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The long run Phillips curve and the role of downward nominal wage rigidity in Tunisia

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Abstract

The long run Phillips curve has been a controversial topic for many economists such as Friedman (1968) and Lucas (1972) among others since the 70's. Many new studies concerning the impact of downward nominal wage resistance hypothesis on the long run Phillips curve have enriched the empirical literature. However, the essential idea of this study is to examine the evidence of this hypothesis and its impact on the long run Phillips curve in Tunisia. Relying on annual data covering the period 1962-2004, we estimate two models of the Phillips curve: one is standard and the other with downward nominal wage rigidity in which we are inspired by the works of Akerlof, Dickens and Perry (1996). We adopt two estimation methods: the Ordinary Least Square and the Non Linear Least Square. Our results suggest a possibility of a long run unemployment inflation trade off incorporating the downward nominal wage rigidity hypothesis.

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1. Introduction

Several economists maintain the idea that nominal wages are downward rigid at a low inflation regime. Thus downward wage rigidity may cause a high rate of the unemployment average when authorities target a low inflation rate. This thesis, according to which the inflation could have a lubricating effect on the labor market, was mainly defended by Schultze (1959), Tobin (1972), Akerlof, Dickens and Perry (ADP) (1996, 2000), Djoudad and Sargent (1997), Fortin et al. (2002), Fortin and Dumont (2000), Lundborg and Sacklem (2001) and Stark and Sargent (2003). The proposal suggesting that the nominal wage rates should be downward rigid is crucial when evaluating the empirical validation of the long run unemployment inflation relationship. This new approach also allows to dash definitely the idea that the Phillips curve is outdated. The empirical studies done in this field focused closely on the role played by the downward nominal wage rigidity in the Phillips curve. These studies borrowed diverse methodologies and led to mitigated empirical results for developed countries. However, Dioudad and Sargent (1997), who use a similar model to that of ADP (1996), came to the conclusion that the downward nominal wage rigidity subject to an absolute drop noticeably ameliorates the performance of the Phillips curve incorporating the predictions for Canada. They demonstrated that the model with wage rigidity has a better significance for the prediction of inflation during the post war period particularly during 1990-1996 than the model without rigidity. Similar results were presented by Fortin and Dumont (2000) for Canada and Lundborg and Sacklem (2001) for Sweden.

Our objective in this paper is to show whether there is a long run unemployment inflation relationship in Tunisia relying on the evidence of the downward nominal wage rigidity. This kind of study has never been done for the case of Tunisia and this is where the originality of our work lies. This study presents a model that has been inspired from that of ADP (1996). These researchers were influenced by the theory and works in psychology to have a thorough idea of the behaviour of the economic agents. They explore two hypotheses which aim at establishing a permanent trade off between unemployment and inflation even at long run when inflation is low or when unemployment is quite high. They built a general stochastic equilibrium model integrating the downward nominal wage rigidity. Inflation, for them, serves as a lubricant in the labor market and is found to be desired at a weak dose. In this model, the long run unemployment rate, compatible with a stable inflation grows sensibly when inflation drops to low levels.

In our research, we try to analyse the unemployment inflation trade off relying on the role of the downward nominal wage rigidity during the period 1962-2004. We adapt two estimation methods: the Ordinary Least Squares and the Non Linear Least Squares where we rely on two types of prediction: adaptive and actual rate of inflation lagged 1 period. We have been able to show that the unemployment inflation trade off is permanent in case the predicted inflation is in its actual rate of inflation lagged 1 period form.

The remaining of this paper is organized as follows: in section two we present ADP (1996) estimation model. Then, in section three, we show, according to macro economic series, the evidence of the downward nominal wage rigidity in Tunisia. In section four, we present the general frame of our model. Our empirical results are shown and discussed in section five. Whereas, section six is allocated to the conclusion.

2. ADP (1996) estimation model

ADP's theoretical model is made up of the following four equations:

$$w_{ti}^{n} = p_{t}^{a}G_{t}\exp(c-au_{t})$$
⁽¹⁾

$$w_{ti} = \max\left(w_{ti}^n, w_{t-1,i}\right) \tag{2}$$

$$S_{t} = \frac{E(w_{ti}) - E(w_{ti}^{n})}{p_{t}^{a}G_{t}}$$
(3)

$$\pi_t = \pi_t^a + c - au_t + S_t M \tag{4}$$

In the first expression (1), w_{ii}^n represents the negotiated nominal wage. The anticipated price p_t^a is represented through its expected inflation which is a moving average of the previous two years' inflation. G_t is the labour productivity and u_t is the unemployment rate. The equation (2) tells us that because of the downward nominal wage inflexibility, the wage of a certain period (t) of a firm (i) will be the maximum of the normally notional wage and that of the preceding period.

Moreover, ADP added a new term reflecting the downward nominal wage rigidity to the Phillips curve augmented by the predictions. To measure this impact on salaries, they defined this additional variable S_t (equation 3) by the difference between the current wages and the notional ones. The expectancy operator shows that we take the average of the whole economy.

Starting from the definition of S_t , ADP obtained a Phillips curve type of equation (4) that contains the term S_t , that's to say that it takes into account the possible presence of a rigidity of wages.

In the equation (4), the term π_t represents the current inflation rate and the term π_t^a is the expected inflation. The unemployment rate is u_t and M is the Mark up of the sales price by the labor unit cost. This new Phillips curve (equation 4) differs from the traditional one only by its last term S_t which reflects the real wage costs engendered by the rigidities and M which converts these costs into prices. The specific property of this new Phillips curve is that the rapid productivity growth improves neatly the relation between unemployment and inflation.

However, the macroeconomic equations to estimate are (3) and (4) for S_t and π_t . The equations (1) and (2) are auxiliary ones serving to the estimation of equation (3). To estimate equations (3) and (4), we, first, have to conceive the expected inflation which is a moving average of the previous two years' inflation. The expected inflation equation is written as follows:

$$\pi_t^a = \alpha \pi_{t-1} + (1 - \alpha) \pi_{t-2} \tag{6}$$

Then, we have to find a suitable expression for S_t . And to estimate it, ADP assume that the observed salaries and the normal notional ones obey to a bivariate density.

Let's define:

$$v_{t} = \frac{E(w_{t-1,i}) - E(w_{ii}^{n})}{p_{t}^{a}G_{t}}$$
(7)

Where $w_{t-1,i}$ replaces w_{ti} , ADP show that

$$S_{t} = \sigma_{0} \rho \left(\frac{v_{t}}{\sigma_{0}} \right) + v_{t} \phi \left(\frac{v_{t}}{\sigma_{0}} \right)$$
(8)

In this equation (8), ρ represents the standard normal density function, ϕ is the cumulative normal distribution function, σ_0 is the standard deviation of the gap between lagged wages and the

notional ones, thought to be constant and written as follows $\frac{w_{t-1,i} - w_{ti}^n}{p_t^a G_t}$ and v_t is the gap between the average wage of last period and average notional wage of this period which is represented by $\frac{w_{t-1,i} - w_{ti}^n}{p_t^a G_t}$.

ADP finally obtain a complex equation connecting v_t to S_{t-1} , which is as follows:

$$v_{t} = \frac{S_{t-1} - \left(\frac{1}{M}\right) \left[\pi_{t}^{aa} + g_{t} - a(u_{t} - u_{t-1})\right]}{1 + \pi_{t}^{aa} + g_{t}}$$
(9)

With g_t the productivity growth rate and $\pi_t^{aa} = \pi_t^a + \pi_{t-1} - \pi_{t-1}^a$ is the rate change of price expectation π_t^a .

The recursive system is, then, formed by the following three equations:

$$S_{t} = \sigma_{0} \rho \left(\frac{v_{t}}{\sigma_{0}} \right) + v_{t} \phi \left(\frac{v_{t}}{\sigma_{0}} \right)$$
(10)

$$\pi_t = \pi_t^a + c - au_t + S_t M \tag{4'}$$

$$v_{t} = \frac{S_{t-1} - \left(\frac{1}{M}\right) \left[\pi_{t}^{aa} + g_{t} - a(u_{t} - u_{t-1})\right]}{1 + \pi_{t}^{aa} + g_{t}}$$
(11)

This system is estimated by the nonlinear least squares, which allows for the simultaneous estimation of the parameters of the Phillips curve and S_t . The parameters to estimate, then, are c, σ_0 and a. the parameter M is fixed at 1.4¹.

3. Evidence of downward nominal wage rigidity in Tunisia

During the last five decades, the pressures exerted on the labour market throughout the world in general, and in Tunisia particularly, were the strongest mainly between 1970 and 2000. Yet, despite the pressure exerted by the young workers, men and women, decreasing the nominal wages is a politically difficult decision. In fact, what we remark is rather an evolution of log change of the nominal wages (w_t) (figure 1) and that of log change of the Inter professional Guaranteed Minimum Wage (Salaire Minimum Inter professionnel Garanti (SMIG))(w_t^n) (figure 2) (see appendix 2).

4. General frame of the model design

Bearing in mind the empirical studies in the developed countries, done by (ADP) (1996-2002), Lundborg and Sacklen (2001), Knoppik (2001), Stark and Sargent (2003), among others, who examined the effect of the downward nominal wage rigidity on the unemployment inflation trade off and its impact on the economy at the macroeconomic level, we retain only one dummy variable. This variable reflects the effect of the liberalisation of prices on the general price level, in particular, and on the unemployment inflation trade off, in general. In addition we have approximated the notional wages to the SMIG in Tunisia, just as it was done in the study of Stark and Sargent (2003), for the SMIG is considered as the notional wage negotiated by the state and the syndicates. Thus, we write the recursive system as follows:

$$\pi_t = \alpha_0 + \alpha_1 \pi_t^a - \alpha_2 u_t + \alpha_3 S_t + \alpha_4 dummy$$
(12)

$$S_{t} = \sigma_{0} \varphi \left(\frac{v_{t}}{\sigma_{0}} \right) + \phi \left(\frac{v_{t}}{\sigma_{0}} \right) v_{t}$$
(13)

$$v_{t} \cong \frac{S_{t-1} - \left(\frac{1}{\alpha_{3}}\right) \left(\pi_{t}^{aa} + g_{t} - \alpha_{2} \left(u_{t} - u_{t-1}\right)\right)}{1 + \pi_{t}^{aa} + g_{t}}$$
(14)

¹ For further details see ADP (1996).

Before estimating the above recursive system, we calculate the downward nominal wage rigidity variable by the following equation:

$$S_t = \frac{w_t - w_t^n}{\pi_t^a G_t} \tag{15}$$

Similarly, we calculate the v_t variable by the following equation:

$$v_{t} = \frac{w_{t-1} - w_{t}^{n}}{\pi_{t}^{a} G_{t}}$$
(16)

The consumer price index is used to measure the inflation.

In order to be able to take into account the effects of the downward nominal wage rigidity on the unemployment inflation trade off, we evaluate the effect of the expected inflation in its adaptive form and in its actual rate of inflation lagged 1 period as well. The comparison of the results of the two forms of the expected inflation allows us to understand the importance of the methodological improvements suggested in this study.

5. Data bases and empirical results

Before proceeding with the econometric estimation, the presentation and the interpretation of the results, we first present the sources of our statistic data which we have collected for the estimation needs.

5.1 Sources and characteristics of the used data

The statistic data we used for the need of our estimation were collected from two national sources. At the Institute of the Quantitative Economy (IQE), we got the chronological series concerning the nominal wages (expressed in Tunisian Dinar), the total unemployment rate and the labor productivity for the economy. As for the consumer price index chronological series and the Inter professional Guaranteed Minimum Wage (SMIG) we got them from the online data basis of the National Institute of Statistics (INS)². All of the series cover the period 1962-2004. The determination of the Inter professional guaranteed minimum wage, in its annual chronological series, was done in the following way:

Monthly wage *12

Yet, the monthly wage is calculated in the following way:

Number of monthly work fours *hour rate³

To build the dummy variable representing the price liberalisation, which has a considerable impact on the general price level evolution we include an 0.75 value from 1991 to 2004. Since the independence and until the mid 1950's, the Tunisian State took in charge the strategic economic sectors. It controlled mainly the commercialization of the basic products and the prices level. A system of grants and authorisations allowed the administration to control the private investments, to regulate the commerce and to limit the imports.

Towards the mid 1980's, the Tunisian authorities, like several developing countries, must have deduced the limitations of such a policy and its inability to deal with a rapid growth and development of the economy.

Since 1986, Tunisia has been committed to carry out an ambitious process of economic reforms. These reforms aimed, mainly, at the liberalisation of the internal and external commerce, the liberalisation of prices and the privatisation of the public enterprises. It had as an objective to install a real competition. Thus, the beginning of an economic liberalisation was accompanied by the enactment of a law concerning the competitive prices. This policy of competition is based on the law of the 29 July 1991. This law emphasizes the principal of the liberty of prices, establishes rules of transparency and good functioning of the market and prohibits all the behaviours that might hinder competition.

² Information about the series is available at the website on URL <u>http://www.ins.nat.tn</u>

³ Computed assuming a 48- hour work week which yields a total of 208 hours per month.

In fact, for a recently liberalised economy, like the case of Tunisia, competition is, in a certain way, still regarded as a complementary instrument to the intervention of the state. Applying the free prices and free competition principal has to take into consideration a certain number of institutional constraints. However, a certain number of goods and services are excluded from this regime of free prices, either because they are first need products, which prices are subsidized by the state or because the concerned activities are characterised by a lack of competition. Besides, the public firms are still very important and the state is present not only in the network industries like telecommunications, energy, transports and banks but also in other sectors which are generally private in developed countries such as fertilizers, mining, electricity, construction materials etc. In general, the volume of the products which prices are regulated is 13% at the production level and 20% at the distribution level. According to the studies of the experts of the National Institute of Statistics in Tunisia, about the quarter of the Tunisian economic activities is not subject to free competition and is still under the state control. This justifies our choice of 0.75 as a value for the dummy variable during the period 1991-2004.

5.2 Empirical results

On the basis of the already presented hypotheses, we proceeded with the econometric estimations adopting log change of time series of the consumer price index ($Logipc_t = \pi_t$), labor productivity

 (G_t) , nominal wages (w_t) , and Inter professional Guaranteed Minimum Wage (SMIG) (w_t^n) . We have used annual data for all our variables- which might unfortunately limit the number of observations- because of the unavailability of monthly or quarterly ones about certain variables like labor productivity and nominal wages. The estimations were calculated using two econometric techniques: ordinary least squares and nonlinear least squares. But before moving to the estimation we have to determine the adaptive expected inflation (see appendix 1).

5.2.1 Estimation through the ordinary least squares method

Using an EVIEWS 5 software, we have estimated, at first, a model without downward nominal wages rigidity described as follows:

$$\pi_t = \alpha_0 + \alpha_1 \pi_t^a - \alpha_2 u_t + \alpha_4 dummy + \varepsilon_t$$
(17)

As a first case we took:

$$\pi_t^a = \pi_{t-1}$$

And for the second case:

$$\pi_t^a = 0.2 \sum_{i=1}^{11} 0.8^{i-n} \pi_{t-i}^{4}$$

The results of the estimation of the equation (17) through the ordinary least squares method are presented in table (I) (see appendix 3). According to the results shown in table (I), we remark the existence of an unemployment inflation trade off but only in case the expected inflation is one year lag retarded.

In the case of adaptive expectations, the relation between unemployment and inflation is positive, which is opposite to the Phillips curve predictions. That is to say, the unemployment rate coefficient is statistically insignificant.

In the case of a one year lag retarded expected inflation the elasticity of the unemployment rate is around 0.003. This coefficient is statistically significant for $\alpha = 5\%$. This implies that each 1% growth of the unemployment rate leads to an increase in the general level of prices of about 0.03%. Similarly, we remark that the general level of prices is influenced by the expected inflation and by a dummy variable that reflects the degree of the liberalisation of prices having elasticities of respectively 1.024 and 0.063. This shows the considerable impact of the expected inflation on

⁴ For further details see appendix 1.

the general level of prices, which conforms to the empirical studies of Fillon and Léonard (1997), Dinardo and Moore (1999), Kichian (2001), Khalaf and Kichian (2004) among others. The expected inflation elasticity is around 1.024, which is statistically very significant. The price liberalisation elasticity is about 0.063. This coefficient is statistically significant even to the point $\alpha = 1$ %. Our empirical results confirm the theoretical ones originating from the standard Phillips curve. In fact, the Durbin Watson (DW) value is weak, which means that the residual terms are auto interrelated. Besides, the explanatory power of the model testified to the value R² is high. In addition, An ADF (Augmented Dickey-Fuller) test proved the non-stationnarity of our time series, except for the unemployment rate, which could have led to spurious estimates and doubtful results. In this case, the ordinary least squares estimations become less certain. Consequently, such an estimation method would be inadequate to be used. Furthermore, incorporating the downward nominal wage rigidity in our model might be advantageous for the long run Phillips curve. Thus, the use of a recursive system which should be estimated by the nonlinear least squares becomes a need.

5.2.2 Estimations through nonlinear least squares method

Using an EVIEWS 5 software, the recursive system of the equations (12), (13), and (14) is estimated by the nonlinear least squares method. This same system admits a simultaneous estimation of the Phillips curve parameters and the downward nominal wage rigidity S_t variable. So, before moving to the recursive model estimation, the time series S_t are generated by an estimation process using equation (15). Similarly, the time series v_t are determined by generating an estimation process using the equation (16). The estimation results are presented in table II (see appendix 3).

A deep reading of our table (II) allows us to draw some interesting conclusions. The most important one is the ability to point out the existence of an unemployment inflation trade off even at long run but only in the case of a one year lag retarded expected inflation. Therefore, our results confirm the empirical ones of ADP (1996, 2000), Dupasquier and Ricketts (1998), among others who proved the existence of a long run asymmetric unemployment inflation relationship with the downward nominal wage rigidity hypothesis. The elasticity of unemployment is around 0.018. Such a coefficient is statistically significant for $\alpha = 10\%$.

However, contrary to ADP (1996, 2000), Dupasquier and Ricketts (1998), Beaudry and Doyle (1999), among others who established an unemployment inflation permanent trade off in the case of adaptive expectation, such a trade off with downward nominal wage rigidity doesn't exist according to our research. In fact, our findings show that there is a positive sign between unemployment and inflation, which doesn't fit in the Phillips curve predictions. In addition the value of R^2 , which is used to testify the explanatory power of the model, is rather high. And such a high value may lead to doubtful erroneous results.

According to our empirical results, the anticipated inflation has a considerable impact on the unemployment inflation trade off even at long run. Our results confirm many empirical studies as old as those of Friedman (1968)⁵ when the expected inflation was in its adaptive form. Similarly, recent empirical ones such as those of Gali and Gertler (1999), Gogley and Sbordone (2005), Karolina (2006), Bakhshi et al. (2007), Kinga and Morley (2007), Basistha and Nelson (2007), Fanelli (2008) and certainly others showed that the expected inflation in its rational form has a considerable influence on the dynamic of the inflation and the robustness of the Phillips curve in general.

Moreover, the liberalisation of prices has an effect on the formation of prices and consequently on the dynamic of inflation. In our study this liberalisation is reflected by the dummy variable which has an impact on the long run unemployment inflation trade off. It has an elasticity of around 0.28. Its coefficient is statistically significant. In deed, according to our estimation, we remark that the

⁵ Friedman (1968): "The Role of Monetary Policy". He started doubting the validity of Phillips curve as an unemployment inflation trade off. He says: "There always exists a temporary inflation unemployment trade off; it is not permanent. The temporary trade off is not due to the inflation itself but to the non expected inflation".

higher the liberty degree is, i.e. when the liberty degree increases with one point, the lower the general level of prices goes; it dimishes with 0.28 point. In our analysis, we have shown that this liberty is not absolute, and this is due to the intervention of the state in some fields of activity.

Concerning the downward nominal wage rigidity, its coefficient is also statistically significant. Our results show a negative sign of the wage rigidity coefficient. This result is opposite to the empirical results of ADP (1996, 2000), Knoppik (2001) and Stark and Sargent (2003)⁶ where the wage rigidity coefficient sign is positive. We can attribute this negativity to many factors such as the mutations witnessed by the employment situation during the previous years, the devaluation and declassement of the young graduates in Tunisia.

In fact, during the previous few years, Tunisia has witnessed some important mutations in the employment situation. Among these mutations we can cite the access of a considerable number of graduate young men and women to the labor market, the rapid development of women's activity and the increase of lay offs due to economic restructurations mainly in the public sector.

These evolutions led to the growth of unemployment among the young and, recently, the unemployment among the young graduates, and the development of a new informal sector. To put up with this situation Tunisia developed certain policies which principally consist of active programmes in the form of subsidized activities or professional training ones together with other passive help measures to help both the laid off and the unemployed. We can cite here 'the promotion programmes to employ the young' and 'the promotion instruments of independent employment and micro firms'. All these elements participate as intermediary on the labor market to bring labor supply and demand closer. However, the job supply growth is still insufficient to satisfy the needs of the demand which caused unemployment to reach high levels.

6. Conclusion

Using the nonlinear least squares method applied to annual macroeconomic data from 1962 to 2004, the empirical results of our model show the viability and the permanent pertinence of the Phillips curve incorporating the downward nominal wage rigidity in Tunisia. The recent literature introducing this kind of hypothesis supposes an adaptive behaviour on the part of the agents when anticipating inflation. Yet, our estimation seems to favour the one year lag retarded expected inflation. Furthermore, the empirical evaluation of the role allocated to the past inflation in our model is subject to a consensus. The elasticity of the expected inflation is around 1.4.

According to our results, the only parameter, with an unpredicted sign in the Phillips curve, is the one representing the downward nominal wage rigidity hypothesis though it is statistically significant. This parameter has an elasticity of about 3.16. We have justified this by the rapid growth of unemployment among the young graduates during the previous few years. The unemployment rate among these people is around 60%. In other words, the unemployment is made up of unemployment of the young whose rate lies between 30 and 40% and that of those who access the labor market for the first time. The latters are 15-24 years of age and they are badly affected by unemployment, worse than the 25-34 years. In addition, new phenomena developed for the young unemployed graduates. These are the phenomenon of devaluation and that of declassement.

However, the above developed examples show that the downward nominal wage rigidity hypothesis has a positive sign imposed by simulation before the estimation of the model. It, also, has a considerable impact on the long run unemployment inflation trade off.

Meanwhile, our empirical results examine the influence of this hypothesis on the validity and pertinence of the long run Phillips curve. Thus, this new approach allows to cross once and for ever the idea that the Phillips curve is a fact of the past.

⁶ Stark et Sargent (2003): "Is there downward nominal wage rigidity in the Canadian Phillips curve?". They showed by the SUR method during the period 1981-1999 including the impact of expected inflation and expected productivity growth that the long run Canadian Phillips curve is non linear. Moreover, they stressed the impact of expected inflation and the expected productivity along with other variables on wages.

Appendix 1

The determination of the expected inflation

We chose to use the adaptive expectations to determine the expected inflation. Their results aren't much realistic as the economic agents are more likely to consider not only the evolution of the predicted variable but also the expectation errors made in the past.

In this case the economic agents correct their expectations of the current period with a fraction of the previously stated gap between the real value and the expected one of inflation. They adapt their expectations to the recently recorded evolution of the expected inflation. We can write it as follows:

$$\pi_{t}^{a} - \pi_{t-1}^{a} = b\left(\pi_{t-1} - \pi_{t-1}^{a}\right) \tag{18}$$

With π_t^a : inflation prediction at the period t-1 for the period t.

 $\pi_{l-1} - \pi_{l-1}^a$: the most recent prediction error; i.e. the gap between the predicted value of inflation in t-1 and its observed value at the same period.

b: the error percentage that the agents correct in their previous prediction in order to formulate new expectations.

Let's develop the first equation; we obtain

$$\pi_t^a = (1-b)\pi_{t-1}^a + b\pi_{t-1} \tag{19}$$

For the adaptive process to be optimal (that's to say minimize prediction errors), we must consider not only the most recent prediction error but also all the previous errors.

Therefore the general form of the adaptive process is as follows:

$$\pi_t^a - \pi_{t-1}^a = \sum_{i=1}^n b_i \left(\pi_{t-i} - \pi_{t-i}^a \right) \tag{20}$$

Let's consider the first order of the adaptive process as follows:

$$\pi_{t}^{a} - \pi_{t-1}^{a} = b \left(\pi_{t-1} - \pi_{t-1}^{a} \right) \text{ with } 0 < b < 1$$

$$\pi_{t}^{a} = \pi_{t-1}^{a} + b \left(\pi_{t-1} - \pi_{t-1}^{a} \right)$$

A successive substitution of $\pi_{t-1}^{a}, \pi_{t-2}^{a}, \dots, \pi_{t-n}^{a}$, will lead us to get

$$\pi_{t}^{a} = b \pi_{t-1} + b(1-b)\pi_{t-2} + b(1-b)^{2} \pi_{t-3} + \dots + b(1-b)^{n-1} \pi_{t-n}$$

$$\Rightarrow \quad \pi_{t}^{a} = b \sum_{i=1}^{n} (1-b)^{i-n} \pi_{t-i}$$
(21)

Where *b* is the unknown adjustment coefficient.

Yet, to calculate π_t^a , we have to know *b*. moreover the value of *b* is determined in such a way that the average expected losses be minimal.

Let, for instance, L the function that characterizes the average quadratic losses:

$$L = \sum_{t=1}^{T} \left(\pi_{t} - b \sum_{i=1}^{\infty} (1 - b)^{i-1} \pi_{t-i} \right)^{2}$$
(22)

Where *t* takes the values from 1962 to 2003.

So the choice of b supposes that L is minimal and then the longer T is, the more the choice of b is optimal.

If we limit the lagging length for π_t to $n < \infty$ and give *b* values ranging between 0 and 1 with a step of 0.1, we can find the value of *b* which minimizes *L* and then π_t^a is estimated.

On the other hand the value of n is determined by the minimization of the Akaike (1974) function given by the following equation:

Alc
$$(i) = Ln\left(\frac{ScRi}{n}\right) + \frac{2i}{n}$$
 (23)

With *ScRi* : the sum of squared residuals;

n: the number of observations.

When we applied this procedure we found that the number of lags is equal to 11 and b=0.2 and then the expected inflation can be determined by the following equation:

$$\pi_t^a = 0.2 \sum_{i=1}^{11} 0.8^{i-n} \pi_{t-i}$$
(24)

Appendix 2 Figures

Figure (1): evolution of log change of the nominal wages (lw_t)



Figure (2): evolution of log change of the SMIG $(l w_t^n)$



Appendix 3 Tables

Dependent Variable (π_t)					
Independent Variable	1 st case : $\pi_t^a = \pi_{t-1}$	$2^{\text{nd}} \operatorname{case} : \pi_t^a = 0.2 \sum_{i=1}^{11} 0.8^{i-n} \pi_{t-i}$			
Constant	0.025 (0.755)	0.036 (0.25)			
π^a_t	1.024 (14)	1.14 (24.9)			
u_t	-0.003 (-2.85)	0.005 (0.65)			
dummy	-0.063 (-4.03)	-0.19 (-3.03)			
R ² DW N	0.99 1.59 42	0.98 0.30 42			

Table (I) Estimation of Phillips curve without rigidity

- Asymptotic t - values in parentheses

- N: observations number

Table (II)) Estimation	of Phillips	curve	with	rigidity
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Dependent Variable (π_t)					
Independent Variable	1 st case $\pi_t^a = \pi_{t-1}$	2 nd case $\pi_t^a = 0.2 \sum_{i=1}^{11} 0.8^{i-n} \pi_{t-i}$			
Constant	-0.61 (-1.98)	0.13 (0.68)			
π^a_t	1.40 (23.59)	1.14 (19.23)			
<i>u</i> _t	$0.018 \\ (1.91)^8$	-0.002 (-0.21)			
$\sigma_0^{\ 7}$	0.11 (1.139)	-0.29 (-2.35)			
S_t	-3.16 (-14.01)	-4.25 (-12.67)			
dummy	-0.28 (-2.44)	-0.16 (-1.98)			
$\frac{R^2}{N}$	0.94 42	0.99 32			

- Asymptotic t - values in parentheses

- N: observations number

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⁷ σ₀ is the standard deviation of the gap between lagged wages and notional wages $v_t = \frac{w_{t-1} - w_t^n}{\pi_t^a G_t}$.

⁸ Denotes significance at 10% level.

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