Economics Bulletin

Volume 29, Issue 2

An Alternative Identification of the Economic Shocks in SVAR Models

Hassan Belkacem Ghassan
USMBA, Morocco and KFU, Saudi Arabia

Mohammed Souissi
UM5, EMI, Rabat Agdal, Morocco

Mohammed Kbiri Alaoui
Ministry of employment and UM5, EMI, Rabat, Morocco

Abstract

The purpose of this paper is to develop a new approach allowing us to identify the structural shocks in the SVAR model. This approach ameliorates substantially the decomposition methods of Bernanke (1986) and Bernanke & Mihov (1998) and improves in the same way the identification procedures pioneered by Blanchard & Quah (1989) and Blanchard & Perotti (2002).

Citation: Hassan Belkacem Ghassan and Mohammed Souissi and Mohammed Kbiri Alaoui, (2009) "An Alternative Identification of the Economic Shocks in SVAR Models ", *Economics Bulletin*, Vol. 29 no.2 pp. 1019-1026.
Submitted: May 05 2009. **Published:** May 16, 2009.

1 Introduction

This paper discusses some technical limits of economic analysis founded on the Structural Vector Autoregression models (SVAR). It presents also an alternative to the existing methods of identification of the structural shocks. The canonical innovations, associated to a VAR model, represent the shocks whose propagation results in the fluctuations of the studied dynamic system. Under the assumption that the innovations are not instantaneously correlated, the contribution of each impulse on the various series of a given system is measurable. If the shocks are not independent, Sims (1980) proceeded by an orthogonalisation of the Choleski type. But, this orthogonalisation is purely statistical and is not associated to an economic theory. Moreover, it skews the economic interpretation of the obtained shocks.

The founders empirical work are mainly those of Blanchard & Watson (1986), Bernanke (1986), Shapiro & Watson (1988), Blanchard & Quah (1989), King, Plosser, Stock & Watson (1991). Their objective was to identify the structural shocks which has an economical interpretation and with a typology that is multiple either of supply or of economic policy. Then these structural shocks were estimated as linear functions of the canonical innovations of the system, subject to some identifying constraints resulting from the economic theory.

In this paper, we suggest to replace this linear relation between these two types of shocks by a differential equation. The solution of this differential equation as well as the identification of the structural shocks will be based on the techniques resulting from the theory of viability developed by Aubin (1992, 1997) and Saint-Pierre (1994). Our main contribution is to give a methodological share by reformulating the relation which ties the canonical innovations to the structural shocks, making therefore the use of orthogonal shocks more flexible. This revision is essential especially to the effect that the determination of impulse functions cannot be done without taking into account the interaction among the shocks of the real economy.

The paper will be organized as follows: the first section sheds light on the importance of the choice of the system variables. At the second section, we present a short outline on the fundamental assumptions of the VAR methodology along with the identification method of Blanchard & Quah (1989). The third section introduces the nonlinear model suggested as well as the primary motivation behind it. The final section is devoted to the methodology herewith suggested related to the structural shock identification.

2 Selection of the variables and VAR methodology

The choice of the variables of the system consists in clarifying the indication that makes it possible to examine and identify the actions of economic policy, including the monetary policy, the budget policy, and the policies of economic growth. For example, the empirical literature using the SVAR methodology is primarily directed towards explaining the various modes of interaction between the real economic growth and the monetary or budgetary variables on the one hand, and between the real economic growth and the rate of unemployment on the other hand. The focus of the modelisation is to determine the effects of an economic policy upon the variables of the system.

The variables of the reduced form must undergo with the precondition of the tests of non-stationnarity and the tests of parsimony to determine the optimal number of delays considered in the model, especially when the data are not to annual frequency. In general all the macroeconomic variables are I(1) whose economic interpretation is important, since it expresses the presence of behaviors with limited rationality. Thus, for instance, the series of the Gross Domestic Product (GDP) and the rate of unemployment noted U are generally integrated of order 1. The system VAR is then given by the pair $(\Delta GDP, \Delta \mathbf{U})$.

The identification of the structural shocks rests on the principle of passing from the shocks resulting from a canonical VAR model to shocks having economic interpretation, where the latter form the subjacent structural VAR model. Let's consider an economic system composed of a vector $X_t = (X_{1t}, X_{2t})$ and let u_t be the canonical innovation which corresponds to the anticipated part of the series observed between the dates t and t-1: $\mathbf{X}_{1t-1}, \mathbf{X}_{2t-1}$. The estimate of these innovations is carried out according to the Sims principle (1980), starting from the vector autoregression representation of the canonical VAR given by:

$$\mathbf{X}_t = \mathbf{A}_1\mathbf{X}_{t-1} + \mathbf{A}_2\mathbf{X}_{t-2} + \ldots\mathbf{A}_p\mathbf{X}_{t-p} + \mathbf{u}_t \quad (2.1)$$

At each date t, the errors $\mathbf{u}_{it}$ are estimated by the residues of the regression corresponding to the individual estimate of each equation of the VAR. If the shocks are not independent, Sims (1980) proceeded by an orthogonalisation of Choleski type, which constitutes a statistical constraint. The disadvantage of this approach is that it does not allow for an economic interpretation. The orthogonalisation obtained by the decomposition of Choleski is largely criticized by the partisans of the SVAR methodology, who recommend an orthogonalisation based on identifying constraints resulting from the economic theory (Shapiro & Watson 1988, Blanchard & Quah 1989, King et *al*. 1992). The methodology of identification assumes the existence of a linear relation between the structural shocks noted ε_t and the canonical shocks $\mathbf{u}_t$ of the form:

$$\boldsymbol{\varepsilon}_t = \mathbf{P}\mathbf{u}_t \quad (2.2)$$

where P is a passage matrix. This method also supposes that the components of ε_t are not correlated and have a unit variance: $\mathbf{E}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t') = \mathbf{I}$.

3 Methodology of nonlinear models

The nonlinear models are various forms which are increasingly used in economy and especially in finance. The founders works are due to Terasvirta (1993, 1994 and 1998) and Franses & Dijk (2000) with the models STAR; Tong (1990) with models TAR (Threshold Autoregressive); Hamilton (1989) with the models of Markov autoregression with change of state i.e. MSVAR and Kuan & White (1994) with the networks of neurons artificial (ANN). With these models it is not possible to know completely the properties of the series and their behavior. It is not easy to interpret a nonlinear model and justify its adequacy in situations where volatility dominates or at least presents unstable behaviors. The nonlinear models quoted above privilege the nonlinearity of the variables of the model. This limitation is certainly restrictive, but it exceeds the traditional scheme of linearity which is less rationalist compared to the evolutions observed.

We introduce a nonlinear relation between the two types of shocks i.e. the structural shocks and the reduced shocks. This nonlinearity makes possible to apprehend and evaluate the responses to the shocks in some better way. The interpretation of the long-run effects - which take into account the interactions between the shocks - will be enriched.

In the first point of this section, one presents the nonlinear relation supposed between the structural shocks and the canonical shocks. The second point presents the impulse functions which will be used and compared with usual ones. The third point consists in introducing the a priori economic ones, the transitory and permanent shocks. The last point presents the set of the constraints relating to the studied economic system and the set of the solutions of the problem formed by the required structural shocks.

3.1 Nonlinearity relation

Let's consider a structural choc of initial value ε_0 (amplitude of the initial shock) known and ~~is~~ subjected to a given force which tends to bring back towards its value average and so subjected to a noise caused by the canonical innovations[1].

$$\frac{d\varepsilon}{dt}(t) = \alpha(m - \varepsilon(t)) + \lambda(t)\frac{du}{dt}(t) \tag{3.1a}$$

α is the intensity of the recall force, λ is the intensity of the noise, which expresses the variability of the parameters associated with the variation with the residues with the VAR model u(t) form the canonical innovations estimates from the VAR model and the parameters α, λ, **m** must be estimated[2].

The first term of the equation (3.1a) finds its explanation in the fact that a stationary series has as a characteristic to turn over to its average when it deviates under the effect of some shocks. The speed to which this return to the average is carried out can vary from an economic system to another. This parameter α can depend on time and the general equation to study will be of the form:

$$\frac{d\varepsilon}{dt}(t) = \alpha(t)(m - \varepsilon(t)) + \lambda(t)\frac{du}{dt}(t) \tag{3.1b}$$

where the recall force α decrease to zero. Concerning the intensity of "noise" $\lambda(t)$, one supposes on the one hand that it is a decreasing function of time (more one moves away in time plus it decreases) and on the other hand that it is enough large to consider only the noise coming from the canonical innovations $u(t)$.

Since the VAR models are treated starting from the stochastic processes $\{\mathbf{X}_{1t}, \mathbf{X}_{2t}\}$, one will consider the canonical innovations estimated and by an interpolation method, one can return to the continuous case[3]. To be able to release the qualitative properties relating to the solutions, we place ourself in the case of the resolution of a differential inclusion of the form:

$$\frac{d\varepsilon}{dt}(t) \in \alpha(m - \varepsilon(t)) + \lambda(t)\frac{du}{dt}(t) =: F(\varepsilon(t)) \tag{3.2}$$

where $F(\varepsilon(t)) = \left\{ \alpha(m - \varepsilon(t)) + \lambda(t)\frac{du}{dt}(t) \,/\, \lambda(t) = \begin{pmatrix} \lambda_{11}(t) & \lambda_{12}(t) \\ \lambda_{21}(t) & \lambda_{22}(t) \end{pmatrix}, \lambda_{ij}(t) \in [0,1], i, j \in \{1,2\} \right\}$

The definition of F expresses that the image by F of the value $\varepsilon(t)$ of the shock at time t is the set of the solutions of the equation (3.1) knowing that $\lambda_{ij}(t)$ course the interval [0, 1]. All the solutions which we look for are in general bounded, so they are exponentially bounded. The choice of these differential inclusions make possible to establish the set of the solutions of the equation (3.1) among which we will choose those which have an economic interpretation i.e. the structural shocks.

[1] It is as if ε_t represents the position of a particle subjected to a recalling force of which brings to an equilibrium position and an unpredictable force modeled by noise.

[2] An equation similar to (3.1) is used in finance by Fouque, Papanicolaou & Ronnie Sircar (2000), which model the stochastic volatility in the Black-Sholes model.

[3] The following approximation $\frac{d\varepsilon_t}{dt} = \Delta\varepsilon_t$ and $\frac{de_t}{dt} = \Delta e_t$ permit the passage between the continuous solution and the discrete solution.

3.2 Impulse functions

The impulse functions are used to measure the response of the variable $\mathbf{X}_{t+h}$ to a shock taking place at time t. In the linear case, the response of $\mathbf{X}_{it}$ to the canonical shocks $\mathbf{u}_{js}$ (s <t) was given by $\frac{\partial \mathbf{X}_{it}}{\partial \mathbf{u}_{js}} = \mathbf{r}_{ij,t-s}$. Also, the impulse function was defined by: $\mathbf{h} \to \mathbf{r}_{ij,h}$.

As the same way, the response of $\mathbf{X}_{it}$ to the structural shocks $\boldsymbol{\varepsilon}_{js}$ (s <t) was given by $\frac{\partial \mathbf{X}_{it}}{\partial \boldsymbol{\varepsilon}_{js}} = \boldsymbol{\theta}_{ij,t-s}$ and the impulse function was given by: $\mathbf{h} \to \boldsymbol{\theta}_{ij,h}$.

For the case of the nonlinear models (and also linear), the traditional impulse functions (see Dijk et *al*. 2001) are defined by:

$$\mathbf{TI}_{X_t}(\mathrm{h}, \varepsilon_t, \omega_{t-1}) = \mathbf{E}(\mathbf{X}_{t+h} \mid \varepsilon_t, \varepsilon_{t+1} = \ldots = \varepsilon_{t+h} = 0, \omega_{t-1}) - \mathrm{E}(\mathrm{X}_{t+h} \mid \varepsilon_t = 0, \varepsilon_{t+1} = \ldots = \varepsilon_{t+h} = 0, \omega_{t-1})$$

where $\boldsymbol{\omega}_{t-1}$ is the set $\mathbf{X}_1, \ldots, \mathbf{X}_{t-1}$ of available information until the time t-1.

In the nonlinear relation (3.1a), the use of the impulse functions $\boldsymbol{\theta}_{ij,h}$ does not seem adequate, for that one takes again the impulse functions introduced by Koop & al. (1996) generalizing the functions TI defined by:

$$\mathbf{GI}_{X_t}(\mathrm{h}, \varepsilon_{jt}, \omega_{t-1}) = \mathbf{E}(\mathbf{X}_{t+h} | \varepsilon_{jt}, \omega_{t-1}) - \mathbf{E}(\mathbf{X}_{t+h} | \omega_{t-1}) \quad (3.3)$$

We also set:

$$\mathbf{GI}^{\infty}_{X_t}(\varepsilon_{jt}, \omega_{t-1}) = \lim_{h \to +\infty} \mathbf{GI}_{X_t}(\mathrm{h}, \varepsilon_{jt}, \omega_{t-1})$$

3.3 A priori economic

With the methodology of Blanchard and Quah (1989), the determination of the matrix P of (2.2) rests on constraints of short term and long term developed from the economic theory and connecting the various variables of the model. In the case of nonlinearity of the structural shocks, one also supposes the existence of two types of effects:

- The permanent effect of a shock on a variable, such as for example the effect of long term of a shock of economic growth on the rate of unemployment.
- The transitory effect of a shock on a variable, as for example the effect of a rise of the prices on the level of employment within the framework of the augmented Phillips curve[4].

Thereafter, we say that $\boldsymbol{\varepsilon}_{it}$ is a shock having a permanent effect on $\mathbf{X}_{it}$ if it verifies:

$$(H_1) \qquad \mathbf{GI}^{\infty}_{X_t}(\varepsilon_{it}, \omega_{t-1}) > 0$$

And by $\boldsymbol{\varepsilon}_{jt}$ a shock having a transitory effect on $\mathbf{X}_{jt}$ if it verifies:

$$(H_2) \qquad GI^{\infty}_{X_t}(\varepsilon_{jt}, \omega_{t-1}) = 0$$

In practice $\mathbf{GI}^{\infty}_{X_t}(\varepsilon_{it}, \omega_{t-1})$ will be identify $\mathbf{GI}_{X_t}(\mathrm{h}^*, \varepsilon_{it}, \omega_{t-1})$ for large $\mathbf{h}^*$.

3.4 Constraints and objectives

It is reasonable to suppose that the shocks $\boldsymbol{\varepsilon}(t)$ at time t are of amplitude lower than a threshold imposed by the studied system. For example, in the case of the system considered $(\Delta Pib, \Delta \mathrm{U})$, the economic growth cannot exceed a value threshold a% which is required by macro-economic equilibrium or the national and international economic situation. When with the shock on unemployment, it should not exceed a threshold b% which makes possible to

[4] More specifically, an increase in prices suggests favorable economic conditions, entrepreneurships recruit and the unemployment rate is falling. Once the monetary illusion disappears, entrepreneurships change their behavior and begin to send away workers.

remain with the top natural unemployment. One defines the whole of the constraints K = [0, a%]$\times$[0, b%].

The problem of identification of the shocks thus consists in solving the equation (3.2) under a set of constraints $\mathbf{K}$ fixed at the beginning (and imposed by the study of the system). Let $\boldsymbol{S_F(\varepsilon_0)}$ be the set of the trajectories (solution of the equation (3.2)) resulting from an initial state $\boldsymbol{\varepsilon_0}$, we will identify the solutions of differential inclusion belonging to $\boldsymbol{S_F(\varepsilon_0)}$ such $\mathbf{X}_{1t}, \mathbf{X}_{2t}$ as constantly or at least until a certain finite time, $\boldsymbol{\varepsilon(t)}$ remains in the set of the constraints $\mathbf{K}$ and moreover satisfying the a priori economic i.e. $(\mathbf{H_1})$ and $(\mathbf{H_2})$.

When trying to resolve the problem, several situations arise:

1. From any point $\boldsymbol{\varepsilon_0}$ of $\mathbf{K}$, any solution always remains in $\mathbf{K}$.
2. From any point $\boldsymbol{\varepsilon_0}$ of $\mathbf{K}$, there exists at least one solution which remains in $\mathbf{K}$.
3. From some points $\boldsymbol{\varepsilon_0}$ of $\mathbf{K}$, there exists at least one solution which remains in $\mathbf{K}$.
4. Every solution starting from a point of $\mathbf{K}$, leave $\mathbf{K}$ in some finite time.

The problem defined by the inclusion (3.2) admits a solution given in the papers of Aubin (1992, 1997) and Saint-Pierre (1994). In the continuation, we will present the sets which we will be useful for the identification of the structural shocks.

We say that $\boldsymbol{\varepsilon(t)}$ verify the condition $(\mathbf{H}_1 - \mathbf{H}_2)$ if the first component of the shocks verify $\mathbf{H}_1$ and the second component verifies $\mathbf{H}_2$.

Let $C = \left\{ \varepsilon_1 \in K \middle| \exists \varepsilon(.) \in S_F(\varepsilon_1) : \varepsilon(t) \in K, \forall t \geq 0 \;\; and \;\; \varepsilon(t) \;\; satisfying \;\; H_1 - H_2 \right\}$ the set of all points of $\mathbf{K}$ from which starts at least one solution of the problem and satisfying a priori economics resulting from the economic theory. Such set is called the target to reach.

Let $C_F(K) = \left\{ \varepsilon_0 \in K \middle| \exists \varepsilon(.) \in S_F(\varepsilon_0) : \exists \tau > 0 \;\; (finite), such : \forall t < \tau, \varepsilon(t) \in K \;\; and \;\; \varepsilon(\tau) \in C \right\}$ the set of initials values from which starts at least one solution of the problem which reach the target $\boldsymbol{C}$ in finite horizon time.

The function $\theta_F(\varepsilon_0) = \min \left\{ \tau \middle| \exists \varepsilon(.) \in S_F(\varepsilon_0) : \forall t < \tau, \varepsilon(t) \in K \;\; and \;\; \varepsilon(\tau) \in C \right\}$ is the minimal time for a solution starting from $\boldsymbol{\varepsilon_0}$, remains in $\mathbf{K}$ behind reaching the target $\boldsymbol{C}$ in finite time.

The identification of the structural shocks will be carried out once the set $\boldsymbol{C_F(K)}$ will be perfectly given. Such a unit makes possible on the one hand to determine the evolution of the structural shocks, and on the other hand it also makes possible to determine the set of the initial amplitudes which the shocks must take to have an economic interpretation.

4 Resolution

The entire problem lies in the determination of an algorithm able as well as possible to approach the set $\boldsymbol{C_F(K)}$ and the value $\boldsymbol{\theta_F(\varepsilon_0)}$. The explicit knowledge of model SVAR (the model with the shocks $\boldsymbol{\varepsilon(t)}$ and allowing their identification) is not necessary since the method presented allows the determination of the evolution $\boldsymbol{\varepsilon(t)}$ and also makes possible to calculate the response of the system to the various shocks. Formally, the SVAR model can be written as where (E)' is the differential inclusion appeared in $F(\varepsilon(t))$:

$$\begin{cases} \mathbf{X}_t = \Phi(\mathbf{X}_{t-1}, \varepsilon(t)) \\ \varepsilon(t) \;\; \text{satisfying} \;\; (E)' \end{cases} \tag{4.1}$$

Amongst other things, this method allows also the identification of the structural shocks in the case of only one equation (differently of the usual case).

$$\begin{cases} \mathbf{X}_t = \mathbf{a}\mathbf{X}_{t-1} + \mathbf{c}\mathbf{Y}_t + \mathbf{e}_t \\ \varepsilon(t) \ \text{ satisfying } \ \mathbf{(E)'} \end{cases} \tag{4.2}$$

The numerical treatment of the methods is in progress in order to approach the set $\boldsymbol{C_F(K)}$. This algorithmic resolution makes possible to analyze in a finer way the shocks of real economic growth on the labor market and conversely, it authorizes to determine the extent of the shocks of qualification on the growth of the real GDP.

5 Conclusion

We presented an approach for the identification of the structural shocks by supposing a nonlinear relation between the structural shocks and the canonical shocks. This relation is controlled by a differential equation and the theoretical base is that of the theory of viability developed by Aubin (1992, 1997) and Saint-Pierre (1994). This approach has the advantage of not being restricted with the assumption of orthogonality of the shocks. It also makes possible to identify the structural shocks and to give the whole of the initial conditions which the shocks must check to be able to be interpreted economically. This method is able to study the modes of interactions between the variables and to thus determine the effect of an economic policy on the variables of the studied system.

References

Aubin, J.P. (1997) *Dynamic Economic Theory: a Viability Approach*, Springer-Verlag: New York.

Aubin, J.P. (1991) *Viability Theory*, Birkhauser, Basel.

Bernanke, B.S. and I. Mihov (1998) "Measuring Monetary Policy" *Quarterly Journal of Economics* **113**(**3**), 869-902.

Bernanke, B.S. (1986) "Alternative Explanations of the Money-Income Correlation" Carnegie-Rochester Conference Series on Public Policy 25, 49–99.

Blanchard, O.J. and R. Perotti (2002) "An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output" *Quarterly Journal of Economics* **117**(**4**), 1329-1368.

Blanchard, O.J. and D. Quah (1989) "The Dynamic Effect of Aggregate Demand and Supply Disturbances" *American Economic Review*, **79**(**4**), 655-673.

Blanchard, O.J. and M.W. Watson (1986) "Are Business Cycles All Alike?" in *The American Business Cycle: Continuity and Change* by R.J. Gordon, Ed., National Bureau of Economic Research Studies in Business Cycles vol. **25**. University of Chicago Press, Chicago, pp. 123–182.

Dijk, D.V., Ph.H. Franses and H.P. Boswijk (2001) "Asymmetric and Common Absorption of Shocks in Nonlinear Autoregressive Models" in www1.fee.uva.nl/ke/…/Boswijk.pdf

Fouque, J.P., G. Papanicolaou and K.R. Sircar (2000) "Derivatives in Financial Markets with Stochastic Volatility" Cambridge University Press.

Hamilton, J.D. (1989) "A New Approach to the Economic Analysis of Non-stationary Time Series Subject to Changes in Regime" *Econometrica* **57**, 357-384.

King, R.G., C.I. Plosser, J.H. Stock and M.W. Watson (1991) "Stochastic Trends and Economics Fluctuations" *American Economic Review* **81**, 819-840.

Saint-Pierre, P. (1994) "Approximation of the Viability Kernel" *Applied Mathematics and Optimisation*, **29**, 187-209.

Shapiro, M.D. and M.W. Watson (1988) "Sources of Business Cycle Fluctuations", NBER Working Paper number 2589.

Sims, C. (1980) “Macroeconomics and Reality” *Econometrica* **48**(**1**), 153-174.
Koop, G., M.H. Pesaran and S.M. Potter (1996) “Impulse Response Analysis in Nonlinear Multivariate Models” *Journal of Econometrics* **74**, 119-147.
Kuan, C.M. and H. White (1994) “Artificial Neural Networks: an Econometric Perspective” *Econometric Reviews* 13, 1-143 (with discussion).
Terasvirta, T. (1998) “Modeling Economic Relationships with Smooth Transition Regressions” in *Handbook of Applied Economic Statistics* by A. Ullah and D.E.A. Giles, Eds., Marcel Dekker: New York, 507-552.
Terasvirta, T. (1994) “Specification, Estimation and Evaluation of Smooth Transition Autoregressive Models” *Journal of the American Statistical Association* **89**, 208-218.
Tong, H. (1990) *Non-linear Time Series: a Dynamical Systems Approach*, Oxford University Press: Oxford.