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Estimation of the elasticity of substitution between oil and capital

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Abstract

The elasticity of substitution between oil and capital is a key parameter when researchers analyze the effect of oil shocks on the economy by using dynamic general equilibrium models. This paper estimates the elasticity of substitution in the U.S. economy, which is consistent with a large class of DGE models. We find that the estimated elasticity of substitution becomes lower than the value estimated by earlier empirical studies. A low elasticity of substitution implies that oil supply shocks have large impacts on the economy.

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1 Introduction

Many researchers analyze the role of oil shocks in the U.S. economy by using dynamic general equilibrium (DGE) models. Finn (2000), Kim and Loungani (1992), and Rotemberg and Woodford (1996) examine the effects of oil prices on the business cycle. Wei (2003) analyzes its effect on the stock market. Backus and Crucini (2000) find that oil accounts for a large part of the variation in the terms of trade. Leduc and Sill (2004) state that the systematic monetary policy to oil price shocks contributes to the fall in output after a rise in oil prices. It is expected that many studies will be carried out in the near future because of the drastic fluctuations in oil prices in recent years.

One of the most popular formulations of oil in production functions is the nested constant elasticity of substitution (CES) function: $Y = L^{\alpha}[\eta K^{\frac{\nu-1}{\nu}} + (1-\eta)O^{\frac{\nu-1}{\nu}}]^{(\frac{\nu}{\nu-1})(1-\alpha)}$. A key parameter is the elasticity of substitution between oil and capital, ν. However, there is no agreement on the value of ν. Table 1 summarizes the values of ν in the related literature. Kim and Loungani (1992) employ high elasticities of substitution, whereas Backus and Crutini (2000) report that a high elasticity of substitution produces strongly counterfactual implications for the time series of prices and quantities in the world oil market. The values used by Kim and Loungani (1992) are based upon empirical studies, whose samples, however, are limited only to the manufacturing sector.[1] In addition, there is inconsistency between the methods in these empirical studies and the models in Backus and Crucini (2000) and Kim and Loungani (1992).

Table 1: Elasticity of substitution between oil and capital

Authors	ν
DGE	
Kim and Loungani (1992)	100 and 0.59
Backus and Crucini (2000)	0.09
Empirical Study	
Berndt and Wood (1979)	–0.5
Morrison and Berndt (1981)	0.59
Griffin and Gregory (1976)	1

Note: The result of Berndt and Wood (1979), $\nu = -0.5$, is outside the admissible range of parameter values. Therefore, Kim and Loungani (1992) set $\nu = 100$.

[1] See Berndt and Wood (1979), Morrison and Berndt (1981), and Griffin and Gregory (1976).

The purpose of this paper is to estimate the elasticity of substitution between oil and capital in the macro economy, which is consistent with a large class of DGE models. To this end, we construct a simple estimation framework and introduce the equation to be estimated from the solution of the firm's cost minimizing problem. The result is that the estimated elasticity of substitution between oil and capital is lower than that in earlier empirical studies and is similar with that in Backus and Crucini (2000).

2 Estimation

2.1 Estimation Framework

We construct a simple estimation framework, which is consistent with various models. The assumptions are that firms are price-takers in the capital rental market and the oil market. Therefore, the estimated value is consistent with the models with price stickiness, wage stickiness, and so on.

A representative firm solves a part of the cost minimizing problem:

$$\min_{K_t, O_t} \; r_t K_t + p_t O_t \tag{1}$$

$$s.t. \; X_t = [\eta K_t^{\frac{\nu-1}{\nu}} + (1-\eta) Z_t^{\frac{1}{\nu}} O_t^{\frac{\nu-1}{\nu}}]^{\frac{\nu}{\nu-1}}, \tag{2}$$

where K_t is the capital stock, O_t is the oil consumption, X_t is the composition of oil and capital, Z_t is the oil augmented technology, r_t is the rental cost of capital, p_t is the oil price, and ν is the elasticity of substitution between oil and capital. Since we assume that K_t and O_t have no influence on the other parts of the cost minimizing problem, we can concentrate only on this problem.

Two intratemporal first-order conditions for K_t and O_t are given by

$$r_t = \eta \left(\frac{X}{K} \right)^{\frac{1}{\nu}}, \tag{3}$$

$$p_t = (1-\eta) \left(Z_t \frac{X_t}{O} \right)^{\frac{1}{\nu}}. \tag{4}$$

Using these two equations, we can introduce the following equation:

$$\log \left(\frac{p_t}{r_t} \right) = \log \frac{1-\eta}{\eta} + \frac{1}{\nu} \log \left(Z_t \frac{K_t}{O_t} \right). \tag{5}$$

2.2 Data and Method

The data of the oil consumption O_t and the oil price p_t are sourced from the "Product Supplied, Total Crude Oil and Petroleum Products" and "Refiner Acquisition Cost of Crude Oil," released by the Energy Information Administration, Department of Energy.[2] The capital stocks K_t are constructed from real gross domestic investment and real net stock of fixed assets sourced from the National Income and Product Accounts (NIPA) of the Bureau of Economic Analysis. The rental rate of capital r_t is also calculated from NIPA. We employ the "exogenous oil supply shocks" constructed by Killian (2008a) as instrument variables.[3] The baseline sample period is 1983:Q3–2003:Q1.

The capital-oil ratio $(\frac{K_t}{O_t})$ displays a trend, which we assume reflects the change in the oil augmented technology. We apply the HP filter with the conventional smoothing parameter $\lambda = 1600$ and employ the deviations from the trend $(\frac{\hat{K_t}}{O_t})$. Therefore, the equation to be estimated is

$$\log\left(\frac{p_t}{r_t}\right) = \log\frac{1-\eta}{\eta} + \frac{1}{\nu}\log\left(\frac{\hat{K_t}}{O_t}\right) + \epsilon_t. \tag{6}$$

We employ two sets of instruments for the robustness check: one is lagged oil prices and lagged capital-oil ratios and the other is exogenous oil supply shocks. We compare the results of the generalized method of moments (GMM) with lagged endogenous variables as instruments,[4] GMM with exogenous oil supply shocks, and OLS.

2.3 Result

Table 2 displays the results. In both the cases of the GMM, both the estimated values of ν are very low, that is, 0.100 and 0.086. These values are significantly different from these in earlier empirical studies and are very similar to the value employed by Backus and Crucini (2000)

These results are robust to the sample period. Since the oil consumption data is available from 1981:Q1, we test the starting point from 1982:Q1 to 1985:Q4. However, the values of ν are not very different from the baseline case, but are significantly different from these in earlier empirical studies in most cases. The results are robust to the lag of instruments.

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[2] Data are available at `http://tonto.eia.doe.gov/dnav/pet/pet_sum_top.asp`.

[3] We are grateful to Lutz Kilian for making this dataset available at his homepage: `http://www-personal.umich.edu/~lkilian/oilshock.txt`.

[4] Lags range are from one to four.

Table 2: Estimation results

Method	ν	$s.e.$[1]	R^2	$Jstat.$
GMM(1)[2]	0.100	0.033	- - -	0.004
GMM(2)[3]	0.086	0.045	- - -	0.000
OLS	0.267	0.146	0.042	- - -

[1] The standard error is calculated by the delta method

[2] Two-step GMM estimation with lagged endogenous variable (lag = 1–4)

[3] Two-step GMM estimation with exogenous oil supply shocks

3 Conclusion

This paper attempted to estimate the elasticity of substitution between oil and capital in the whole U.S. economy, which is consistent with a large class of DGE models. To this end, we constructed a simple cost minimizing problem of a representative firm and introduced the estimation equation from the first-order conditions. The result is that the estimated elasticity of substitution is lower than that in earlier empirical studies and is consistent with the assumption by Backus and Crucini (2000).

We think that this difference arises from the difference in the samples. The samples employed in earlier studies are limited only to the manufacturing sector. On the other hand, our sample pertains to the whole economy. The oil demands of the nonmanufacturing sector, such as gasoline for transportation or electricity for retail, might be more inelastic than that of manufacturing productions.

A low elasticity of substitution implies that oil supply shocks have large impacts on oil price, output, and other important variables. We believe that this study provides the basis for researches that analyze the effects of oil shocks on the macroeconomy.

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