Criteria of Comparison for Current Economic Analysis

The performance of the above trend-cycle estimators is evaluated on the basis of four major criteria for current economic analysis: (1) the identification of true turning point, (2) time delay, (3) number of false turning points produced, and (4) the size of revisions to the last data point (concurrent) trend-cycle estimate as new data are added. The method to be preferred is the one that gives the smallest values for each of these four criteria. If no method ranks first in all

[3]See Chhab et al. (1999) for a discussion

four criteria, the order of preference is given as follows: First, the number of true turning points identified and their time delay, second, the number of false turning points introduced by each method, and third the size of the revisions.
Relative to the criteria considered in Dagum (1996) we added, here, the identification of true turning points, since not always the model-based trend-cycle estimators identified a true turning point. In this study, a true turning point is considered missed if never identified or detected with more than 6 months delay.

We follow the definition of a turning point in the context of smoothed data largely accepted in the literature (Zellner et al., 1991) according to which a downturn occurs at time t if: $y_{t-k} \leq \cdots \leq y_{t-1} > y_t \geq y_{t+1} \geq \cdots \geq y_{t+m}$; and a upturn if $y_{t-k} \geq \cdots \geq y_{t-1} < y_t \leq y_{t+1} \leq \cdots \leq y_{t+m}$ with $k = 3$ and $m = 1$.

An unwanted ripples arises whenever two (false) turning points occur within a 10 months period implying short cycles of 10 month periodicity. We also consider turning points generated by shorter cycles from 3 to 7 month periodicity. These latter are seldom confuse as false turning points but produce a more variable trend from which it becomes difficult to assert its direction in the short-term.
The time lag to detect a true turning points is obtained by calculating the number of months it takes for the revised trend series to signal a turning point in the same position as in the final trend series.
The size of total revision of the concurrent trend estimate is calculated by means of the mean absolute percentage error (MAPE) over four complete years, defined by

$$MAPE = \frac{1}{48} \sum_{t=1}^{48} \left| \frac{\widehat{y}_t^{(F)} - \widehat{y}_t^{(0)}}{\widehat{y}_t^{(0)}} \right| \tag{3}$$

where $\widehat{y}_t^{(0)}$ is the last estimate of the trend when the series is truncated in t and then adding one point at a time till $t + 48$. The $\widehat{y}_t^{(F)}$ denotes the corresponding final estimate, i.e. the one obtained for each point from the full span of the series.

We also compute criteria (1) and (2) in probability terms, with α the probability of identifying a true turning point and γ_i the probability of identifying a true turning point with up to i month delays (Chhab et al., 1999).

4. Data sets

The trend-cycle estimators are applied to a number of non-seasonal simulated data with different levels of variability. The simulated non-seasonal series are composed of trend, cycle and irregulars, ranging from January 1948 till December 2000. We chose two levels of signal to noise ratio, one low and one high, to generate series of high and low variability, respectively. The trend component is assumed to be a random walk model without drift: $T_t = T_{t-1} + \nu_t$ with $\nu_t \sim N(0, \sigma_\nu^2)$. The cyclical component is modelled by: $C_t = \rho[\cos(2\pi t/\lambda) + \sin(2\pi t/\lambda)]$

where ρ is a damping factor ($0 < \rho < 1$) and λ is the cycle frequency. The irregulars are assumed to be white noise with mean zero and variance σ_e^2: $I_t = e_t$ with $e_t \sim N(0, \sigma_e^2)$.

For the simulations we vary the σ_ν^2, σ_e^2, ρ and λ parameters. For the series with a low signal to noise ratio (high variability), $\sigma_\nu^2 = 0.08$, $\sigma_e^2 = 0.40$, $\lambda = 60$ and $\rho = 0.50$, 0.7, 0.80. For the series with a high signal to noise ratio (low variability), $\sigma_\nu^2 = 0.08$, $\sigma_e^2 = 0.20$, $\lambda = 60$ and $\rho = 3.0$, 3.5, 4.0[4].

5. Empirical results

We perform a comparative analysis of the new trend-cycle estimator with that of Dagum (1996) and the estimators available in X11/X12-ARIMA with the H13 filter, TRAMO-SEATS with the ARIMA model-based (nonlinear) parametric trend-cycle estimator (Gómez and Maravall, 1997), and STAMP with the structural trend-cycle parametric estimator (Koopman et al., 1995)[5]. The comparison is done on the basis of six non-seasonal simulated data and from the four major criteria for current economic analysis.

5.1. Identification of true turning points and time delay

The ARIMA model-based and the structural trend-cycle predictors not always detected a true turning point present in the series analyzed. There are 17 true turning points in each simulated series[6]. As shown in Table 1, the nonparametric trend-cycle predictors, X11/X12-ARIMA (H13), DMH and DMH-D identified all of them correctly ($\alpha = 100\%$) whatever the level of variability. On the contrary, the ARIMA trend-cycle parametric estimator from TRAMO-SEATS, and that from STAMP, performed very poorly in both number of true turning point identified and associated probabilities. This is particularly so for series with a high signal to noise ratio where the short-term trends estimated by both parametric procedures follow closely the input data and do not smooth, thus, leaving most of the noise.

Table 2 displays the time delays for the simulated non-seasonal series. It is apparent that, independently of the data variability, the nonparametric estimators identify the true turning points more quickly than the parametric ones. For series of high variability, STAMP and TRAMO-SEATS identify the turning points with a time lag of three and four months, respectively. No average is shown for

[4]The level of variability has been verified from the irregular-trend-cycle ratio (I/C) available in the Census X-11 seasonal adjustment method and all its variants.

[5]More precisely, we apply the basic structural model [BSM] developed by Harvey (1981).

[6]There are 21 turning points from the generated series, however, only 17 are identifiable due to losing some of them with some of the trend-cycle methods.

Table 1: Number of true turning points identified.

	H13	DMH	DMH-D	STAMP	TS
low I/C	17 (1.00)	17 (1.00)	17 (1.00)	14 (0.82)	11 (0.65)
high I/C	17 (1.00)	17 (1.00)	17 (1.00)	4 (0.23)	8.3 (0.49)

H13: X11/X12-ARIMA (H13); DMH: Dagum's modified 13-term Henderson; DMH-D: modified DMH; TS: TRAMO-SEATS. The α probabilities are given in brackets.

those of small variability because both procedure missed a large number of true turning points. The average true turning point delay for DMH and DMH-D is less than two months for the series with a high signal to noise ratio, and of two and a half months for those of with a low signal to noise ratio. In all cases they show an improvement respect to H13 as applied in X11/X12-ARIMA.

Table 2: Average time delay of true turning point detection (in month).

	H13	DMH	DMH-D	STAMP	TS
low I/C	2.59	2.49	**2.45**	4.01	3.02
γ_1	0.06	0.20	0.22	0.02	0.06
γ_2	0.51	0.45	0.47	0.14	0.31
γ_3	0.90	0.84	0.82	0.27	0.47
γ_4	0.96	0.94	0.96	0.51	0.51
high I/C	2.41	1.92	**1.90**	-	-
γ_1	0.06	0.29	0.29	0.24	0.39
γ_2	0.61	0.75	0.77	0.25	0.43
γ_3	0.92	0.96	0.96	0.25	0.45
γ_4	1.00	1.00	1.00	0.25	0.45

H13: X11/X12-ARIMA (H13); DMH: Dagum's modified 13-term Henderson; DMH-D: modified DMH; TS: TRAMO-SEATS. γ_i denotes the probability of identification with i month delay.

Table 2 gives also the probability γ_i of true turning point detection with a time lag ranging from one to four months, respectively. For the series with a low I/C, the probability γ_1 of detecting a true turning point, within one month, with DMH and DMH-D is around 0.20 whereas for the remainder estimators is much lower. For series with high I/C, all the estimators, except H13 and TRAMO-SEATS, can detect a true turning point with a probability ranging from 0.24 to 0.29. The nonparametric estimators based on the 13-term Henderson filter identify true turning points, within two months, with a probability γ_2 close to 0.50 if I/C is low and 0.70 when I/C is high. The parametric estimators from

TRAMO-SEATS and STAMP only identify 0.14 and 0.31 of all possible turning points, respectively when the variability is high and 0.25 and 0.43 when it is low. The probability γ_4 of detecting a turning point with a maximum time lag of four months, for highly variable data, reaches around 0.95 for H13, DMH and DMH-D, while this probability is of 0.50 for the both parametric estimators. For series of low variability the latter have even a lower probability of true turning point identification whereas the non parametric methods reach 1. In general, the probability of detecting a true turning point with a nonparametric trend-cycle predictor increases with the time delay but this is not so for the parametric predictors.

5.2. Number of unwanted ripples

The presence of false turning points resulting from 10-month cycles (unwanted ripples) seems to be more a problem of the non parametric than the parametric methods, due to the use of the H13 filter. On the other hand, the parametric method filters tend to under- or over-smooth, producing then a large number of turning points from short cyclical variations (3-7 month periodicity) or very few turning points (whether true or false).

Table 3 displays the number of turning points from short cycles and indicates that the parametric methods either oversmooth or undersmooth. The largest number of very short cyclical fluctuations is given by TRAMO-SEATS independently of the signal to noise ratio. On the contrary, STAMP oversmooth series of high variability (I/C low) and produces a large number of turning points from short cycles when I/C is high.

Table 3: Number of turning points from short cycles.

	H13	DMH	DMH-D	STAMP	TS
low I/C	20	12	12	**8**	28
high I/C	14	**8**	**8**	16	27

H13: X11/X12-ARIMA (H13); DMH: Dagum's modified 13-term Henderson; DMH-D: modified DMH; TS: TRAMO-SEATS.

5.3. Revisions of Concurrent Trend-Cycle Estimates

Table 4 displays the revisions calculated over four full years, from January 1983 till December 1986. For the series with a low signal to noise ratio (high variability), the smaller average revision is given by TRAMO-SEATS (0.98) followed by DMH and DMH-D that give an average revision of 1.06. The largest revision is given by STAMP. For the series with a high signal to noise ratio (low variability), the revisions produced by STAMP are extremely low relative to the other methods (similarly by TRAMO-SEATS for two series). In fact, this is to be expected for the trend curves of the concurrent estimates from STAMP and TRAMO-SEATS are almost superimposed with their historical (final) estimates. Consequently, for these parametric methods, the difference between their concurrent and historical trend-cycle estimates is very small (sometimes null). The trend estimates are practically not smoothed with a large number of short-cyclical fluctuations present. In fact, we cannot dissociate the problem of revisions with the type of final trend produced by a given method. In our study, when the series are of low variability (high I/C) both STAMP and TRAMO-SEATS produce very distinct final trends to those from the non parametric methods.

Table 4: Concurrent trend-cycle revisions (MAPE).

	H13	DMH	DMH-D	STAMP	TS
low I/C	1.12	1.06	1.06	1.19	0.98
high I/C	1.17	1.23	1.21	-	0.66

H13: X11/X12-ARIMA (H13); DMH: Dagum's modified 13-term Henderson; DMH-D: modified DMH; TS: TRAMO-SEATS.

6. Conclusion

We performed a comparative analysis of trend-cycle estimators available in seasonal adjustment methods, namely, X11/X12-ARIMA, TRAMO-SEATS and STAMP, and other two obtained by post-processing seasonally adjusted data. These latter are the Dagum modified Henderson filter (DMH), and a new modified version (DMH-D) developed to improve on the size of the revisions to most recent trend-cycle estimates.
The performances of the trend-cycle estimators were evaluated on the basis of four major criteria for current economic analysis. These criteria were evaluated on the basis of six simulated non-seasonal data of varying degrees of variability. The results showed that the proposed modification to the Dagum filter produced smaller revisions while maintaining DMH good properties of true turning point detection, short-time delay and small number of unwanted ripples. The revision gains introduced by DMH-D were more significant when the revisions were large in magnitude. Moreover, the non-parametric trend-cycle predictors performed systematically better than the parametric ones available in TRAMO-SEATS and STAMP. Further research should investigate the performance of the DMH and DMH-D with others trend-cycle estimators and from a larger simulation including others types of models with varying degree of variability to confirm our preliminary results.

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