Abstract
We examine how consumption externalities affect R&D activities by using a simple innovation-driven growth model where the sources of growth are both horizontal and vertical innovations. We show that if there are negative (positive) consumption externalities, the economy attains a higher (lower) variety expansion rate than the economy without consumption externalities, whereas the quality enhancement rate becomes lower (higher). If the elasticity of substitution between any two goods is high (low), the economy with positive consumption externalities tends to attain a higher (lower) output growth rate than the economy without consumption externalities.

I would like to express my sincere gratitude to Koichi Futagami for his helpful comments. All remaining errors are mine.

1. Introduction

The recent studies have shown that the external effects of consumption may have significant effects on growing economy both qualitatively and quantitatively. Carroll et al. (1997, 2000), for example, investigate effects of consumption externalities on savings and growth pattern in an endogenous growth model with Ak technology.¹

Most of the existing studies about the effects of consumption externalities on economic growth assume a homogeneous consumption good and perfect competition. Departing from these existing studies, Doi and Mino (2008) investigate effects of consumption externalities in a model where there are a variety of consumption goods that are produced by monopolistically competitive firms. The range of variety of consumption goods can be increased by R&D activities. They assume that consumers set a benchmark consumption level for each consumption good. In their model, therefore, there exists commodity-specific external effects. They argue that each monopolistic firm may exploit commodity-specific external effects because commodity-specific external effects influence consumer’s demand for its own product. They therefore assume that each firm can internalize commodity-specific externalities when maximizing its operating profits. Consequently, commodity-specific externalities have influences on the pricing decision of the monopolistic firms in their model. However, commodity-specific externalities do not have any direct effects on the incentive for R&D activities. In their model, the intensity of R&D activities is affected by commodity-specific externalities through the effects on the pricing decision.

In this note, we introduce into the model quality differentiation of consumption goods that is not considered by Doi and Mino (2008). And then, we show that commodity-specific externalities have a direct effect on the incentive for the quality-enhancing R&D activities.

We obtain the following results. If there are negative (positive) consumption externalities, that is, each household’s instantaneous utility is negatively (positively) affected by the benchmark level of consumption, the economy attains a higher (lower) variety expansion rate than the economy without consumption externalities. However, the quality enhancement rate becomes lower (higher) in the presence of negative (positive) consumption externalities. If the elasticity of substitution between any two goods is high (low), the economy with positive consumption externalities tends to attain a higher (lower) output growth rate than the economy without consumption externalities.

The rest of the note is organized as follows: In Section 2, we present the structure of our model. A general equilibrium is derived in Section 3. Section 4 considers the growth effects of commodity-specific externalities. Concluding remarks are in Section 5.

2. The Model

We introduce consumption externalities into the model of Young (1998), where the sources of growth are both horizontal and vertical innovations.²

2.1. Households

¹The recent studies have examined the effects of consumption externalities on indeterminacy (Chen and Hsu (2007) and Alonso-Carrera (2007)), on growth pattern (Alonso-Carrera (2005a)), on income inequality (Hori (2009)), and, on the efficiency of the dynamic equilibrium in an economy with dynastic altruism (Alonso-Carrera (2005b)). Using a two-period overlapping generations model with consumption externalities, Knell (1999) examines the condition under which income inequality has a negative impact on growth. Mino (2007) also uses a simple two-period overlapping generations model to examine the role of consumption externalities.

²The purpose of Young (1998) is to present a model of growth without scale effects. Because our purpose is to examine the effects of consumption externalities on horizontal and vertical innovation, we do not eliminate scale effects.
The economy is populated by identical consumers whose number is normalized to one. Each household inelastically supplies $L$ units of labor in any periods. As in Young (1998), the utility function of the representative household is given by

$$U_0 = \sum_{i=0}^{\infty} \beta^t \ln C(t),$$

where $\beta \in (0, 1)$ is the subjective discount factor, and $C(t)$ represents an index of consumption that is specified as:

$$C(t) = \left\{ \int_0^{n(t)} \left[ (q_j(t)c_j(t)) x_j(t)^{-\alpha} \right]^\alpha d \bar{j} \right\}^{\frac{1}{\alpha}}, \quad 0 < \alpha < 1. \quad (1)$$

In the above equation, $c_j(t)$ denotes consumption of good $j$ whose quality is $q_j(t)$, and $n(t)$ denotes the number of consumption goods. The elasticity of substitution between any two consumption goods is $1/(1 - \alpha)$. Besides the level of household’s own consumption, utility depends on the benchmark level of consumption $x_j(t)$ that is equal to $q_j(t) \tilde{c}_j(t)$ where $\tilde{c}_j(t)$ is the average (total) consumption of good $j$ whose quality is $q_j(t)$. The presence of $x_j(t)$ represents commodity-specific externalities.\(^3\) We assume that the parameter $\theta$ satisfies $1 - 1/(2\alpha) < \theta < 1$. This assumption ensures that problems of firms are well-defined. In this note, we say that there are negative (positive) consumption externalities, if each household’s instantaneous utility is negatively (positively) affected by the benchmark level of consumption, $\theta > 0 \ (\theta < 0)$. When there are no commodity-specific externalities ($\theta = 0$), our utility function is exactly the same as the one employed by Young (1998).

Consumption expenditure of the representative household is given by

$$E(t) = \int_0^{n(t)} p_j(t)c_j(t) d \bar{j}. \quad (2)$$

For a given consumption expenditure $E(t)$, the representative household maximizes $C(t)$ in any periods. The solution for the problem is

$$c_j(t) = \frac{E(t)x_j(t)^{-\alpha} q_j(t)^{\alpha} p_j(t)^{1-\alpha}}{\int_0^{n(t)} (p_j(t)q_j(t)^{-1} x_j(t)^{\theta})^{-\frac{\alpha}{1-\theta}} d \bar{i}}. \quad (3)$$

The above equation represents the demand for good $j$ of a single household.\(^4\) Because $\alpha$ is in the interval of $(0, 1)$, (3) shows that private consumption of good $j$ decreases (increases) with benchmark consumption $x_j$ if there are negative (positive) consumption externalities, $\theta > 0 \ (\theta < 0)$. As pointed out by Doi and Mino (2008), therefore, negative consumption externalities are associated with ”running away from the Joneses” (RAJ), while positive consumption externalities are associated with ”keeping up with the Joneses” (KUJ) in our specification.\(^5\)

\(^3\)Our specification of commodity-specific externalities in (1) follows Doi and Mino (2008).

\(^4\)Using the first order conditions, we can derive $q_j(t)c_j(t) = \left\{ \hat{p}_j(t)/\bar{p}_j(t)(x_j(t)/\tilde{x}_j(t))^{\omega - \alpha} \right\}^{1/(1-\alpha)} q_j(t)c_j(t)$ where $\hat{p}_j(t) = p_j(t)/q_j(t)$. By substituting this relationship into (2) and after some manipulations, we can derive (3).

\(^5\)Dupor and Liu (2003) point out that if the marginal utility of private consumption increases (decreases) with the benchmark level of consumption, consumers’ preferences show KUJ (RAJ). Doi and Mino (2008) show that given the specification (1), the marginal utility of private consumption increases (decreases) with the benchmark level of consumption when $\theta$ is negative (positive).
By using (3), we obtain \( C(t) = E(t)/p_C(t) \), where \( p_C(t) \equiv \left\{ \int_0^{e(t)} \left( p_i(t)q_i(t)^{-1}x(i)^\theta \right)^{\frac{1-\alpha}{1-\alpha(1-\theta)}} dt \right\}^{-\frac{1-\alpha}{1-\alpha(1-\theta)}} \) represents the price index. The aggregate consumption expenditure then evolves according to the following Euler equation:

\[
\frac{E(t + 1)}{E(t)} = \beta(1 + r(t)),
\]

where \( r(t) \) is the interest rate between periods \( t \) and \( t + 1 \).

### 2.2. Firms

Each good is produced by the monopolistically competitive firms. Production of one unit of each good requires one unit of labor. The operating profits in period \( t \) is equal to \( \pi_j(t) = (p_j(t) - w(t))\bar{c}_j(t) \), where \( p_j(t) \) is the price of good \( j \) and \( w(t) \) is the wage rate. We take labor as the numeraire and set \( w(t) \) equal to one.

In each period, each firm charges a price that maximizes its operating profits. Doi and Mino (2008) argue that each monopolistic firm may exploit commodity-specific external effects because commodity-specific external effects influence consumer’s demand for its own product. They therefore assume that each firm can internalize commodity-specific externalities when maximizing its operating profits. This assumption means that each firm knows the fact that \( c_j(t) \) is equal to \( \bar{c}_j(t) \) because households are identical. We follow Doi and Mino (2008) and internalize commodity-specific externalities by substituting \( c_j(t) = \bar{c}_j(t) \) into (3) and solving for \( \bar{c}_j(t) \). We then obtain

\[
\bar{c}_j(t) = \frac{E(t)}{\int_0^{e(t)} \left( p_i(t)q_i(t)^{-1}x(i)^\theta \right)^{\frac{1-\alpha}{1-\alpha(1-\theta)}} dt} \left( q_j(t)^{\alpha(1-\theta)} p_j(t)^{-1} \right) \bar{c}_j(t)^{\alpha(1-\theta)}. (5)
\]

The above equation represents the relationship between the price of good \( j \) and the total demand for good \( j \). The price elasticity \( \varepsilon_p \equiv 1/[1 - \alpha (1 - \theta)] > 0 \) decreases with \( \theta \). When the preference satisfies KUJ \( (\theta < 0) \) (RAJ \( (\theta > 0) \)), the firms face a more (less) elastic demand curve. Faced with a decrease in price, each household increases private consumption \( c(j) \), which leads to an increase in the average consumption \( \bar{c}_j(t) \). An increase in \( \bar{c}_j(t) \) yields an external effect. When KUJ prevails, each household has an incentive to increase private consumption further. As a result, the demand curve becomes more elastic. The elasticity with respect to the quality \( \varepsilon_q \equiv \alpha(1 - \theta)/(1 - \alpha (1 - \theta)) > 0 \) also decreases with \( \theta \). Faced with an increase in the quality, each household increases private consumption \( c(j) \), which leads to an increase in the average consumption \( \bar{c}_j(t) \). When KUJ prevails, an increase in \( \bar{c}_j(t) \) positively affects each household’s consumption through commodity-specific external effects. As a result, the demand curve becomes more elastic with respect to the quality. Given (5), each firm maximizes the operating profits \( \pi_j(t) = (p_j(t) - 1)\bar{c}_j(t) \) by charging the price below:

\[
p_j(t) = \frac{1}{\alpha(1 - \theta)} \equiv p. \quad (6)
\]

Each firm charges the same price \( p \) in any periods.\(^6\) In the economy with positive (negative) consumption externalities \( \theta < 0 (\theta > 0) \), the firms charge a lower (higher) price than in the

---

\(^6\)In Doi and Mino (2008), the benchmark level of consumption \( x_j \) is equal to a weighted sum of the past average consumption of good \( j \) up to the present. They also derive a monopolistic price, and show that the firm charges the same monopolistic price as (6) when the benchmark level of consumption depends on today’s average consumption alone as in our model.
economy without consumption externalities because the firms face a more (less) elastic demand curve. Our assumption with respect to $\alpha$ and $\theta$ ensures that $p$ is larger than the wage rate ($w(t) = 1$). The production level of firm $j$ and the maximized profits are given by

$$
\bar{c}_j(t) = \frac{E(t)}{p} \int_0^{t(n(t))} q_i(t)^{-\alpha(1-\theta)/\theta} di.
$$

(7)

$$
\pi_j(t) = \{1 - \alpha(1 - \theta)\} E(t) \int_0^{t(n(t))} q_i(t)^{-\alpha(1-\theta)/\theta} di.
$$

(8)

In order to produce good $j$ in period $t$, the firms must engage in R&D activities in period $t - 1$. Our modeling of R&D activities follows Young (1998). R&D activities require the fixed amounts of labor that is given by

$$
F(q_j(t), \bar{q}(t - 1), n(t - 1)) = \frac{1}{n(t - 1)} a \left( \frac{q_j(t)}{\bar{q}(t - 1)} \right).
$$

(9)

where $a(\cdot)$ is an increasing and convex function that takes positive values, and $\bar{q}(t - 1)$ is the average of $q_j(t - 1)$. The presence of $\bar{q}(t - 1)$ and $n(t - 1)$ in (9) reflects the intertemporal knowledge spillover that sustains growth. Because firms must incur R&D costs in each period and can not appropriate the intertemporal knowledge spillover, the planning horizon of each firm is only one period. Each firm therefore chooses $q_j(t)$ so as to maximize

$$
\Pi_j = \frac{\pi_j(t)}{1 + r(t - 1)} - \frac{1}{n(t - 1)} a \left( \frac{q_j(t)}{\bar{q}(t - 1)} \right).
$$

The first order condition is given by

$$
\frac{\partial \pi_j(t)}{\partial q_j(t)} = \frac{1}{n(t - 1) \bar{q}(t - 1)} a' \left( \frac{q_j(t)}{\bar{q}(t - 1)} \right).
$$

(10)

If $\varepsilon_q$ is smaller than one, the second order condition is satisfied. It is easily verified that the assumption $1 - 1/(2\alpha) < \theta < 1$ ensures the inequality $\varepsilon_q < 1$. The free entry implies that $\Pi_j$ is equal to zero in equilibrium.

$$
\frac{\pi_j(t)}{1 + r(t - 1)} = \frac{1}{n(t - 1)} a \left( \frac{q_j(t)}{\bar{q}(t - 1)} \right).
$$

(11)

Dividing the both sides of (10) by (11), we obtain the following equation:

$$
\varepsilon_q = \frac{g_j(t)a'(g_j(t))}{a(g_j(t))}.
$$

(12)

where $g_j(t) \equiv q_j(t)/\bar{q}(t - 1)$ denotes the growth rate of the quality of good $j$.

Because all firms are identical, $q_j(t) = \bar{q}(t)$ holds for all $j$. The above equation therefore determines the growth rate of the average quality that we henceforth denote by $g_{q^*}$. Although the right-hand side of (12) can be either increasing or decreasing, the second order condition ensures that the right-hand side of (12) must have a positive slope in equilibrium. The figure below presents the case where $g_{q^*}$ is uniquely determined. By using this figure, we can prove the existence of $g_{q^*}$.

---

5 Young (1998) specifies $a(q_j(t)/\bar{q}(t - 1)) = f e^{\mu q_j(t)/\theta(t - 1)}$ where $f$ and $\mu$ are parameters.

6 Young (1998) derives the same condition as (12).

7 The derivative of the right-hand side of (12) takes a positive value when $g_{j a'(g_j)}/a(g_j) < 1$ holds. The second order condition, together with (12), ensures that $g_{j a'(g_j)}/a(g_j)$ is smaller than one in equilibrium.
Figure. The Quality Enhancement R&D Activities

Proposition 1
Suppose that \( a'(1)/a(1) \) is strictly smaller than \( \varepsilon_q \) and \( \lim_{x \to \infty} xa'(x)/a(x) \) is strictly larger than \( \varepsilon_q \), and that \( xa'(x)/a(x) \) is a strictly increasing function of \( x \). There exists a unique \( g_q^* \) that is larger than one.

Because \( \varepsilon_q \) decreases with \( \theta \), we can examine the effects of consumption externalities on \( g_q^* \).

Proposition 2
The economy with positive (negative) consumption externalities, \( \theta < 0 (\theta > 0) \), attains a higher (lower) quality enhancement rate \( g_q^* \) than the economy without consumption externalities.

From (7) and (8), we know that an improvement in the quality increases the demand and the operating profits. In the economy with positive consumption externalities, firms face a demand curve that is more elastic with respect to the quality of good. The operating profits also becomes more elastic with respect to the quality of good. Each firm has a stronger incentive to improve the quality of its own product. The quality enhancement rate \( g_q^* \) therefore becomes higher in the economy with positive consumption externalities. Consumption externalities apparently have direct impacts on the incentive for the quality-enhancing R&D activities.

3. The General Equilibrium

The labor allocated to the production is given by

\[
L_p(t) = \int_{0}^{n(t)} \bar{c}_j(t) dj = \frac{E(t)}{p(t)} = \alpha(1 - \theta)E(t). \tag{13}
\]

In the above, we use (6) and (7). Because \( q_i(t) = q_j(t) \) holds for all \( i, j \), (8) implies that the operating profits of each firm is

\[
\pi_j(t) = \{1 - \alpha(1 - \theta)\} \frac{E(t)}{n(t)}. \tag{14}
\]
By using (4), (11) and (14), we can obtain the labor allocated to R&D activities:

\[ L_R(t) = \frac{n(t+1)}{n(t)} a(g^*_q) = \beta[1 - \alpha(1 - \theta)]E(t). \]  

(15)

The labor market equilibrium requires \( L_R(t) + L_p(t) = L \). By substituting (13) and (15) into this equilibrium condition and solving for \( E(t) \), we obtain

\[ E(t) = \frac{L}{\beta + \alpha(1 - \beta)(1 - \theta)}. \]  

(16)

From (15) and (16), we can derive the growth rate of \( n(t) \).

\[ g^*_n \equiv \frac{n(t+1)}{n(t)} = \frac{\beta[1 - \alpha(1 - \theta)]}{\beta + \alpha(1 - \beta)(1 - \theta) a(g^*_q)} \frac{L}{a(g^*_q)}. \]  

(17)

It is easy to examine the effects of consumption externalities on the variety expansion rate \( g^*_n \).

**Proposition 3**

The economy with positive (negative) consumption externalities, \( \theta < 0 (\theta > 0) \), attains a lower (higher) variety expansion rate \( g^*_n \) than the economy without consumption externalities.

(Proof) The term \( [1 - \alpha(1 - \theta)]/[\beta + \alpha(1 - \beta)(1 - \theta)] \) increases with \( \theta \). Because \( g^*_q \) decreases with \( \theta \) and \( a(\cdot) \) is an increasing function, the term \( L/a(g^*_q) \) increases with \( \theta \). □

Consumption externalities affect \( g^*_n \) through two channels. As discussed in Subsection 2.2, firms face a less elastic demand curve in the economy with negative consumption externalities, \( \theta > 0 \).

In the economy with negative externalities, therefore, each firm charges a higher price than in the economy without consumption externalities, which implies that the production of each monopolistic firm becomes smaller. Compared with the economy without consumption externalities, therefore, each firm employs a small amount of labor for production in the economy with negative consumption externalities. The second effect works through the quality-enhancing R&D activities. The economy with negative consumption externalities, \( \theta > 0 \), attains a lower quality enhancement rate than the economy without consumption externalities, which implies that labor requirement for the quality-enhancing R&D activities is smaller in the economy with negative consumption externalities. With reduced labor demand in production and reduced labor requirement in the quality-enhancing R&D activities, a large number of firms can enter the market in the economy with negative consumption externalities. As a result, the variety of consumption goods grows at a high rate. Consumption externalities affect the variety expansion rate through the pricing decision of firms and the quality-enhancing R&D activities. Although consumption externalities have direct impacts on the incentive for the quality-enhancing R&D activities, consumption externalities have only indirect influences on the intensity of the horizontal R&D activities. \(^{10}\)

4. The Growth Effects

In the symmetric equilibrium, consumption index can be written as:

\[ C(t) = \left\{ \frac{\alpha(1 - \theta)L}{\beta + \alpha(1 - \beta)(1 - \theta)} \right\}^{1-\theta} q(t)^{1-\theta} n(t)^{\frac{1}{2}-(1-\theta)}. \]

\(^{10}\)In Doi and Mino (2008), consumption externalities indirectly affect the variety expansion rate only through the first channel, that is, the effects on the pricing decision.
The growth rate of consumption index is given by
\[ g_C^* = g_q^* \frac{1-\theta}{\theta} g_n^{\frac{1}{\gamma}-(1-\theta)}. \]  
\[ \text{(18)} \]

We can interpret \( g_C^* \) as the growth rate of output. The output growth is driven by quality enhancements and variety expansion. By using (18), we can examine the growth effects of consumption externalities. In Doi and Mino (2008), the quality-enhancing R&D activities are not considered. The output growth is then driven by the variety expansion. The variety expansion rate increases with \( \theta \). If there do not exist the quality-enhancing R&D activities as in Doi and Mino (2008), the economy with positive (negative) consumption externalities unambiguously attains a lower (higher) growth than the economy without consumption externalities.

Once quality improvement is taken into consideration, however, consumption externalities have an ambiguous effect. Differentiating \( g_C^* \) with respect to \( \theta \), we obtain
\[ \frac{\partial g_C^*}{\partial \theta} = g_C^* \left\{ \ln \frac{g_n^*}{g_q^*} + \frac{1}{\beta + \alpha(1-\beta)(1-\theta)} \right\}. \]

In deriving the above expression, we use equation (12). The sign of \( \partial g_C^* / \partial \theta \) can be positive or negative, depending on the parameters and the functional form of \( a(\cdot) \). If \( g_q^* \) is larger (smaller) than \( g_q^* \exp[1/(\beta + \alpha(1-\beta)(1-\theta))] \), an increase in \( \theta \) has a negative (positive) marginal effect on \( g_C^* \). In other words, a rise in \( \theta \) decreases (increases) \( g_C^* \) in the economy where the output growth is mainly driven by quality enhancement (variety expansion), that is, \( g_q^* > (<) g_q^* \exp[1/(\beta + \alpha(1-\beta)(1-\theta))] \). This result is very intuitive. From Propositions 2 and 3, we know that an increase in \( \theta \) has a negative effect on \( g_q^* \) while \( g_n^* \) is positively affected by an increase in \( \theta \). Then, in the economy whose output growth is mainly driven by quality enhancement (variety expansion), an increase in \( \theta \) decreases the output growth rate. The next proposition summarizes the results.

**Proposition 4**

Suppose that in the economy without consumption externalities (\( \theta = 0 \)), the output growth is mainly driven by quality enhancement (variety expansion). As long as \( \theta \) is sufficiently close to zero, the economy with positive consumption externalities (\( \theta < 0 \)) attains a higher (lower) \( g_C^* \) than the economy without consumption externalities, whereas the economy with negative consumption externalities (\( \theta > 0 \)) attains a lower (higher) \( g_C^* \) than the economy without consumption externalities.

In what economy is the output growth mainly driven by quality enhancement (variety expansion)? Because \( \varepsilon_q \) increases with \( \alpha \), a rise in \( \alpha \) increases \( g_q^* \) (see the figure just above Proposition 1). A large \( \alpha \) indicates a high elasticity of substitution between any two goods. The quality elasticity \( \varepsilon_q \) therefore increase with \( \alpha \). Faced with a more elastic demand curve, each firm has a stronger incentive for the quality-enhancing R&D activities. As a result, \( g_q^* \) increases with \( \alpha \). In contrast, a rise in \( \alpha \) decreases \( g_n^* \) because an increase in \( \alpha \) intensifies the quality-enhancing R&D activities and then raises the fixed labor cost (9). This can be verified by (17). In addition, the term \( \exp[1/(\beta + \alpha(1-\beta)(1-\theta))] \) also decreases with \( \alpha \). The discussion so far suggests that in the economy where \( \alpha \) is high (low), it is likely that the output growth is mainly driven by quality

---

11Consumption index (1) can be interpreted as a production function, where the average of each input \( q(t)\bar{c}(t) \) yields external effects.

12The term \( (1-\alpha(1-\theta))/(\beta + \alpha(1-\beta)(1-\theta)) \) decreases with \( \alpha \). Because \( g_q^* \) increases with \( \alpha \) and \( a(\cdot) \) is an increasing function, the term \( L/\alpha(g_q^*) \) decreases with \( \alpha \). Therefore, a rise in \( \alpha \) decreases \( g_q^* \).
enhancement (variety expansion), that is, \( g_q^* > (\leq)g_q^\ast \exp\left[1/(\beta + \alpha(1 - \beta)(1 - \theta))\right] \). We then conclude that if the elasticity of substitution between any two goods is high (low), the economy with positive consumption externalities tends to attain a higher (lower) \( g_q^* \) than the economy without consumption externalities.

5. Conclusion

We examine how consumption externalities affect R&D activities by using a simple innovation-driven growth model. We obtain the following results. If there are positive (negative) consumption externalities, that is, each household's instantaneous utility is positively (negatively) affected by the benchmark consumption, the economy attains a higher (lower) variety expansion rate than the economy without consumption externalities. However, the quality enhancement rate becomes lower (higher) in the presence of negative (positive) consumption externalities. If the elasticity of substitution between any two goods is high (low), the economy with positive consumption externalities tends to attain a higher (lower) output growth rate than the economy without consumption externalities.
References


