Abstract

Using the basic overlapping generations one-sector model of endogenous growth we show that unionisation of labour markets may be growth-enhancing with respect to the standard competitive equilibrium economy with full employment, provided the capital's weight in technology and the replacement rate are both high enough. Moreover, a growth-maximising value of the union’s relative wage intensity does exist. In particular, a wage-oriented rather than an employment-oriented union should be preferred as an inducement to a higher per capita income growth. Therefore, an appropriate combination of both union’s behaviour and government policies may trigger a virtuous growth mechanism. A policy implication is that the government could follow the union’s growth-maximising rule simply by choosing properly the replacement rate. Moreover, along the balanced growth path, individuals can be better off in a unionised economy with unemployment rather than in the competitive economy with full employment.

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1. Introduction

The persistently high rates of unemployment in Europe, which, together with the low rates of per capita income growth – at least relative to those experienced in both the US and several Asian countries –, represent a major concern for current policies, are often associated with the existence of unionised labour markets. Although the centrality of unemployment and economic growth in macroeconomics is undoubted, most of the related economic literature has devoted to each of them separately, and only a few papers have investigated the effects of unemployment on economic growth in the same breath.

On the empirical ground, the sign of the correlation between economic growth and unemployment is controversial, either across countries or over long periods of time in the same country. In particular, to sum up the results of the empirical literature we may distinguish three cases: (i) the correlation is essentially zero, e.g., Aghion and Howitt (1992), who reported that both high and low growth countries experienced lower unemployment rates relative to those with intermediate rates of productivity growth among the 20 OECD countries included in their study. Bean and Pissarides (1993), instead, found no correlation between unemployment and the measures of productivity growth across OECD economies; (ii) the correlation is negative, e.g., Hoon and Phelps (1997), Muscatelli and Tirelli (2001); and (iii) the correlation is positive, e.g. Caballero (1993) who reported a positive time series relationship between growth and unemployment both in the UK and US between 1966 and 1989, whereas Muscatelli and Tirelli (2001) accounted for a negative correlations for the five G7 economies but UK and US.

On the theoretical side, the common wisdom framed in the basic one-sector overlapping generations (OLG) model of growth argued essentially either for a negative relationship between unemployment and the rate of per capita income growth, e.g. Daveri and Tabellini (2000), or at most for a neutral one (e.g. Corneo and Marquardt, 2000). An exception is represented by Irmen and Wigger (2002), which instead found, under some rather restrictive technological conditions, that a positive relationship between economic growth and unemployment, as detailed below.

It must be noted, however, that in models at all different from the basic Cobb-Douglas one-sector OLG setting analysed here, some authors predict that the unionisation of labour markets might not necessarily be growth-reducing, owing to the existence of an interaction between heterogeneous agents and sectors, frictions, uncertainty, irreversibilities, agents’ infinite horizon, and so on. For instance: (i) Agell and Lommerud (1993) analysed a multi-sector economy where a trade union pursuing an egalitarian wage policy can increase the productivity growth by favouring a reduction in the wage differentials between low-productivity and high-productivity sectors; (ii) Bean and Pissarides (1993), building on a search model where unemployment is due to matching frictions in the labour market, showed that the relationship between economic growth and the relative workers’ bargaining power is ambiguous to the extent that a shift in income from entrepreneurs to workers is compensated by an increase in the saving rate and, hence, in the growth rate; (iii) Aghion and Howitt (1994), exploiting the Schumpeterian idea of creative disruption, concluded that the unemployment induced by labour market imperfections could enhance economic growth; (iv) de la Croix and Licandro (1995), analysing a model with irreversible decisions, showed that a rise in the union power, on the one hand, reduces the physical capital and, on the other hand, increases both the value of the firm and the physical capital, so that the final effect of unionisation on growth is ambiguous; (v) Palokangas (1996) analysed a two-sector model (the final good and the R&D sectors) with skilled and unskilled workers, and argued that under some rather stringent conditions (separated skilled and unskilled labour markets; skilled and unskilled workers are complements in

1 As a matter of fact, it is worth noting that the unionisation of labour markets may be growth-reducing even when an interaction between sectors exists. For instance, Quang and Vousden (2002), building on a four-sector OLG model (a competitive final goods sector; two intermediate goods – competitive and monopolistic – sectors; a competitive R&D sector), found that a rise in the union bargaining power in the monopolised intermediate-good sector reduces the growth rate of the economy.
the production of final goods; the R&D sector employs only skilled labour), a positive relationship between unionisation and economic growth may be possible when production in the R&D sector increases enough; (vi) developing an OLG model with two sectors (the consumption goods and the educational sectors), inter-generational altruism and human capital, Ramos-Parreno and Sánchez-Losada (2002) showed that the unionisation of labour markets in the consumption goods sector: (a) causes unemployment in that sector and (b) reduces the wage rate in the educational sector. The latter effect in turn implies a higher return from human capital investments leading definitively to (c) a rise in the production of human capital and (d) a higher rate of per capita income growth; (vii) assuming a static two-sector economy (the intermediate and the final goods sectors) with skilled and unskilled workers, Cardona and Sánchez-Losada (2006) showed that in some cases the unionisation of both skilled and unskilled labour may have a positive impact on the final goods production.

The present paper is framed in the basic one-sector OLG growth theoretical literature and investigates how unions affect economic growth and welfare in context where a Romer-type capital investment externality (assumed here to be the average per capita stock of capital installed in the whole economy) represents the engine of endogenous growth. In contrast with the prevailing past literature, we show that if the government provides an unemployment benefit to the young people when unemployed, the unionisation of labour markets can lead to a more rapid per capita income growth than that of a competitive equilibrium economy with full employment, in spite of a reduced employment rate.

Our model differ from the above mentioned literature framed in the one-sector OLG growth context mainly as regards the following two features: (i) the type of production externality, and (ii) the type of union behaviour. As regard the former, we assume that the Romer-type capital investment externality is given by the per capita stock of capital installed in the whole economy (e.g. Daveri and Tabellini, 2000; Irmen and Wigger, 2002) rather than expressed by the stock of capital in per worker terms (e.g. Corneo and Marquardt, 2000). As regards the latter, we follow both Corneo and Marquardt (2000) and Irmen and Wigger (2002) by considering a right-to-manage union which cares about wages and employment and then chooses unilaterally the wage rate, rather than postulating an only wage-interested monopoly union (e.g. Daveri and Tabellini, 2000).

In contrast with both the above cited literature and the present paper, where unions are assumed to be of either the right-to-manage type or the monopoly union type with firms being free to choose unilaterally the employment rate, some authors postulating the “efficient bargaining” model (McDonald and Solow, 1981; Clark, 1990), where firms and unions bargain over both wage and employment and the wage rate is only a fraction of total output with the rest being a pure profit, have argued that both in a two-period OLG context with either constant (Devereux and Lockwood, 1991) or increasing (Coimbra et al., 2005) returns to scale and in an infinite horizon context (Chang et al., 2007), an economy with unionised labour may have a better economic performance than a competitive labour market economy.

It must be emphasised that the beneficial effect of unionisation described in this paper hinges upon a different mechanism than those described above. In particular, while the results of our paper are intrinsically not comparable with those obtained in papers abstracting from the one-sector OLG structure or assuming an “efficient bargaining” framework, they may be related to the main results obtained by Irmen and Wigger (2002), and hence they deserve a brief discussion as to the comparison between them: different from Irmen and Wigger (2002), we assume a double Cobb-Douglas OLG economy and a government which redistributes resources within the same working

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2 We note that, although so far not evidenced in the economic literature, these two seemingly similar formulations lead to strongly different outcomes as regards the relationship between unionisation and economic growth, as shown in another companion paper in progress.

3 In particular, to note other distinctive features of these articles, Devereux and Lockwood (1991) also allowed for the existence of a stock market in a standard double Cobb-Douglas OLG economy, while Coimbra et al. (2005) assumed an OLG economy with a linear rather than with the usual well-behaved utility function also allowing for a labour-leisure choice. Chang et al. (2007), using a growth model with agents’ infinite horizon showed that a positive relationship between unionisation and economic growth may emerge by the nature of internal conflict within a political union.
age generation with an unemployment benefit policy at a balanced budget. We show that if both the technology of production is relatively capital-oriented, rather than labour-oriented (that is, the relative weight of capital is high enough), and the unemployment benefit is sufficiently high, then a relatively wage-oriented union may promote economic growth in spite of a reduced employment rate. Irmen and Wigger (2002), instead, found that an economy with unionised labour and unemployment may growth faster than the competitive equilibrium economy with full employment if and only if the sum of the elasticity of substitution between capital and labour in efficiency units and the output elasticity of labour in efficiency units is smaller than unity. Therefore, we may note that when production takes place according to a Cobb-Douglas technology, the beneficial effect of unionisation on economic growth described by Irmen and Wigger would not hold any longer, while our result would hold, a fortiori, by assuming, in line with Irmen and Wigger, a production function where capital and labour are relatively complement.

To sum up, the union-growth-enhancing mechanism described in this paper depends on the mutual relationship between the weight that the union attaches to wages and the size of the replacement rate (as part of the unemployment benefit system). In particular, if the latter is sufficiently high the saving rate increases along with the union’s relative wage intensity, leading definitively to a higher rate of per capita income growth in spite of a higher unemployment rate.

Our finding is, to the best of our knowledge, a novelty and contributes to fill a gap in the existing theoretical literature on endogenous growth with imperfect labour markets. Noteworthy, our conclusions are reached within the basic one-sector OLG model of growth with Cobb-Douglas utility and production functions where the usual Romer-type technological externality represents the engine of endogenous growth.

The policy implications are straightforward: depending on the mutual relationship between the capital’s weight in technology and the replacement rate, a growth-maximising value of the relative intensity between wages and employment in the union’s objective function exists; therefore, in order to promote both economic growth and welfare, the government should follow the union’s growth-maximising rule choosing appropriately the replacement rate.

The remainder of the paper is organised as follows. In Section 2 we develop the model and the main growth and welfare effects of unionisation are analysed and discussed; Section 3 concludes.

2. The model

2.1. Individuals

Consider an OLG economy populated by identical two-period lived individuals (Diamond, 1965). Life is divided into youth (working period) and old-age (retirement period) and, for simplicity, population is assumed to stay constant over time. Individuals belonging to generation $t$ ($N_t$) are endowed with a homothetic and separable lifetime utility function ($U_t$) defined over $c_{1, t}$ and $c_{2, t+1}$, i.e. working period consumption and retirement period consumption, respectively. Only young individuals join the workforce and supply inelastically one unit of labour on the labour market, while receiving a unitary wage income at the non-competitive rate $w_u$. Therefore, the labour market does not clear and involuntary unemployment occurs. The aggregate unemployment rate (defined in terms of fractions of time not worked) is $u_t = (N_t - L_t)/N_t$, where $L_t$ is the labour demand. Moreover, young age individuals are entitled to an unemployment benefit – defined to be

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4 In this model there is no uncertainty, hence each young agent will be employed for $1 - u_t$ units of time and unemployed for the remaining fraction $u_t$ (see Fanti and Gori, 2007). In other words, the unionisation reduces the employed time of each individual, rather than causing some individuals to be employed at higher wages and others to be unemployed. It is important to note, however, that this hypothesis is just the same as assuming two types of individuals,
a fraction of the prevailing competitive wage, that is, $b_t := zw_{e,t}$ with $0 < z < 1$ being the so-called replacement rate $\tau$, for the time left unemployed by the unionisation of the labour market.\(^5\) When old agents are retired and live on the proceeds of their savings ($s_t$) plus the accrued interest at the rate $r_{t+1}$.

Therefore, the representative individual born at time $t$ faces the following programme:

$$\max_{\{c_{1,t}, c_{2,t+1}\}} U_t = \ln(c_{1,t}) + \beta \ln(c_{2,t+1}),$$

subject to

$$c_{1,t} = w_{a,t}(1 - u_t) + b_t u_t,$$

$$c_{2,t+1} = (1 + r_{t+1}) s_t,$$

where $\tau_t > 0$ is a consumption tax and $0 < \beta < 1$ is the subjective discount factor, i.e., the higher $\beta$ is the more individuals prefer to smooth consumption over time. In other words, $\beta$ represents the degree of individual’s (im)patience to consume over the life cycle.

Maximisation of (P) thus gives the following first and second period of life consumption functions:

$$c_{1,t} = \frac{1}{(1 + \beta)(1 + \tau_t)} \left[ w_{a,t}(1 - u_t) + b_t u_t \right],$$

$$c_{2,t+1} = \frac{\beta}{1 + \beta} \left[ w_{a,t}(1 - u_t) + b_t u_t \right],$$

whereas the saving function is:

$$s_t = \frac{\beta}{1 + \beta} \left[ w_{a,t}(1 - u_t) + b_t u_t \right].$$

### 2.2. Firms

As in Romer (1986), and similarly with Daveri and Tabellini (2000) and Irmen and Wigger (2002), we assume the technology of production faced by each firm as:

$$Y_{it} = B k_{i,t}^{1 - \alpha} K_{i,t}^\alpha L_{i,t}^{1 - \alpha},$$

where the index $i$ denotes the typical firm, $Y_i$ is total output produced by firm $i$, $K_{i,t}$ and $L_{i,t}$ are the capital and the labour inputs hired in that firm, respectively, $k_t := K_t / N_t$ is the average capital per capita installed in the overall economy,\(^6\) which is taken as given by each single firm, $B > 0$ represents a scale parameter and $0 < \alpha < 1$ is the capital’s weight in technology. Setting $L_{i,t} = L_t$, $K_{t,i} = K_t$, and $Y_{i,t} = Y_t$, the time-$t$ aggregate production function takes place according to

$$\bar{y}_t = B k_t^{1 - \alpha} K_t^\alpha L_t^{1 - \alpha},$$

where $L_t = (1 - u_t) N_t$ is the total labour force employed at the aggregate level. Therefore, the intensive-form (per capita) aggregate production function may be written as

$$y_t = B k_t^{1 - \alpha} K_t^\alpha L_t^{1 - \alpha},$$

where $y_t := Y_t / N_t$. Knowing that (i) the stock of capital depreciates fully at the end of each period, and (ii) the price of final output is normalised to unity, profit maximisation leads to the following marginal conditions for capital and labour, respectively:

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\(^5\) Notice that we consider the replacement rate to be an exogenous policy parameter, while the unemployment rate is endogenously determined.

\(^6\) It is important to note that different from Daveri and Tabellini (2000) and Irmen and Wigger (2002), Corneo and Marquardt (2000) assumed the productivity parameter of the learning-by-investing externality to be equal to the average capital per worker rather than equal to the per capita stock of capital installed in whole economy.
\[ r_t = \alpha B (1-u_t)^{1-\alpha} - 1, \]  
\[ w_{a,t} = (1-\alpha)Bk_t(1-u_t)^{-\alpha}. \]  

Exploiting Eq. (6) the endogenous current unemployment rate is given by:

\[ u_t = 1 - \left( \frac{w_{c,t}}{w_{a,t}} \right)^{\frac{1}{\alpha}}, \]  

where \( w_{c,t} = (1-\alpha)Bk_t \) is the equilibrium competitive wage.

### 2.3. Unions: The right-to-manage hypothesis

The right-to-manage union programme is characterised as usual (see, for instance, Pencavel, 1984; Mezzetti and Dinopoulos, 1991; Booth 1995; Corneo and Marquardt, 2000; Layard et al., 2005) by the following constrained utility maximisation of the union’s members, where the preferences of the union are expressed over pairs of wages and levels of employment at time \( t \), that is:

\[
\max_{[w_{a,t}]} V_{a,t} = (w_{a,t} - w_{c,t})^\gamma \cdot (1-u_t)^{1-\gamma},
\]  

subject to the aggregate labour demand (7). \( \gamma > 0 < 1 \) measures the excess wage elasticity in the union’s objective function, i.e. it captures the union intensity in fixing the wage relative to employment, thus trading-off between high wages and low unemployment, i.e. the higher (lower) \( \gamma \), the more the union is wage-oriented (employment-oriented). \( \gamma < 1/(1+\alpha) = \tilde{\gamma} \).

Maximisation of (PP) yields:

\[ w_{a,t} = \frac{1-\gamma}{1-\gamma(1+\alpha)} w_{c,t} = \mu(\gamma) \cdot w_{c,t}, \]  

implying that the wage fixed by the union is a mark up over the equilibrium competitive wage. Notice that Eq. (8) implies that a finite positive solution to programme (PP) requires \( \gamma < 1/(1+\alpha) = \tilde{\gamma} \).

Exploiting Eq. (8), the constant unemployment rate reads as:

\[ u = 1 - \left[ \mu(\gamma) \right]^{\frac{1}{\alpha}}. \]  

From Eqs. (9) and (5) it is easy to see that the interest rate is a constant and, in particular, it is always lower than the corresponding value in the competitive equilibrium economy with full employment.

### 2.4. Government

The government runs an unemployment benefit policy at a balanced budget. We assume that only a proportional (non-distorting) tax on the consumption of the young at the rate \( \tau_y > 0 \) is levied and adjusted in every period to finance the unemployment benefit expenditure. \( \tau_y > 0.5 \) (\( \gamma < 0.5 \)). Therefore, the per capita time-\( t \) government constraint reads as:

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7 These right-to-manage rules imply that the union cares about wages and employment, while choosing unilaterally the wage rate, and under such a choice, firms unilaterally choose employment.

8 Notice that, given the OLG structure of the economy, programme (PP) implies that in each period only young-aged people are unionised. Moreover, the union cannot commit future generations to a certain wage policy.

9 The reference wage of the union is the competitive wage.

10 Following the terminology used by Mezzetti and Dinopoulos (1991), the union is assumed to be “wage oriented” (“employment oriented”) if \( \gamma > 0.5 \) (\( \gamma < 0.5 \)).

11 We have deliberately chosen a tax levied exclusively on the consumption of the young for two reasons: (i) a better analytical tractability, and (ii) in this way the nature of the unemployment benefit policy is purely redistributive, that is, consumption taxed away from the young rebated to the same individuals as a benefit for the unemployed time. This
\[ b_t u_t = \tau_t c_{1,t}, \]  
where the left-hand side represents the unemployment benefit expenditure and the right-hand side the tax receipts. Collecting Eqs. (1), (8) and (9) and knowing that \( b_t := z w_{t,j} \), the (constant) equilibrium consumption tax which balances the government budget is:

\[ \tau = \frac{(1 + \beta)z\left[\mu(\gamma)\beta - 1\right]}{\mu - \beta z\left[\mu(\gamma)\beta - 1\right]} . \]  

\[ 2.5. \, \text{Balanced growth} \]

We now close the model with the analysis of the balanced equilibrium growth. Given Eq. (10) and knowing that \( N_{t+1} = N_t \), the market-clearing condition in goods as well as in capital markets is expressed by the equality \( k_{t+1} = s_t \), i.e. the per capita stock of capital installed at time \( t+1 \) equals the amount of resources saved at time \( t \). Using Eq. (3) to substitute out for \( s_t \) into the equilibrium condition we get:

\[ k_{t+1} = \frac{\beta}{1 + \beta} \left[ w_{t_j}(1-u_t) + b_t u_t \right] . \]  

(12)

We are now wondering about the effects of a rise in the relative wage intensity of the union (and thus in the rate of unemployment) on the rate of economic growth. To this purpose, let us rewrite Eq. (12) as a generic function of the union wage preference parameter \( \gamma \) as:

\[ k_{t+1} = k_t \left[ w_{t_j}[\mu(\gamma)], u[\mu(\gamma)] \right] . \]  

(13)

The total derivative of Eq. (13) with respect to \( \gamma \) gives:

\[ \frac{dk_{t+1}}{d\gamma} = \frac{\partial k_{t+1}}{\partial w_{t_j}} + \frac{\partial \mu}{\partial \gamma} + \frac{\partial k_{t+1}}{\partial u} + \frac{\partial \mu}{\partial \gamma} + \frac{\partial \mu}{\partial \gamma} . \]  

(14)

Eq. (14) reveals that a rise in the union’s relative wage intensity has an ambiguous effect on capital accumulation and, hence, on the rate of per capita income growth. In particular, a rise in \( \gamma \) (i) increases the wage earned by the young when employed, thus promoting both savings and capital accumulation, and (ii) given the neoclassical labour market context of supply and demand, it reduces the employment rate and, hence, the saving rate. The final effect, therefore, depends on which of the two opposite forces (i.e. higher wages and lower employment) dominates. If the relative weight of capital in production and the replacement rate are both high enough, the positive wage-effect overcompensates the negative unemployment-effect, and hence the rate of per capita income growth in an economy with unionised labour and unemployment is definitively higher than the growth rate we had before unionisation in spite of a lower employment rate.

\[ \text{implies that the positive effect of unionisation on both economic growth and welfare – as shown in this paper – does not rely on any inter-generational transfer mechanisms which redistribute resources across generation, i.e. from the old-dissavers to the young-savers (as it would have been the case if, for instance, the unemployment benefit system was financed entirely or partially by either capital income taxes or lump-sum taxes on the elderly). The justification for taxes levied only upon the young people’s consumption may be more convincing, however, if the elderly are assumed live on tax-exempt pension funds. We acknowledge that analysis of social security in an economy with non-competitive labour is a promising direction of research.} \]

\[ ^{12} \text{It can readily be shown that the denominator of Eq. (11) is positive for any } \gamma < \gamma^* . \]

\[ ^{13} \text{Details are given in Appendix A.} \]
To analyse ultimately which of the two forces dominates and to characterise how the union’s relative wage intensity affects economic growth in this simple stylised economy, we combine Eqs. (8), (9) and (12) to obtain:

\[ k_{t+1} = \frac{\beta}{1+\beta} (1-\alpha)B \cdot H(\gamma)k_t, \]

(15)

where \( H(\gamma) := \left[ \frac{1-\gamma(1+\alpha)}{1-\gamma} \right]^{1-\alpha} + z - z \left[ \frac{1-\gamma(1+\alpha)}{1-\gamma} \right]^{1-\alpha}. \)

From Eq. (15) the growth rate of the per capita stock of capital in the unionised-wage economy with unemployment (which obviously coincides with the rate of per capita income growth since the unemployment rate is constant, see Eq. 9)\(^{14}\) may be expressed as:

\[ g_u(\gamma) = (1+g_c) \cdot H(\gamma) - 1, \]

(16)

with \( g_c = \frac{\beta}{1+\beta} (1-\alpha)B - 1 \) being the growth rate in the competitive equilibrium economy with full employment. Notice that \( g_u(\gamma) \) is independent of time so that the model does not show transitional dynamics, and thus a marginal change in the wage fixed by the union – as expressed by a marginal change in the union’s relative wage intensity \( \gamma \) – automatically implies an instantaneous adjustment of the economy to a new balanced growth path. It is worth noting that in this model both the unemployment rate and the interest rate (and hence the growth rate of the economy) are constant over time, that is, the model does not exhibit the convergence property. Therefore, a marginal change in the exogenous parameters of the model will imply an instantaneous jump of the economy from an old to a new balanced growth path. In this sense our model does not show transitional dynamics just like any \( Ak \)-typed models. In this paper we used the terminology widely adopted in the economic literature (e.g., amongst many others, Barro and Sala-i-Martin, 2004). Note, however, that a simple jump from a balanced growth path to another due to a permanent change in some exogenous parameters of the model may be thought to be a special type of transitional dynamics. In particular, a constant rate of per capita income growth does not guarantee any short-run effect (i.e., out of the balanced growth path) of changing the key parameters of the model.\(^{15}\)

From Eq. (16) it can readily be seen that if \( \gamma = 0 \) (i.e., no union’s power in fixing wages), then the growth rate in the unionised-wage economy and the growth rate in the competitive-wage economy coincide irrespective of the value of the replacement rate, that is \( H(0) = 1 \) and \( g_u(0) = g_c \) for any \( 0 < z < 1 \).

Below we show that depending on the mutual relationship between the replacement rate, the capital’s weight in production and the union relative wage intensity, a unionised-wage economy with unemployment may grow faster than a competitive-wage economy with full employment.

Analysis of Eq. (16) gives the following proposition:

**Proposition 1.** A necessary and sufficient condition for the existence of a positive union’s relative wage intensity such that a unionised-wage economy with unemployment grows faster than a competitive-wage economy with full employment is \( z > \bar{z} \).

\(^{14}\) In fact, the growth rate of per capita income in the unionised-wage economy can easily be expressed as:

\[ g_u, y(\gamma) = \frac{y_{t+1} - y_t}{y_t} = \frac{B(1-u)^{-\alpha} \cdot (k_{t+1} - k_t)}{B(1-u)^{-\alpha} \cdot k_t} = \frac{k_{t+1} - k_t}{k_t} = g_{u,k}(\gamma) = g_c(\gamma). \]

\(^{15}\) Assume, for instance, that the union’s relative wage intensity increases. As a consequence, the unemployment rate raises and the interest rate shrinks. Hence, a lower interest rate reduces the material consumption of the current old age people. We thank an anonymous referee for suggesting to clarify this point.
Proof. The proof straightforwardly derives from differentiating Eq. (16) with respect to $\gamma$ and evaluating it at $\gamma = 0$, that is:
\[
\left. \frac{\partial g_u(\gamma)}{\partial \gamma} \right|_{\gamma=0} = (1 + g_c)(\alpha + z - 1).
\]
Therefore,
\[
\begin{align*}
\left. \frac{\partial g_u(\gamma)}{\partial \gamma} \right|_{\gamma=0} < 0 & \iff z < \overline{z} \\
\left. \frac{\partial g_u(\gamma)}{\partial \gamma} \right|_{\gamma=0} > 0 & \iff z > \overline{z}
\end{align*}
\]
where $\overline{z} := 1 - \alpha$. Q.E.D.

Proposition 1 reveals that if the replacement rate is high enough, the unionisation of labour markets promotes economic growth. In this case, the positive wage-effect (i.e. the higher wages earned by the young when employed) which increases the saving rate overcompensates the negative unemployment-effect which instead acts negatively on savings. Definitively, under the condition stated in Proposition 1, a higher $\gamma$ implies a higher saving rate and, hence, a higher rate of per capita income growth.

Proposition 1 showed that a relatively high wage-oriented union can let a unionised-wage economy with unemployment grow faster than a competitive-wage economy with full employment. In the following proposition we show the existence of (i) a whole range of the union’s objectives which respect to which a Cobb-Douglas economy with unionised labour and unemployment grows faster than a Cobb-Douglas competitive equilibrium economy with full employment, and (ii) a growth-maximising value of the union’s relative wage intensity.

Proposition 2. (1) Let $z \leq \overline{z}$ hold. Then $g_u(\gamma) < g_c$ for any $0 < \gamma < \overline{\gamma}$. (2) Let $z > \overline{z}$ hold. Then $g_u(\gamma)$ is an inverted U-shaped function of the union’s relative wage intensity and $g_u(\gamma)$ is maximised at $\gamma = \hat{\gamma}$ with $g_u(\gamma) > g_c$ for any $0 < \gamma < \gamma^\circ$ and $g_u(\gamma) < g_c$ for any $\gamma^\circ < \gamma < \overline{\gamma}$ where $\hat{\gamma} < \gamma^\circ < \overline{\gamma}$.

Proof. The proof uses the following derivative:
\[
\frac{\partial g_u(\gamma)}{\partial \gamma} = \left(1 + g_c\right)\left[1 + \gamma(1 + \alpha) \right]^{-\frac{1}{2}} \cdot \alpha + z - 1 - \gamma[\alpha + z(1 + \alpha) - 1] \right) \left[1 - \gamma(1 + \alpha)\right]^{-\frac{1}{2}}.
\]
If $z \leq \overline{z}$ then $\frac{\partial g_u(\gamma)}{\partial \gamma} < 0$ for any $0 < \gamma < 1$.

If $z > \overline{z}$ then
\[
\frac{\partial g_u(\gamma)}{\partial \gamma} > 0 \iff \gamma < \hat{\gamma},
\]
where
\[
\hat{\gamma} := \frac{\alpha + z - 1}{\alpha + z(1 + \alpha) - 1} > 0,
\]
is the growth-maximising relative wage intensity of the union. Notice that $\hat{\gamma} < \overline{\gamma}$ holds for any $z > \overline{z}$. Since $g_u(0) = g_c$, $g_u(\gamma)$ is a positive (negative) monotonic function of $\gamma$ for any $0 < \gamma < \hat{\gamma}$ ($\hat{\gamma} < \gamma < \overline{\gamma}$) and $\lim_{\gamma \to \overline{\gamma}} [1 + g_u(\gamma)] = (1 + g_c)z < 1 + g_c$, then there always exists a threshold value.
γ^<\in\(\hat{\gamma}, \bar{\gamma}\)\) such that \(g_u(\gamma_\rho) = g_c\), and thus \(g_u(\gamma) > g_c\) for any \(0 < \gamma < \gamma_\rho\) and \(g_u(\gamma) < g_c\) for any \(\gamma_\rho < \gamma < \bar{\gamma}\). Q.E.D.

From Propositions 1 and 2 the following remark holds:

**Remark 1.** Although unionisation can always be desirable as an inducement to per capita income growth (provided the condition stated in Proposition 1), the existence of a growth-maximising relative wage intensity of the union, \(\hat{\gamma}\), implies that a unionised-wage economy may jump instantaneously over the highest possible balanced growth path. Moreover, there exists a whole range of the relative weight attached by the union to wages such that an economy with unionised labour and unemployment grows faster than an economy with competitive labour and full employment.

Moreover, it is easy to show also that: (i) when the government chooses a high replacement rate, the union should be relatively wage-oriented rather than employment-oriented to pursue higher growth rates in spite of a higher unemployment rate. Conversely, this means that when the union is relatively wage-oriented, the rate of economic growth is increased when the replacement rate is high enough. This follows from \(\frac{\partial \hat{\gamma}}{\partial z} = \frac{\alpha(1-\alpha)}{\alpha + z(1+\alpha)-1} > 0\); and (ii) when the technology of production is relatively capital-oriented rather than labour-oriented (i.e., the parameter \(\alpha\) is sufficiently high), a relatively wage-oriented rather than employment-oriented union speeds up economic growth, that is \(\frac{\partial \hat{\gamma}}{\partial \alpha} = \frac{z(1-z)}{\alpha + z(1+\alpha)-1} > 0\).

The policy implications of our findings are straightforward. In particular, depending on the size of the union’s relative wage intensity, the government should follow the union growth-maximising rule and then choose appropriately the replacement rate, as long as the existence of a finite positive consumption tax which balances the unemployment benefit expenditure is guaranteed for any \(z > \bar{z}\) and \(\gamma < \bar{\gamma}\).

2.6. Welfare

Analysis of both growth and welfare effects of public policies in endogenous growth models is long lasting. For instance, Barro (1990) showed – in a model with public services used as an input in production – that the growth-maximising tax rate used to finance the government expenditure at a balanced budget coincides with the welfare maximising one, whereas Futagami et al. (1993) showed that if public services are assumed to be a stock variable, then the growth-maximising and the welfare-maximising policies are different. In this section, we will analyse how the lifetime welfare of the representative generation will react following a rise in the power of the union to set the wage rate. In particular, we will try to give an answer to the following questions.

(i) How unionisation affects the individual lifetime welfare when the economy switches from a competitive equilibrium growth path with full employment to a new equilibrium growth path with unemployment?

(ii) Does the unionisation of labour markets create a trade-off between growth-maximising and welfare-maximising policies?
Comparison of the growth-maximising objective of the union with the welfare-maximising objective of the government in this simple stylised OLG economy implies that both policies exactly coincide along the balanced growth path.

Assuming as a measurement of the individual welfare the utility attained by the representative generation over the life cycle, below we investigate how the degree of unionisation of labour markets (as measured by the relative weight that the union attaches to wages) affects the lifetime indirect utility index of the representative generation in a unionised-wage economy with unemployment ($W_u$) in comparison with that attained in a competitive-wage economy with full employment ($W_c$).

In particular, we assume a benevolent government whose purpose is to maximise the lifetime indirect utility index of the representative generation with respect to the power of the union to set the wage ($\gamma$). Since the economy is always in a position of balanced growth and both young-aged and old-aged consumptions grow without transition at the (constant) rate $g_{y,c} = g_{y,c} = g_n(\gamma)$, the lifetime welfare grows steadily at the same rate along the balanced growth path, i.e. $g_{u,v}(\gamma) = g_u(\gamma)$. Therefore, the following proposition holds:

**Proposition 3.** (1) Let $z \leq \gamma$ hold. Then $W_{a,1}(\gamma) < W_{c,1}$ for any $0 < \gamma < \gamma$. (2) Let $z > \gamma$ hold. Then $W_{a,1}(\gamma)$ is an inverted $U$-shaped function of the union’s relative wage intensity and $W_{a,1}(\gamma)$ is maximised at $\gamma = \hat{\gamma}$ with $W_{a,1}(\gamma) > W_{c,1}$ for any $0 < \gamma < \gamma^\circ$ and $W_{a,1}(\gamma) < W_{c,1}$ for any $\gamma^\circ < \gamma < \gamma$. where $\hat{\gamma} < \gamma^\circ < \gamma$.

**Proof.** See Appendix B.

Proposition 3 reveals that the growth-maximising relative wage intensity of the union coincides with the government welfare-maximising policy, and the higher the replacement rate is the more likely a positive relationship between economic growth and unemployment (i.e. welfare and unemployment) exists. Therefore, under the hypotheses and the conditions stated in point 2 of Propositions 3, there exists a whole range of the union’s preference in fixing the wage rate relative to employment with respect to which individuals are better off in an economy with unionised labour and unemployment rather than in a competitive equilibrium economy with full employment along the balanced growth path.

### 3. Conclusions

Using the basic one-sector model of endogenous growth with overlapping generations and Cobb-Douglas utility and production functions, we showed that the unionisation of labour markets may be growth- and welfare-enhancing if both the replacement rate (as part of the public provided unemployment benefit system) and the weight of capital in production are high enough. In particular, we assumed the standard right-to-manage model and showed the existence of (i) a growth-maximising relative wage intensity of the union with respect to which the economy jumps instantaneously over the highest possible balanced growth path in equilibrium, and (ii) a whole range of the union’s relative wage intensity such that an economy with unionised labour and unemployment grows faster than an economy with competitive labour and full employment.

Moreover, we argued that the more the union is wage-oriented rather than an employment-oriented the higher is the rate of per capita income growth in spite of a higher unemployment rate.

These results are rather unusual within the existing endogenous growth literature framed in the basic OLG context where a Romer-type externality (assumed here to be the average *per capita* stock of capital installed in the whole economy, rather than the stock of capital in *per worker* terms).
represents the engine of endogenous growth, and indicate that a positive unemployment rate may be beneficial for both economic growth and welfare provided that (i) the technology is relatively capital-oriented and (ii) the unemployment benefit system is sufficiently generous.\footnote{It is worth noting that although the (non-distorting) young-aged consumption tax used to finance the benefit system acts (just like a wage tax) as a redistributive device within the same working age generation, the endogenous growth literature has argued that such a tax, different from a wage tax which is growth-reducing, may be growth-neutral as shown by Rebelo (1991).} Therefore, a policymaker may favour the occurrence of the latter condition turning on unionisation from a threat to an opportunity for growth.

Appendix A

In this appendix we present details of Eq. (14) in the main text.

\[
\frac{\partial k_{t+1}}{\partial w_{u,t}} = \frac{\beta}{1 + \beta} (1 - u) > 0 , \tag{A1}
\]

\[
\frac{\partial w_{u,t}}{\partial \mu} = w_{c,t} > 0 , \tag{A2}
\]

\[
\frac{\partial k_{t+1}}{\partial u} = -\frac{\beta}{1 + \beta} (w_{u,t} - b_t) < 0 , \tag{A3}
\]

\[
\frac{\partial u}{\partial \mu} = \frac{\mu}{\alpha \mu} > 0 , \tag{A4}
\]

\[
\frac{\partial \mu}{\partial \gamma} = \frac{\alpha}{[1 - \gamma (1 + \alpha)]^2} > 0 . \tag{A5}
\]

Appendix B

Proof of Proposition 3.

Since the consumption tax is constant in equilibrium (see Eq. 11 in the main text), and \( c_1 \) is a monotonic increasing function of the per capita stock of capital, then the growth rate of \( c_1 \) is given by:

\[
g_{c_1} (\gamma) = \frac{c_{1,t+1} - c_{1,t}}{c_{1,t}} = \frac{(1 - \alpha)BH(\gamma) \cdot (k_{t+1} - k_t)}{(1 - \alpha)BH(\gamma) \cdot k_t} = \frac{k_{t+1} - k_t}{k_t} = g_{u,k} (\gamma) = g_u (\gamma). \tag{B1}
\]

Moreover, since the unemployment rate and the interest rate are both constant, the growth rate of the retirement period consumption (\( c_2 \)) may easily be expressed as:

\[
g_{c_2} (\gamma) = \frac{c_{2,t+1} - c_{2,t}}{c_{2,t+1}} = \frac{\beta (1 - \alpha) BH(\gamma) (1 + r) k_{t+1} - k_t}{\beta (1 - \alpha) BH(\gamma) (1 + r) k_t} = \frac{k_{t+1} - k_t}{k_t} = g_{u,k} (\gamma) = g_u (\gamma). \tag{B2}
\]

Knowing that \( c_1 \) and \( c_2 \) grow without transition at the constant rate \( g_u (\gamma) \), then the lifetime welfare of the representative generation (see Eq. P in the main text) grows steadily at the same constant rate along the balanced growth path.
Now, consider the following homogeneous of degree one utility function, which represents homothetic preferences over the life cycle consumption bundles of the generation born in period $t$, that is:

$$U_t := (1 + \beta) \ln W_t(c_{1,t}, c_{2,t+1}) = \ln c_{1,t} + \beta \ln c_{2,t+1},$$

where

$$W_t(c_{1,t}, c_{2,t+1}) = c_{1,t}^{1+\beta} \cdot c_{2,t+1}^{1+\beta},$$

and $\beta := \frac{1}{1 + \sigma}$ with $\sigma \in (0, +\infty)$ being the subjective discount rate. From Eq. (B3) individual welfare in the unionised-wage economy grows at the rate

$$g_{u,w}(\gamma) = g_{u,w}(\gamma) = \frac{W_{t+1} - W_t}{W_t} = \frac{1}{c_{1,t+1}^{1+\beta} \cdot c_{2,t+2}^{1+\beta} - c_{1,t}^{1+\beta} \cdot c_{2,t+1}^{1+\beta}}$$

$$= \left(\frac{c_{1,t+1}}{c_{1,t}}\right)^{1+\beta} \cdot \left(\frac{c_{2,t+2}}{c_{2,t+1}}\right)^{1+\beta} - 1$$

$$= \left[1 + g_u(\gamma)\right]^\frac{1}{1+\beta} \cdot \left[1 + g_u(\gamma)\right]^\frac{\beta}{1+\beta} - 1$$

$$= 1 + g_u(\gamma) - 1$$

$$= g_u(\gamma).$$

Exploiting Eq. (B3) the dynamic evolution of individual welfare as a function of the union’s relative wage intensity can be described, after some algebraic manipulations, as

$$W_{u,t+1}(\gamma) = \left[1 + g_u(\gamma)\right] \cdot W_{u,t}(\gamma). \tag{B4}$$

The government objective is to maximise the solution of Eq. (B4) with respect to $\gamma$, that is:

$$\max_{\gamma} W_{u,t}(\gamma) = W_0 \cdot \left[1 + g_u(\gamma)\right], \tag{B5}$$

where $W_0 > 0$ is the initial value of individual utility and $g_u(\gamma)$ is determined by Eq. (16) in the main text. Differentiating Eq. (B5) with respect to $\gamma$ yields

$$\frac{\partial W_{u,t}(\gamma)}{\partial \gamma} = t W_0 \left[1 + g_u(\gamma)\right]^{-1} \cdot \frac{\partial g_u(\gamma)}{\partial \gamma},$$

where $\frac{\partial g_u(\gamma)}{\partial \gamma}$ is given by Eq. (17) in the main text. Therefore,

if $z \leq \overline{z}$ then $\frac{\partial W_{u,t}(\gamma)}{\partial \gamma} < 0$ for any $0 < \gamma < 1$. This proves point (1);

if $z > \overline{z}$ then

$$\frac{\partial W_{u,t}(\gamma)}{\partial \gamma} > 0 \iff \gamma \leq \hat{\gamma},$$

where $\hat{\gamma}$ is the welfare-maximising relative wage intensity in the government’s objective function (which coincides with growth-maximising relative wage intensity in the union’s objective function – see Eq. 18 in the main text). Since $W_{u,t}(0) = W_{c,t}$, $W_{u,t}(\gamma)$ is a positive (negative) monotonic function of $\gamma$ for any $0 < \gamma < \hat{\gamma}$ ($\hat{\gamma} < \gamma < \overline{\gamma}$) and $\lim_{\gamma \to \gamma} W_{u,t}(\gamma) = W_0 \left[1 + g_u(\gamma)\right]^{-1} < W_0 \left[1 + g_u(\gamma)\right]'$, then there always exists a threshold value $\gamma^0 \in (\hat{\gamma}, \overline{\gamma})$ such that $W_{u,t}(\gamma^0) = W_{c,t}$, and thus $W_{u,t}(\gamma) > W_{c,t}$ for any $0 < \gamma < \gamma^0$ and $W_{u,t}(\gamma) < W_{c,t}$ for any $\gamma^0 < \gamma < \overline{\gamma}$. This proves point (2). Q.E.D.
Propositions 2 and 3 reveal that both per capita income and individual welfare grow steadily at the same constant rate along the balanced growth path. Therefore, under the hypotheses and the conditions stated in points 2 of Propositions 2 and 3, both the rate of per capita income growth and the indirect utility index of the representative individual are higher in a unionised-wage economy with unemployment than in a competitive-wage with full employment. Moreover, the growth-maximising and the welfare-maximising objectives are exactly the same along the balanced growth path. Therefore, for any \( z > \overline{z} \) and \( 0 < \gamma < \gamma^* \) individuals living in a unionised-wage economy with unemployment are better off than those living in a competitive-wage economy with full employment. In particular \( \gamma = \gamma^* \) implies an instantaneous adjustment of both the rate of per capita income growth and individual welfare to the highest possible balanced growth path.

References


