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An examination of U.S. Phillips curve nonlinearity and its relationship to the business cycle

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Abstract

We test for and model nonlinearity of the reduced-form U.S. Phillips curve using the smooth transition regression (STR) framework. We find evidence of two regimes: a “high inflation regime” associated with fast rising food and energy prices and a “low inflation regime” associated with slower rising or falling food and energy prices. This suggests that the U.S. Phillips curve varies asymmetrically over the business cycle. Particularly, the U.S. Phillips curve has a tendency to shift in and flatten towards the end of expansion periods and in recessions. This result implies that the non-accelerating inflation rate of unemployment (NAIRU) varies over the short-run or business cycle.

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1. Introduction

The shape of the Phillips curve is a basic issue in empirical studies. For example, studies of the U.S. Phillips curve such as Gordon (1998) and Staiger, Stock, and Watson (2001) rely on a linear curve while allowing for the nonaccelerating inflation rate of unemployment (NAIRU) to vary in the long-run (to have a trend). Of course, the original specification (Phillips, 1958) was a concave function showing an inverse relation between wage growth and unemployment. Many have examined a concave function that shows an inverse relation between inflation and the unemployment rate (Clark, Laxton, and Rose 1996 and Akerlof and Yellen 2006). Stiglitz (1997) suggests that the inverse relation between inflation and the unemployment rate is actually convex and Eisner (1997) effectively finds the same thing. We test for nonlinearity in the U.S. Phillips curve using quarterly data from 1960:1 to 2006:1. First we specify a linear Phillips curve and test against a nonlinear alternative using the methodology associated with the smooth transition regression (STR) literature as in Luukkonen, Saikkonen, and Terasvirta (1988), Terasvirta (1994), and Escribano and Jorda (1998). Specifically we use the more powerful version of the nonlinearity test from Escribano and Jorda (1998). Results indicate the presence of nonlinearity in the Phillips curve. Nonlinearity is then modeled using the STR model, which is a regime-switching model where switches between regimes are continuous and smooth rather than discrete. Switches are endogenously determined. We find evidence that ties the nonlinearity to the U.S. business cycle. Specifically, the U.S. Phillips curve tends to shift inward and flatten towards the end of expansion periods and in recession periods. In effect this supports the idea of convexity of the Phillips curve. Late in business cycle expansions when the unemployment rate is relatively low, the Phillips curve is relatively flat. The result also implies NAIRU varies over the short-run or business cycle frequency.

In an earlier study, Eliasson (2001) conducts similar tests and fails to reject linearity of the U.S. Phillips curve (though finding nonlinearity for Australian and Swedish Phillips curves). There are at least three reasons that can explain why we find a different result. One is that we use a more powerful version of the nonlinearity testing strategy developed by Escribano and Jorda (1998) whereas Eliasson (2001) relied on the version in Terasvirta (1994). Second, we use a longer time series dataset with different variables. Third, we specify the reduced-form Phillips curve using the accelerationist version similar to Staiger, Stock, and Watson (2001). A further study to identify which of those reasons generates the difference would be useful.

Why is the STR framework attractive for addressing this issue? One advantage is the flexibility of the STR model over simply adding basic nonlinear terms such as the squared unemployment rate. A convex or concave Phillips curve is not likely to be invariant over time any more than a linear Phillips curve and the STR model can reflect that possibility. The STR model's advantage over other types of threshold (or regime dependent) models is that it allows for a smooth transition period between regimes. As a result, the STR model can be viewed as encompassing other regime dependent models. Further, the model has an easy to implement testing strategy to facilitate non-linearity testing. Finally, if the economic factors driving non-linearity are firm level or individual level, once those behaviors are aggregated, the resulting aggregate behavior is likely to be smoothed. This makes the STR model attractive from a microfoundations standpoint.

Section 2 specifies and estimates a linear Phillips curve as a benchmark. Section 3 explains the STR model and nonlinearity testing in detail. Section 4 applies the technique to the U.S. Phillips curve. Section 5 concludes and suggests future research.

2. Baseline Phillips Curve

To start, we estimate a linear Phillips curve as a benchmark. A typical Phillips curve includes inflation, unemployment, and supply “shocks”, which are Phillips curve shifters such as food and energy prices.

$$\Delta\pi_t = \alpha + \varphi_\pi(L)\Delta\pi_{t-1} + \varphi_m(L)[\pi_{t-1} - (w_{t-1} - \lambda_{t-1})] + \varphi_U(L)U_{t-1} + \varphi_{FE}(L)FE_{t-1} + \varepsilon_t \quad (1)$$

Equation (1) is a standard reduced-form accelerationist Phillips curve, similar to Staiger, Stock, and Watson (2001). Specifying the dependent variable as the first difference of inflation rather than the level makes it “accelerationist”. In the equation, $\Delta\pi_t$ is the first difference of inflation (between time t and $t-1$), $[\pi_{t-1} - (w_{t-1} - \lambda_{t-1})]$ is the growth rate of the markup (or the error correction term), w is nominal wage growth, λ is productivity growth, U is the unemployment rate, FE is the growth rate of food and energy prices, $\varphi(L)$ are the lagged coefficients, α is the intercept, and ε is the error term. By assumption, the growth rate of the markup, which captures the effects of wages and productivity that underlay the Phillips curve, and food and energy prices do not directly affect NAIRU. That is, they are constructed as mean zero variables. NAIRU depends on the value of the intercept relative to the negative of the Phillips curve slope. By defining those variables as mean zero, they do not affect the estimated intercept. *Appendix A* details the data sources.

We first test for unit roots of each variable and the results are in *Table 1*. From *Table 1*, evidence suggests inflation is integrated of order one or I(1) and the other variables are stationary or I(0) over the sample. A unit root test of the first difference of inflation or $\Delta\pi$ suggests it is I(0). These results support the specification in equation (1) as each variable enters the equation as I(0). We estimate equation (1) by OLS with eight lags of each explanatory variable as selected using the Akaike information criterion and results are in *Table 2*.

Notice from *Table 2* that all of the coefficients are significant at a 1% significance level. NAIRU is the intercept divided by the negative of the sum of lagged coefficients of

the unemployment variable and calculated as $\frac{0.38886}{0.06457} = 6.02230\%$ using the *Table 2*

results. It is important to note this raises a question about the lack of inflationary pressure during the late 1990s and early 2000s. Since 1995, the unemployment rate was below 6% every quarter except for two. Surprisingly, inflation fell from about 2% in 1995 to a little over 1% in 1998 before rising to a little over 3% by the start of 2006. In essence if this baseline Phillips curve is to be believed, a near decade long unemployment rate below NAIRU (about 1% below) led to roughly a 1% rise in inflation. This raises the issue of NAIRU’s relevance that has long been questioned by Galbraith (1997) and others, as sustained unemployment below NAIRU did not generate much inflation. Either that, or NAIRU itself has fallen (trended downward), which is the common empirical solution (Gordon 1998 or Staiger et al 2001). Or, there were a decade long set of shocks that kept inflationary pressure in check (which does not seem likely). Without

denying the possibility that NAIRU itself trended downward, this lack of the baseline Phillips curve to provide an explanation for the US recent historical experience suggests the possibility that a non-linear model might do better.

3. STR Model and Testing Strategy

The alternative to the linear Phillips curve in the previous section is a nonlinear curve that takes the following form, which is the STR model.

$$\Delta\pi_t = \phi_0 + \phi_\pi(L)\Delta\pi_{t-1} + \phi_m(L)[\pi_{t-1} - (w_{t-1} - \lambda_{t-1})] + \phi_U(L)U_{t-1} + \phi_{FE}(L)FE_t + \{\theta_0 + \theta_\pi(L)\Delta\pi_{t-1} + \theta_m(L)[\pi_{t-1} - (w_{t-1} - \lambda_{t-1})] + \theta_U(L)U_{t-1} + \theta_{FE}(L)FE_t\}F(z_{t-d}, \gamma, c) + v_t \quad (2)$$

In equation (2), the ϕ and θ terms are the coefficients to be estimated (including lags), v is the error term, and the function $F(z_{t-d}, \gamma, c)$ is the “transition function”, which defines the nonlinearity. Notice that if the transition function were a simple indicator function taking on values of 0 or 1, then equation (2) collapses to a simple discrete regime-switching model as in Hamilton (1994). Here, the functional form is continuous. This allows the economy to not be fully in either regime. At any point in time, the model will be a weighted average of the two regimes where the weight is determined endogenously through the transition function. This is a particularly attractive feature here as the Phillips curve is presumably derived from micro-level wage-setting and price-setting behavior. Once that behavior is aggregated to the macro-level, it is likely to be smoothed.

Typical functional forms for $F(z_{t-d}, \gamma, c)$ are the logistic or the exponential function (Anderson and Terasvirta 1992 and Escribano and Jorda 1998). For example, with the logistic function, equation (2) would be equation (3).

$$\Delta\pi_t = \phi_0 + \phi_\pi(L)\Delta\pi_{t-1} + \phi_m(L)[\pi_{t-1} - (w_{t-1} - \lambda_{t-1})] + \phi_U(L)U_{t-1} + \phi_{FE}(L)FE_t + \{\theta_0 + \theta_\pi(L)\Delta\pi_{t-1} + \theta_m(L)[\pi_{t-1} - (w_{t-1} - \lambda_{t-1})] + \theta_U(L)U_{t-1} + \theta_{FE}(L)FE_t\} \frac{1}{1 + e^{-\gamma(z_{t-d} - c)}} + v_t \quad (3)$$

The variable z_{t-d} is the transition variable driving the transition function. The variable z_{t-d} will be one of the lagged explanatory variables where d is the lag length. This allows for the transition between regimes to be endogenously determined. The parameter γ measures the slope or speed of the transition between regimes and c is the threshold parameter that defines the location of the transition function. When z_{t-d} equals c , the economy is defined as halfway or an evenly weighted average between the two regimes.

In effect, equation (2) and equation (3) are linear Phillips curves with interactive, nonlinear terms appended to them with the properties of the nonlinearity defined by $F(z_{t-d}, \gamma, c)$. A simple testing methodology would be to estimate equation (2) and conduct a hypothesis test of whether the nonlinear function $F(z_{t-d}, \gamma, c)$ is 0 or not. That test would be whether γ , the slope parameter, is zero or not. However, with that hypothesis test, the θ terms and c are not identified under the null. Alternatively, we could jointly test whether the θ terms are zero or not, but then γ and c are not identified

under the null (Luukkonen, Saikkonen, and Terasvirta 1988, Terasvirta 1994, and Escribano and Jorda 1998).

Luukkonen et al (1988) suggest replacing $F(z_{t-d}, \gamma, c)$ with its Taylor series approximation. They construct a Lagrange Multiplier test and show that it is asymptotically distributed χ^2 . In practice, the test is usually approximated as an F-distribution because of its small sample properties (Escribano and Jorda 1998). The most parsimonious approximation to use would be a first-order Taylor series approximation. However, as discussed in Luukkonen et al (1988), the test has low power when θ_0 (see equation 2) and the other θ terms are small in magnitude. Rather than add a full higher order approximation and reduce power of the test by using up degrees of freedom, the solution advocated in Luukkonen et al (1988) is to augment the first order approximation with a few selected higher order Taylor series terms. Building on Luukkonen et al (1988), Escribano and Jorda (1998) suggest augmenting the first order approximation with up to fourth-order terms for increased power. That is the testing strategy we follow and is described in Escribano and Jorda (1998) as test NL4A.

Replacing $F(z_{t-d}, \gamma, c)$ with its first-order Taylor series approximation, augmented with up to fourth order terms, defined as $T(z_{t-d})$, changes equation (2) to equation (4).

$$\Delta\pi_t = \delta_0 + \delta_\pi(L)\Delta\pi_{t-1} + \delta_m(L)[\pi_{t-1} - (w_{t-1} - \lambda_{t-1})] + \delta_U(L)U_{t-1} + \delta_{FE}(L)FE_t + \{\psi_0 + \psi_\pi(L)\Delta\pi_{t-1} + \psi_m(L)[\pi_{t-1} - (w_{t-1} - \lambda_{t-1})] + \psi_U(L)U_{t-1} + \psi_{FE}(L)FE_t\}T(z_{t-d}) + \omega_t \quad (4)$$

where the δ and ψ terms are the coefficients to be estimated and ω is the error term. The hypothesis test is then whether the estimated ψ coefficients are jointly zero or not and is evaluated using the F -statistic. Rejection of the null is a rejection of linearity. Equation (4) is estimated by OLS for each possible transition variable z_{t-d} and this hypothesis test conducted. Again the possible transition variables are each of the lagged explanatory variables. Results from these tests are in *Table 3*.

The most striking result from *Table 3* is that linearity is rejected at least at a 10% level when the first difference of inflation ($\Delta\pi$) and when the growth rate of food and energy prices (FE) are the transition variables. This is true up through the 5th lag of each variable. This is fairly strong evidence in favor of nonlinearity. The other striking result from *Table 3* is that the unemployment rate (U) is not a good candidate for the transition variable. The best candidate is FE_{t-2} as the F-statistic is largest (p-value is the smallest) for the corresponding hypothesis test. Of course there is an economic question as to why this variable drives the nonlinearity. One possibility is that monetary policy is particularly sensitive to food and energy price moves. Given that monetary policy affects the economy with a lag, their response to this variable may be what we capture. This is an area where further research is needed.

Based on the evidence from the nonlinearity tests that a nonlinear model is appropriate, the next step is to choose a functional form for the transition function, $F(z_{t-d}, \gamma, c)$. As mentioned earlier, the typical functional forms are the exponential and logistic functions. To decide between these two, we implement the suggested procedure in Escribano and Jorda (1998), which is a variation of the procedure outlined in Terasvirta (1994). The procedure is to estimate equation (4) using our selected transition

variable, conduct two separate hypotheses tests, and then compare the results. The tests are of the joint significance of the coefficients on the even terms and the odd terms in the Taylor series approximation respectively. Recall that the Taylor series approximation is a first-order approximation with higher order terms added. Thus an “even” term would be the coefficient on z_{t-2}^2 and an “odd” term would be the coefficient on z_{t-2}^3 . Each test is an F-test. If the p-value is smaller for the test on the coefficients of the even terms, then the exponential is the appropriate choice. If it is the reverse then the logistic is the appropriate choice. When we conducted the test, the logistic function was chosen (p-value of 0.09 on the “odd” test versus a 0.16 p-value on the “even” test).

4. Results and Analysis

Based on the results from *Section 3*, equation (5) is the asymmetric Phillips curve.

$$\Delta\pi_t = \phi_0 + \phi_\pi(L)\Delta\pi_{t-1} + \phi_m(L)[\pi_{t-1} - (w_{t-1} - \lambda_{t-1})] + \phi_U(L)U_{t-1} + \phi_{FE}(L)FE_t + \{\theta_0 + \theta_\pi(L)\Delta\pi_{t-1} + \theta_m(L)[\pi_{t-1} - (w_{t-1} - \lambda_{t-1})] + \theta_U(L)U_{t-1} + \theta_{FE}(L)FE_t\} \frac{1}{1 + e^{-\gamma(FE_{t-2} - c)}} + v_t \quad (5)$$

We estimate equation (5) by nonlinear least squares (NLS) and the results are in *Table 4*. To aid in the presentation and analysis of the results, define the “low inflation regime” to

be when $\frac{1}{1 + e^{-\gamma(FE_{t-2} - c)}} = 0$ and the “high inflation regime” to be when

$\frac{1}{1 + e^{-\gamma(FE_{t-2} - c)}} = 1$. Again, it is important to bear in mind that the Phillips curve will be

a weighted average of these two regimes at any point in time. We report coefficient sums based on these two regimes in *Table 4*.

Table 4 shows stark differences between the low inflation regime and the high inflation regime. In the low inflation regime, the Phillips curve is downward-sloping (coefficient sum of -0.15) whereas in the high inflation regime, the Phillips curve is slightly upward-sloping (coefficient sum of 0.04). Also, note that based on the estimated intercepts, the Phillips curve in the high inflation regime is shifted inward relative to the low inflation regime. In the low inflation regime, the markup growth rate is not statistically significant (p-value of 0.87) but it is in the high inflation regime (p-value of 0.02). Rising food and energy prices actually decelerate inflation in the low inflation regime. This contrasts with the high inflation regime where rising food and energy prices accelerate inflation. Finally it is worth noting that the slope of the transition function (gamma) is relatively imprecisely estimated. Gamma is the speed or how quickly the economy moves between the high and low regimes.

The coefficient, c , is the location of the transition, which is statistically significant at a 1% level. The interpretation is that if $FE_{t-2} = 0.19$, the economy is characterized as halfway between the two regimes or an evenly weighted average of the two regimes. For example, the Phillips curve slope would be -0.055 at that instant in time. If $FE_{t-2} > 0.19$, more weight is placed on the high inflation regime. If $FE_{t-2} < 0.19$, more weight is placed on the low inflation regime. Finally, it is worth noting that in the low inflation regime,

the estimate of NAIRU is 5.06% and in the high inflation regime, the estimate of NAIRU is 7.98%. NAIRU at any point in time will be a weighted average of those values. When $FE_{t-2} = 0.19$, NAIRU will be an equally weighted average or 6.52%.

What is the relationship between this Phillips curve (which is a weighted average of two linear Phillips curves) and the business cycle? To see this relationship, we graph the estimated transition function, which is equation (6). The graph of the function is *Figure 1*.

$$\frac{1}{1 + e^{-9.68(FE_{t-2} - 0.19)}} \quad (6)$$

In *Figure 1*, the shaded areas (gray bars) are recession periods as defined by the National Bureau of Economic Research (NBER). There are a few notable facts from *Figure 1*.

First, there is a tendency for recession periods to be associated with the high inflation regime or a move towards the high inflation regime. This move often starts prior to the onset of a recession. In other words, there is a tendency for the Phillips curve to become relatively flatter and shift inward toward the end of an expansion and into a recession compared to the start of an expansion. One exception to that is the 1981-1982 recession, where the Phillips curve became flatter in the brief 1980 recession beforehand and remained so through the brief expansion in-between. At the start of an expansion, the economy tends to be in the low inflation regime, where the Phillips curve is relatively steeper and shifted outward compared to the high inflation regime. This is especially prevalent at the start of the long 1980s expansion.

In the latter half or end of an expansion, there appears to be increased volatility in the transition function. This indicates more frequent movements from one regime to another. However, it does seem that the high inflation regime is more prevalent in the second half of expansion periods than not. Also, it is interesting to note that the long 1960s expansion and the long 1990s expansion exhibit a very similar pattern, with a noticeable spike. This implies a quick flattening of the Phillips curve partway through the expansion period.

Table 5 shows summary statistics for the slope of the estimated Phillips curve during different portions of the business cycle (again using the NBER business cycle dates). Each expansion phase is split in two and denoted “early expansion” and “late expansion” respectively. In expansion phases where there are an odd number of quarters, the middle quarter is considered part of the early expansion phase.

Table 5 largely confirms what we observed in *Figure 1*. In the late expansion phase of the business cycle and in a recession, the Phillips curve is relatively flat compared to the early expansion phase. Further, the standard deviation of the transition function is larger in the late expansion phase and in the recession phase compared to the early expansion phase. Hence, the slope of the Phillips curve is more prone to change during the end of an expansion or a recession than during the beginning of an expansion.

5. Conclusion

To conclude we discuss future avenues of research that may follow from these preliminary results. It will be useful to check for robustness using different measures of inflation (such as core inflation), different measures of aggregate demand (such as the output gap), and even different supply shocks (such as import prices or exchange rates). It may be useful to check robustness over sub-samples as well. Additionally, it will be of interest to incorporate the so-called time-varying NAIRU that has come to dominate reduced-form specifications. The forecasting ability of this nonlinear model relative to other models is of interest as well.

Beyond the mentioned empirical issues, there are theoretical issues too. For example, a better understanding of the underlying economic forces driving the transition and driving the asymmetry is needed. One possibility is that monetary policy is related to this Phillips curve nonlinearity and a better understanding of how is needed. Further, the implications of the nonlinearity for the conduct of monetary policy is also needed.

Overall, using the STR nonlinearity testing strategy, we have found evidence that the U.S. Phillips curve is nonlinear. We have found evidence that the nonlinearity is tied to the business cycle in that the Phillips curve tends to flatten and shift inward during late expansion periods and recessions. A direct implication is that NAIRU varies with the business cycle.

A. Data Appendix

Data is from the Haver Analytics database. Monthly series were converted to quarterly frequency using Eviews. The common lag-adjusted quarterly sample is 1960:1 to 2006:1.

Price Level: Gross Domestic Product, Implicit Price Deflator Index (SA, 2000=100), 1947:1 to 2006:1, Bureau of Economic Analysis (BEA), *gdgp*.

Price Level without Food and Energy: CPI-U: All Items Less Food and Energy (SA, 1982-84=100), 1957:01 to 2006:04, Bureau of Labor and Statistics (BLS), *pcuslfe*.

Productivity: Nonfarm Business Sector: Output per Hour/All Persons (SA, 1992=100), 1947:1 to 2006:1, BLS, *lnfa*.

Unemployment Rate: Civilian Unemployment Rate (Percent), 1948:01 to 2004:05, BLS, *lr*.

Wage Level: Nonfarm Business Sector, Unit Labor Cost Index (SA, 1992=100), 1947:1 to 2006:1, Bureau of Labor and Statistics (BLS) *lnfu*.

If the level of a variable is X_t , then its growth rate is calculated as $400 \ln \left(\frac{X_t}{X_{t-1}} \right)$. Food

and energy price growth is constructed as the growth rate of the price level minus the growth rate of the price level without food and energy.

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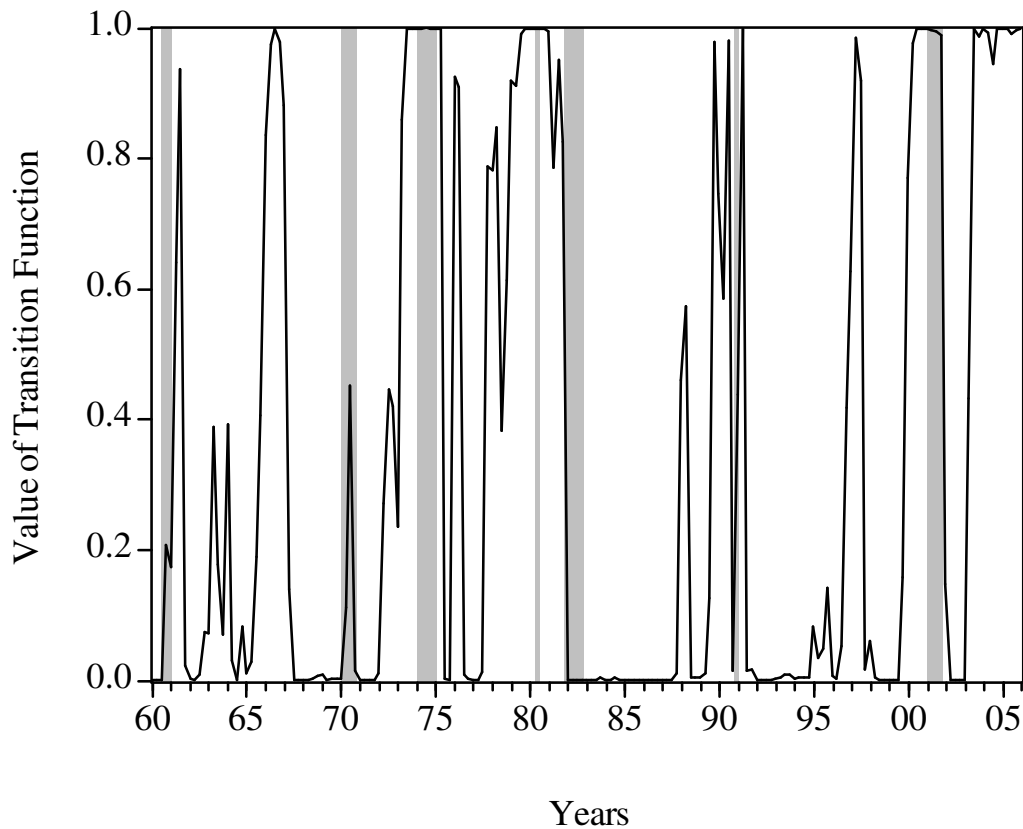
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Figures

Figure 1
Estimated Transition Function
1960:1 to 2006:1



Tables

Table 1
Unit Root Tests, 1960:1 to 2006:1

Variable	π	$\Delta\pi$	$\pi - (w - \lambda)$	U	FE
T-Statistic	-1.48	-6.23	-3.25	-3.03	-3.35
[p-value]	[0.54]	[0.00]	[0.02]	[0.03]	[0.01]

Notes: Unit root test is an augmented Dickey-Fuller test that includes a constant. The null hypothesis is that the series contains a unit root.

Table 2
Baseline Phillips Curve

Variable	Coefficient or Coefficient Sum [p-value]
α	0.39 [0.00]
$\Delta\pi$	-0.11 [0.00]
$\pi - (w - \lambda)$	-0.03 [0.01]
U	-0.06 [0.00]
FE	0.07 [0.00]
$NAIRU$	6.02
Sample	1960:1 to 2006:1
Adjusted R-squared	0.58
Jarque-Bera	[0.55]
ARCH	[0.94]

Notes: The null hypothesis of the Jarque-Bera test is that the error term is normally distributed. The null hypothesis of the ARCH test is that the model contains no ARCH(1) effects. Both p-values indicate a failure to reject the null.

Table 3
P-Values from Nonlinearity Tests

Delay Parameter (<i>d</i>)	Transition Variable (<i>z</i>)			
	$\Delta\pi$	$\pi - (w - \lambda)$	<i>U</i>	<i>FE</i>
1	0.03	0.01	0.36	0.09
2	0.02	0.25	0.51	0.00
3	0.02	0.13	0.58	0.00
4	0.02	0.10	0.60	0.00
5	0.00	0.01	0.53	0.06
6	0.19	0.42	0.56	0.14
7	0.29	0.44	0.53	0.10
8	0.64	0.31	0.75	0.42

Notes: The p-values are from F-tests where the test statistic is distributed $F(35,117)$. There are 185 observations. Rejection of the null is a rejection of linearity.

Table 4
STR Phillips Curve Estimation Results

Coefficients	Estimated Coefficient or Sum of Coefficients [p-value]	Coefficients	Estimated Coefficient or Sum of Coefficients [p-value]
γ	9.68 [0.10]	<i>c</i>	0.19 [0.01]
Low Inflation Regime	$\frac{1}{1 + e^{-\gamma(FE_{t-2} - c)}} = 0$	High Inflation Regime	$\frac{1}{1 + e^{-\gamma(FE_{t-2} - c)}} = 1$
ϕ_0	0.76 [0.00]	$\phi_0 + \theta_0$	-0.29 [0.00]
$\phi_\pi(L)$	-0.28 [0.00]	$\phi_\pi(L) + \theta_\pi(L)$	-0.59 [0.00]
$\phi_m(L)$	0.03 [0.87]	$\phi_m(L) + \theta_m(L)$	-0.03 [0.02]
$\phi_U(L)$	-0.15 [0.00]	$\phi_U(L) + \theta_U(L)$	0.04 [0.00]
$\phi_{FE}(L)$	-0.22 [0.03]	$\phi_{FE}(L) + \theta_{FE}(L)$	0.20 [0.00]
<i>NAIRU</i>	5.06	<i>NAIRU</i>	7.98
Adjusted R-squared	0.68	Sample	1960:1 to 2006:1
Jarque-Bera	[0.30]	ARCH	[0.35]

Notes: The null hypothesis of the Jarque-Bera test is that the error term is normally distributed. The null hypothesis of the ARCH test is that the model contains no ARCH(1) effects. Both p-values indicate a failure to reject the null.

Table 5
Transition Function Summary Statistics

Business Cycle Phase	Mean	Median	Standard Deviation	Median Phillips Curve Slope	Observations
Recession	0.51	0.45	0.46	-0.07	24
Expansion	0.33	0.02	0.42	-0.15	161
Early Expansion	0.19	0.01	0.35	-0.15	84
Late Expansion	0.48	0.45	0.43	-0.07	77