Economics Bulletin

Volume 29, Issue 2

A malthusian model for all seasons

Paul R Sharp
University of Copenhagen

Jacob L Weisdorf
University of Copenhagen

Abstract

An issue often discussed in relation to agricultural development is the effect on agricultural labour productivity of more intensive land-use. Introducing aspects of seasonality into a stylized Malthusian model, we unify two diverging views by showing that labour productivity may go up or down with agricultural intensification, depending on whether technological progress emerges in relation to cultivation or harvesting activities. Our result rests on evidence reported by Boserup (1965) and others, which suggests that harvest seasons in traditional agriculture are characterized by severe labour shortage.

Citation: Paul R Sharp and Jacob L Weisdorf, (2009) "A malthusian model for all seasons", *Economics Bulletin*, Vol. 29 no.2 pp. 769-774.
Submitted: Feb 15 2009. **Published:** May 03, 2009.

1 Introduction

The so-called 'Malthusian' model is frequently used by development economists, growth economists and economic historians to capture development in a traditional agricultural society. An issue often discussed in relation to agricultural development—but about which the Malthusian model is silent—is the effect on agricultural labour productivity of more intensive land-use. According to Hunt (2000), scholars divide into two camps. On the one hand, Boserup (1965) makes a strong case that agricultural intensification raises labour costs per unit of food produced. In her view, therefore, agricultural labour productivity is *negatively* correlated with land productivity, a notion referred to by Hunt as the 'decline thesis'. Other scholars, however, have pointed to the nearly fourfold increase of England's population level between 1500 and 1800 and compared it to the halving of the share of England's population employed in agriculture over the same period as convincing evidence that agricultural labour productivity is *positively* correlated with land productivity. Following Hunt, this position has become known as the 'rise thesis'.

In this paper, we demonstrate that the 'rise thesis' and the 'decline thesis' can both be understood within the context of a single framework through an acceptance of the importance of seasonality in agriculture. This we demonstrate by adding two salient features about traditional agriculture to a stylized Malthusian model. The first feature involves making a distinction between two types of seasonal activities in agriculture: cultivation and harvesting. The second feature builds on evidence reported by Boserup (1965) and others, which suggests that land productivity in traditional agriculture is limited, not by how much food can be grown, but by how much labour is available in the harvest season. In our seasonality-augmented version of the Malthusian model, the sign of the correlation between land and labour productivity depends simply on the relative rate of growth of technological progress in cultivation and harvesting activities.

2 The model

Following Clark (2007), a defining characteristic of a traditional agricultural society is that technological progress over the long run permits a larger population but does not affect income per capita which remains at the level of subsistence. The stylized Malthusian model captures this through the use of two components: (i) a positive relationship between income and population growth, and (ii) diminishing returns to labour in production. Our modified version of the Malthusian model builds on both components. The former is described in detail in the following; the latter is described further below.

Consider a one-sector, one-good agricultural economy. Economic activities extend over infinite (discrete) time. Unless explicitly noted, all variables are considered in period t. In period t, the population consists of N_t identical individuals. Consistent with the stylized Malthusian model, the reproductive success of an individual is 'checked' by the size of its income. Symbolically, the number of surviving children of an individual is given by the functional relationship $n = n(w)$, where w is income per individual and where $n(\cdot)$ is continuous and monotonic, with $n(0) = 0$ and $n(\infty) > 1$. This implies that a 'subsistence' income level exists, defined as the income level at which an individual is able to raise exactly one surviving offspring. Since all individuals are identical, change in the size of the labour force between any two periods is given by $N_{t+1} = n_t N_t$.

To introduce aspects of seasonality, suppose that goods are generated according to a two-step procedure. This involves first the production of intermediate goods (think: crops cultivated for the purpose of harvesting), then the production of final goods (think: harvested crops). Thus, there are two types of seasonal activities in the model: cultivation (seeding, weeding, harrowing, hoeing etc) and harvesting.

Suppose that the production of output is subject to constant returns to land and labour. Each individual is endowed with one unit of labour, which, as will be explained below, is divided endogenously (but, for the individual, parametrically) between output-generating activities and unemployment. Consistent with the stylized Malthusian model, land, measured by X, is in fixed supply, and is normalized to one ($X \equiv 1$). For simplicity, there is no property rights over land, and each period the land mass is divided equally between the individuals. In symbolic terms, the land mass per individual is thus given by $x = X/N \equiv 1/N$.

Let subscript C and H denote *Cultivation* (intermediate goods) and *Harvesting* (final goods) activities, respectively. Output per individual in season $i \in \{C, H\}$ can thus be written as

$$y_i = A_i \left(\gamma_i e_i\right)^{\alpha} \left(x\right)^{1-\alpha} \equiv A_i \left(\gamma_i e_i\right)^{\alpha} \left(\frac{1}{N}\right)^{1-\alpha}, \quad \alpha \in (0,1), i \in \{C, H\}, \tag{1}$$

where $\gamma_i \in (0,1)$ is the fraction of the year for season $i \in \{C, H\}$ (implicitly, therefore, $\gamma_C + \gamma_H \leq 1$); $e_i \in (0,1)$ is the labour input of an individual in season $i \in \{C, H\}$; and A_i is the level of technology (total factor productivity) in season $i \in \{C, H\}$. It follows that income per individual is equal to the individual's final output, i.e. $w = y_H$.

Time-budget studies done in traditional agricultural societies suggest that harvest seasons are normally characterized by severe labour shortage (hence the term *peak* season), and that the harvest season labour supply, therefore, sets the upper limit to agricultural output (see e.g. Boserup 1965; Jones 1964). Following this approach, individuals in the model employ their entire labour resources in the harvesting season (i.e. $e_H = 1$). Assuming absence of uncertainty, the number of intermediate goods produced equals the number of final goods that can be harvested, i.e. $y_C = y_H$. It then follows that the labour input per individual needed in the cultivation season (for intermediate goods production) is given by

$$e_C = \frac{\gamma_H}{\gamma_C} \left(\frac{A_H}{A_C}\right)^{\frac{1}{\alpha}}.$$

Notice the season-specific effects of technological progress on the demand for labour in cultivation activities. Technological progress that occurs in relation to *harvesting* activities is labour-*demanding* (when more final output can be generated in the harvest season, it takes more labour input in the cultivation season to produce more intermediate goods). Technological progress that occurs in relation to *cultivation* activities, by contrast, is labour-*saving* (it needs less labour input to generate the number of intermediate goods needed in the harvest season).

With reference to seminal work by Lewis (1954), the labour resources *not* used for output-generating activities, i.e. $\gamma_c (1 - e_c)$, can be thought of as surplus labour or disguised unemployment.

3 Stability and Comparative Statics

We want to analyze the correlation between land and labour productivity in a steady state. In a steady state, all variables grow at a constant rate (possibly zero). Specifically, this means that the steady state population growth rate is constant. A constant rate of growth of population implies that income per individual must be constant over time. For tractability of the analysis, we parameterize the Malthusian relationship between income and population growth, so that $n(w) = w$. Then, using (1), it follows that the gross rate of growth in income per individual is given by

$$\frac{w_{t+1}}{w_t} = \frac{A_H (\gamma_H)^{\alpha} \left(\frac{1}{n_t N_t}\right)^{1-\alpha}}{A_H (\gamma_H)^{\alpha} \left(\frac{1}{N_t}\right)^{1-\alpha}} = \left(\frac{1}{w_t}\right)^{1-\alpha}.$$

Hence, in steady state (marked with an asterisk), income per individual is $w^* = 1$. Notice that stability of the steady state requires that $\partial w_{t+1}/\partial w_t \mid_{w_t = w^*} < |1|$, a condition that is fulfilled when $\alpha \in (0,1)$, which is equivalent to having diminishing returns to labour in production.

By use of the parameterization above, having $w^* = 1$ means that each individual is able to raise exactly one surviving offspring, i.e. $n^* = 1$. In steady state, therefore, there is no population growth, so the population level is constant ($N_{t+1} = N_t$). Thus, by construction, $w^* = 1$ is the subsistence income. Such a steady state is normally referred to as a 'Malthusian' equilibrium.

In the following, we calculate land and labour productivity in steady state. Using (1), it follows that land productivity—defined as final output per unit of land—in steady state is

$$D^* \equiv \frac{y_H}{x} = ((\gamma_H)^{\alpha} A_H)^{\frac{1}{1-\alpha}}. \tag{2}$$

Labour productivity—defined as final output per unit of land divided by labour input per unit of land—in steady state is

$$E^* \equiv \frac{y_H/x}{(1/x) \cdot \sum_{i=\{C,H\}} \gamma_i e_i} = \frac{1}{\gamma_H (1 + (\frac{A_H}{A_L})^{1/\alpha})} \tag{3}$$

Notice once again the season-specific effects of technological progress. Starting in a steady state, a positive shock to harvesting technology means that individuals can produce more final output, and, therefore, need to cultivate more intermediate goods. In the absence of technological progress in cultivation activities, individuals thus have to put in more labour in the cultivation season. Altogether, therefore, the annual labour input of an individual goes up in response to technological progress in harvesting activities.

At the same time, though, because each individual also generates more final output, at least to begin with, income per individual increases. This leads to growth in the size of the population. However, more people means less land per individual, which ultimately drives down income per individual until population growth is brought to a halt. Eventually, therefore, the economy returns to a steady state with a higher population level, but where income per individual is back at the level of subsistence, a result identical to that of the stylized Malthusian model (see Clark 2007).

However, the introduction of seasonal aspects allows us to take the analysis one step further. Since output per individual over the long run is unaffected whereas individual labour input has gone up, a positive shock to harvesting technology in the current model *increases* land productivity but *reduces* labour productivity. According to the present framework, therefore, if technological progress occurs only in relation to harvesting activities, then the 'decline thesis' holds true.

Meanwhile, we still need to analyze the effects on land and labour productivity of technological progress in relation to cultivation activities. It follows from (2) that a positive shock to cultivation technology has no effect on land productivity. However, technological progress in cultivation means that the amount of intermediate goods needed for the harvesting season can now be generated using less labour. Obtaining the same output using less labour input, labour productivity, therefore, has gone up as a result of technological progress in cultivation.

The latter result helps shed light on how land and labour productivity can be *positively* correlated. To see this, let $g_i \geqq 0$ denote the net rate of growth of technology in season $i \in \{C, H\}$. It then follows from (2) and (3) that, if $g_H > g_C \geq 0$, then the 'decline thesis' applies. By contrast, if $g_C > g_H > 0$, then the 'rise thesis' applies. In words: if the labour-demanding effect of technological progress in harvesting is dominated by the labour-saving effect of technological progress in cultivation, then land and labour productivity are *positively* correlated, and the 'rise thesis' is valid. If, by contrast, the labour-saving effect is dominated by the labour-demanding effect, then land and labour productivity are *negatively* correlated, and the 'decline thesis' is valid.

4 Conclusion

This paper introduces aspects of seasonality in agriculture to a stylized Malthusian model. The analysis shows that labour productivity in agriculture may go up or down with agricultural intensification, depending on the relative rate of growth of technological progress in harvesting and cultivation activities, respectively. The theory should be testable through an analysis of changes in relative seasonal wage rates following season-specific technological progress in traditional agriculture.

References

[1] Boserup, E. (1965) *The Conditions of Agricultural Growth*, Earthscan: London.

[2] Clark, G. (2007) *A Farewell to Alms: A Brief Economic History of the World*, Princeton University Press: Princeton.

[3] Hunt, R.C. (2000) 'Labor Productivity and Agricultural Development: Boserup Revisited' *Human Ecology* **28**, 251-77.

[4] Jones, E.L. (1964) *Seasons and Prices: The Role of the Weather in English Agricultural History*, George Allen & Unwin: London.

[5] Lewis, W.A. (1954) 'Economic development with unlimited supplies of labour' *Manchester School of Economic and Social Studies* **22**, 139-191.