Abstract
This paper explores the growth effects of the enlargement of an integrated economy by making use of a many-country equilibrium growth model with heterogeneous labor. We prove that the impact of the number of member countries on the integrated economy’s growth rate is positive. In addition, we also analyze the growth effects of the individual countries after integration. We find that enlargement may be detrimental to the economic growth of advanced countries.

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1. Introduction

The growth effects of economic integration have in recent years received considerable attention. Most studies suggest that increased economic integration can cause a permanent increase in the rate of economic growth. The economic implication is that dismantling barriers to the flow of all goods, ideas, and factors among the member countries would accelerate long-term economic growth. However, casual observation suggests that the enlargement of the European Union leads to a decrease in the growth rates of the advanced member countries, for example, France, Germany, the Netherlands, Finland, Sweden, and Italy. The purpose of this paper is to explain the phenomenon.

We draw on the ideas of Grossman and Maggi (2000) and Das (2005), in which the assumption of heterogeneous human capital is introduced into an equilibrium growth model and consider that the R&D sector produces new blueprints or ideas which provide the engine of growth, as in Romer (1990) and Jones (2005). We focus on examining the relationship between the enlargement of an integrated economy and the equilibrium growth rates. Our main finding is that the enlargement of the integrated economy will be conducive to economic growth. In addition, we also analyze the growth effects of the individual countries after integration. We show that the enlargement may be detrimental to the economic growth of advanced countries.

This paper is organized as follows. Section 2 establishes the equilibrium growth model with heterogeneous labor. Section 3 considers the impact of the deeper economic integration on growth. Section 4 is concluding remarks.

2. The Model

There are $k$ small open countries ($k \geq 2$), 1, 2, ⋯, and $k$, each with a continuum of workers. Let $L_j$ be the measure of labor forces for country $j$ ($j \in \{1, 2, \ldots, k\}$). Every worker’s talent $n$ is heterogeneous and perfectly observable and could represent a worker’s endowment and years of schooling. Assume that talent’s distribution is uniform with probability density function $\phi(n)$ as shown below:

The variable $b_j$ represents the diversity of talent. The larger the variable $b_j$ is, the more diverse the distribution of talent will be. We assume that $n_{\min}^j$ and $n_{\max}^j$ are the minimum and maximum talent levels respectively and $\bar{n}^j$ is the average talent level.

Each country has two sectors including the consumption-good sector and R&D sector. Suppose that those countries are similar in their production technologies, referring to the supermodular and submodular technologies, as derived by Milgrom and Roberts (1990), Kremer (1993) and Grossman and Maggi (2000). The production process involves two tasks, task $x$ and task $v$ in each sector. The tasks are indivisible and each task is performed by exactly one worker. In the consumption-good sector, which we denote the $C$ sector, a pair of workers performs complementary tasks. Let $\eta^j F_C^j (n_x, n_v)$ be the supermodular production function in the $C$ sector when the first task (task $x$) is performed by a worker with talent $n_x$ and the second (task $v$) by a worker with talent $n_v$. For simplicity, we assume that the complementarity is extreme. Hence, the production function of $C$ sector can be specified as: $\eta^j F_C^j (n_x, n_v) = \eta^j \min\{n_x, n_v\}$. On the other hand, the R&D sector produces the new blueprints $\dot{\eta}^j$ (the time derivative of $\eta^j$), which accelerates technology improvement for producing the consumption-good. In the meantime, as in Romer (1990) and Das (2005), the level of existing technology or the stock of blueprints has a positive influence on the output of the R&D sector. In contrast to $C$ sector, in the R&D sector, which we denote the $S$ sector, the talent of the superior worker fully dominates the effective output, and the workers toil on substitutable tasks. Let $\eta^j F_S^j (n_x, n_v)$ be the submodular production function in the $S$ sector. For simplicity, we also assume that the substitutability is extreme. Thus, the production function of $S$ sector can be specified as: $\eta^j F_S^j (n_x, n_v) = \eta^j \max\{n_x, n_v\}$.

Grossman and Maggi (2000) prove that in equilibrium the $C$ sector employs workers with similar abilities, i.e., “skill-clustering”, and the $S$ sector attracts the most-talented and least-talented workers, i.e., “cross-matching”. We define the variable $\hat{n}^j$ as representing the least-talented worker and $m^j(\hat{n}^j) = 2\bar{n}^j - \hat{n}^j$ as representing the most-talented worker in the $C$ sector. Consequently, the level of
output per capital of good $C$ (denoted by $y_C^j$) is

$$y_C^j = \frac{Y_C^j}{L^j} = \int_{\hat{\eta}^j}^{m^j(\hat{n}^j)} \eta^n F_C^j(n, n)\phi(n)dn = \frac{\eta^n\hat{n}^j}{b^j}(\hat{n}^j - \hat{\eta}^j), \quad (1)$$

where the variable $Y_C^j$ represents the total output of good $C$. As in equation (3) of Das (2005), we assume that the level of output per capital of good $S$ must be equal to $\eta^j$. Therefore, the level of output per capital of good $S$ (denoted by $y_S^j$) is

$$y_S^j = \frac{Y_S^j}{L^j} = \hat{\eta}^j = \int_{\eta_{\text{min}}}^{\hat{\eta}^j} \eta^n F_S^j[n, m^j(n)]\phi(n)dn = \frac{\eta^n}{2\hat{h}^j}(\frac{b^j}{2} - \hat{n}^j + \hat{\eta}^j)(\frac{b^j}{2} + 3\hat{n}^j - \hat{\eta}^j), \quad (2)$$

where the variable $Y_S^j$ represents the total output of good $S$.

The production possibility frontier of country $j$ is strictly concave and its marginal rate of transformation (MRT$^j$) can be calculated as following:

$$\text{MRT}^j = -\frac{\partial y_C^j}{\partial y_S^j} = -\frac{\partial y_C^j}{\partial \hat{n}^j} = \frac{\hat{n}^j}{2\hat{n}^j - \hat{\eta}^j}. \quad (3)$$

Equation (3) should be equal to the relative supply price of good $S$, say $1/p^j_{\text{supply}}$.

That is,

$$\frac{1}{p^j_{\text{supply}}} = \frac{\hat{n}^j}{2\hat{n}^j - \hat{\eta}^j}, \quad (4)$$

where $p^j_{\text{supply}}$ represents the relative supply price of good $C$.

Following Das (2005), we assume that the government imposes the income-tax and fully funds the new blueprints in a competitive market. Meanwhile, these new blueprints would be freely offered to the consumption-good sector. The tax proceeds equal $\tau^j[y_C^j + (1/p^j_{\text{demand}})y_S^j]$. Thus, $\tau^j[y_C^j + (1/p^j_{\text{demand}})y_S^j] = (1/p^j_{\text{demand}})y_S^j$. Or,

$$\frac{1}{p^j_{\text{demand}}} = \Gamma^j \frac{y_C^j}{y_S^j}, \quad \Gamma^j = \frac{\tau^j}{1 - \tau^j}, \quad (5)$$

where $\tau^j$ is the income tax rate. The variable $p^j_{\text{demand}}$ is the relative demand price of good $C$ and hence $1/p^j_{\text{demand}}$ represents the relative demand price of good $S$.

In the free-trade equilibrium, the world relative price of good $C$, $p$, is given and

$$p = p^j_{\text{supply}} = p^j_{\text{demand}}. \quad \text{By substituting } p^j_{\text{supply}} = p \text{ into equation (4), we obtain}$$
\( \hat{n}^j = (2 - p)\bar{n}^j \) (time-invariant) and hence solve the relative supply of good \( C \).\(^2\)

Again, substituting \( p^j_{\text{demand}} = p \) into equation (5) can get the relative demand of good \( C \). Therefore, we can determine the pattern of trade which would rely on \( p \), \( \tau^j \), \( b^j \), and \( \bar{n}^j \).\(^3\) We obtain similar results in Grossman and Maggi (2000).\(^4\)

2.1 Growth

As analysis earlier, in the free-trade equilibrium \( \hat{n}^j \) is independent of time. By differentiating equation (1) with respect to time, we can derive that the growth rate of consumption goods is \( g^j = \hat{\eta}^j / \eta^j \). Combining equation (2) with \( \hat{n}^j = (2 - p)\bar{n}^j \) and eliminating \( \hat{n}^j \) can find the growth rate of country \( j \) as follows:

\[
g^j = \frac{1}{2b^j} [\frac{b^j}{2} + (1 - p)\bar{n}^j] [\frac{b^j}{2} + (1 + p)\bar{n}^j]. \tag{6}
\]

There is no transitional dynamics. As we can see from equation (6), the factors affecting the growth rate include \( b^j \), \( p \), and \( \bar{n}^j \).\(^5\) These results are similar to Das (2005).\(^6\)

3. Economic Integration and Growth

In this section, we will analyze the impact of the integration on economic growth. Consider that countries 1, 2, \( \ldots \), and \( k \) join together to create a common market. Namely, workers could be freely mobile between member countries. For simplicity and without losing generality, suppose that the diversities of talent and the measures

\(^2\) We believe that every worker’s talent is positive, i.e., \( n^j_{\text{min}} > 0 \). Therefore, in order to purge off the corner solution in the free-trade equilibrium, we have the relationship of \( 1 < p < 1 + (b^j / 2\bar{n}^j) < 2 \), implying that \( \hat{n}^j = (2 - p)\bar{n}^j > 0 \) holds.

\(^3\) The proof is stated in Appendix 1.

\(^4\) Equation (A.1) shows that the relationship between \( p \) and the export of good \( C \) is positive. Equation (A.2) indicates a positive link between \( \tau^j \) and the export of good \( C \). The economic intuition is that a rise in \( \tau^j \) will lead to a rise in the relative demand of good \( S \), i.e., a decrease in the relative demand of good \( C \). Equation (A.3) demonstrates a negative link between \( b^j \) and the export of good \( C \). Finally, equation (A.4) postulates that an increase in \( \bar{n}^j \) will lead to an increase in the export of good \( C \).

\(^5\) The proof is stated in Appendix 2.

\(^6\) Equation (A.5) indicates that a rise in the diversity of talent will stimulate the growth rate, as proven in Das (2005). Equation (A.6) postulates that an increase in the world relative price of good \( C \) will be detrimental to economic growth. The economic intuition behind the story is that a rise in the diversity of talent or a decrease in the world relative price of good \( C \) will lead to more output of good \( S \) and thereby speed up the growth rate. The ambiguous result expressed in equation (A.7) can be explained through the following two aspects. First, a rise in the average talent level increases the productivity of workers, and hence raises the output of good \( S \). Second, as in Grossman and Maggi (2000), a rise in the average talent level leads to less aggregate talent allocated to sector \( S \), and thereby reduces the output of good \( S \). In this paper, from \( \hat{n}^j = (2 - p)\bar{n}^j \), we can derive \( 0 < \hat{c}\hat{n}^j / \hat{c}\bar{n}^j = (2 - p) < 1 \) implying that a rise in the average talent level will lead to less aggregate talent employed in sector \( S \). It is obvious that the two aspects summarize an ambiguous response in output of good \( S \). Therefore, the impact of the average talent level on the growth rate is also ambiguous.
of labor forces in countries 1, 2, $\square$, and $k$ are identical, i.e., $b^1=b^2=\square=b^k=b$ and $L^1=L^2=\square=L^k=L$. In this paper, country 1 ($k$) is the most backward (advanced) country with the smallest (largest) average talent level. Assume that $n^1_{\text{max}}=n^2_{\text{min}}$, $n^2_{\text{max}}=n^3_{\text{min}}$, $\square$ and $n^{k-1}_{\text{max}}=n^k_{\text{min}}$. Thus, the difference of the average talent levels of countries 1 and 2, countries 2 and 3, $\square$ or countries $k-1$ and $k$ is the diversity of talent, i.e., $\bar{n}^1=\bar{n}$, $\bar{n}^2=\bar{n}+b$, $\square$, $\bar{n}^k=\bar{n}+(k-1)b$.

We make use of the superscript “$I$” to denote the variables after the integrated economy formed by countries 1, 2, $\square$, and $k$. Therefore, $L^I$ is the measure of labor forces ($L^I=L^1+L^2+\square+L^k$) and $\phi^I(n)$ is the probability density function of talent after integration. As assumed earlier, $\phi^I(n)$ is a uniform distribution as follows:

$$\phi^I(n) = \begin{cases} \frac{1}{kb}, & \text{if } n \in [n^I_{\text{min}}, n^I_{\text{max}}], \\ 0, & \text{otherwise}, \end{cases}$$

where

$$n^I_{\text{min}} = n^1_{\text{min}} = \bar{n} - \frac{b}{2}, \quad n^I_{\text{max}} = n^k_{\text{max}} = \bar{n} + \frac{(k-1)b}{2}.$$ 

The variables $n^I_{\text{min}}$ and $n^I_{\text{max}}$ are the minimum and maximum talent levels respectively. At the same time, $\bar{n}^I = (n^I_{\text{min}} + n^I_{\text{max}})/2 = \bar{n} + (k-1)b/2$ is the average talent level, and $b^I=kb$ denotes the diversity of talent. Therefore, we can use the analytical method of Section 2 and then derive the $g^I$ representing the growth rate of the integrated economy as shown below:7

$$g^I = \frac{1}{2b^I} [\frac{b^I}{2} + (1-p)\bar{n}^I] [\frac{b^I}{2} + (1+p)\bar{n}^I]. \quad (7)$$

By substituting the relationships of $b^I=kb$ and $\bar{n}^I = \bar{n} + (k-1)b/2$ into equation (7), we can find the growth rate of the integrated economy as follows:

$$g^I = \frac{1}{2kb} \left[ \frac{kb}{2} + (1-p)[\bar{n} + \frac{(k-1)b}{2}] \right] \left[ \frac{kb}{2} + (1+p)[\bar{n} + \frac{(k-1)b}{2}] \right]. \quad (8)$$

It will not be difficult to analyze the impact of the integration on economic

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7 Deprez (2003, p.378) indicates that, “As with many aspects of internationalization and economic integration, Europe has led the way. Currently, a process for tax harmonization has gained steam within the European Union (Kaye, 1996). If one has one integrated market with one currency, then it is a logical extension to have a relatively harmonized tax system.” Therefore, it is reasonable to assume that the income tax rates of countries 1, 2, $\square$, and $k$ after integration are the harmonized tax rate, $\tau^I$. 

5
growth. By differentiating equation (8) with respect to \( k \), we can derive the relationship between the number of member countries and economic growth as:

\[
\frac{\partial g^I}{\partial k} = \frac{(p^2 - 1)(\bar{\pi} - b/2)^2}{2k^2b} + \frac{(4 - p^2)b}{8} > 0. \tag{9}
\]

Equation (9) claims that the impact of the number of member countries on the growth rate for the integrated economy is positive. The economic intuition is that a rise in \( k \) will lead to a rise in the diversity of talent and hence increases the output of good \( S \), which will stimulate the growth rate for the integrated economy. Therefore, this result is formalized in the following proposition.

**Proposition 1.** The enlargement of the integrated economy will be conducive to economic growth.

Next, we will explore the growth effects of the integration on the backward and advanced countries. We find that the deeper economic integration will lead to both rises in the diversity of talent and the average talent level, from the backward country’s point of view, and hence speed up the economic growth.

Finally, by substituting the relationships of \( b^k = b \) and \( \bar{\pi}^k = \bar{\pi} + (k - 1)b \) into equation (6), we can find the \( g^k \). Therefore, the impact of the integration on the growth rate of the most advanced country is as follows:

\[
g' - g^k = \frac{(k - 1)b}{8[\Omega^2(b, \bar{\pi}, k) - 1]} [p + \Omega(b, \bar{\pi}, k)][p - \Omega(b, \bar{\pi}, k)], \tag{10a}
\]

where

\[
\Omega(b, \bar{\pi}, k) = [1 + \frac{kb^2}{4\pi^2 + 4(2k - 1)b\bar{\pi} + (k - 1)(4k - 1)b^2}]^{0.5}, \tag{10b}
\]

and

\[
1 < \Omega(b, \bar{\pi}, k) < 1 + \frac{b^k}{2\bar{\pi}^k} < 1 + \frac{b'}{2\bar{\pi}'} < 2.
\]

Hence, we get

\[
g' - g^k \begin{cases} > 0, & \text{if } \Omega(b, \bar{\pi}, k) < p < 1 + (b^k / 2\bar{\pi}^k) < 1 + (b^k / 2\bar{\pi}^k) < 2 \\ < 0, & \text{if } \Omega(b, \bar{\pi}, k) < 1 < p < 1 \end{cases}. \tag{10c}
\]

Equation (10c) indicates that whether the most advanced country’s growth rate after
the integration rises or not depends on the world price $p$, which in turn can be described in Figure 1.\footnote{In order to purge off the corner solution before and after the integration, we have the relationship of $1 < p < 1 + (b^k / 2 \pi^k) < 1 + (b^l / 2 \pi^l) < 2$.} As we can see from equation (10b), the factors affecting the critical point $\Omega(b, \pi, k)$ include $b$, $\pi$, and $k$. However, under a certain situation as will be considered in this paper, we will analyze the impact of the number of member countries on the critical point. From equation (10b), we can find that an increase in $k$ would lead to a lower critical value.\footnote{Generally speaking, we believe that every worker’s talent is positive and the minimum talent level is not beyond an upper bound. We assume that the relationship of $0 < n_{\text{min}}^l < k b$ holds. Therefore, $\partial \Omega(b, \pi, k) / \partial k = [(2 \pi - b + 2 k b)(2 \pi - b - 2 k b)[\Omega^2(b, \pi, k) - 1]^2] / [2 k^2 b^2 \Omega(b, \pi, k)] < 0$.} As we can observe from Figure 1, when the number of member countries rises, the critical point $\Omega(b, \pi, k)$ will shift left. That is to say, after the integration, the larger $k$ is, the more possible an increase in the growth rate of the advanced country will be. The economic intuition is that the integration will lead to an increase in the diversity of talent and a decrease in the average talent level, from the most advanced country’s point of view. As discussed in Section 2.1, the two aspects summarize an ambiguous effect on economic growth. However, when $k$ rises, the effects of the diversity on the growth rate may be bigger than those of the average talent level on the growth rate. Therefore, these features will be summarized as Proposition 2.

**Proposition 2.** From the backward country’s point of view, the integration will speed up the economic growth. From the advanced country’s point of view, the higher the number of member countries, the more likely it is that the integration will stimulate economic growth.

![Figure 1. Terms of trade and growth rate](image)

\[g^l - g^k < 0 \quad \text{g}^l - g^k > 0\]

1 \quad \Omega(b, \pi, k) \quad 1 + \frac{b^k}{2 \pi^k}

4. Concluding Remarks

In this paper, we prove that the impact of the enlargement on the integrated economy’s growth rate is positive. In addition, for the individual countries, we demonstrate that the deeper economic integration will speed up the backward
country’s economic growth. Finally, this paper finds that, from the advanced country’s point of view, the deeper integration may be detrimental to economic growth.

A noteworthy property of this model is that the growth rates are independent of labor endowments, i.e., there is no scale effect. The original Romer (1990) model suffers from the problem of scale effects, but this paper contributes to literature by overcoming it. In addition, we can describe the welfare effects of the integration as follows. The enlargement of the integrated economy will lead to a rise in the diversity of talent and hence increases the output of good \( S \), which will stimulate the output of good \( C \). Therefore, after the integration, none of the workers is worse off; instead some people earn higher wages than before.\(^{10}\)

There is no transitional dynamics in this model. Namely, labor reallocation occurs instantaneously so that we could not distinguish the long- and short-run effects. However, this model is well suited to extend the following direction. We can set up a growth model incorporating the implications of standard growth-theoretic features, such as capital accumulation, private holding of blue prints as assets and endogenous savings rate via either an infinite-horizon or an overlapping generations household framework, see Das (2005).\(^{11}\)

**Appendix 1** Derivations of the pattern of trade

Substituting \( \hat{n}^j = (2 - p)\tilde{n}^j \) into equations (1) and (2) can obtain the relative supply of good \( C \) \((RS)\). By substituting \( p_{\text{demand}}^j = p \) into equation (5), we get the relative demand of good \( C \) \((RD)\). Therefore, we have:

\[
\Psi(p, \tau^j, b^j, \tilde{n}^j) = RS - RD = \frac{2(p - 1)}{[1 + (b^j / 2\tilde{n}^j)]^2 - p^2} - \frac{(1 - \tau^j)}{\tau^j p},
\]

where \( \Psi(p, \tau^j, b^j, \tilde{n}^j) \) represents the export function of good \( C \). The comparative statics results which are similar to Grossman and Maggi (2000) can be stated as follows:

\[
\frac{\partial \Psi(p, \tau^j, b^j, \tilde{n}^j)}{\partial p} = \frac{2\{[1 + (b^j / 2\tilde{n}^j)]^2 - p^2\} + 4p(p - 1)}{\{[1 + (b^j / 2\tilde{n}^j)]^2 - p^2\}^2} + \frac{(1 - \tau^j)}{\tau^j p^2} > 0, \quad (A.1)
\]

\[
\frac{\partial \Psi(p, \tau^j, b^j, \tilde{n}^j)}{\partial \tau^j} = \frac{1}{(\tau^j)^2 p} > 0, \quad (A.2)
\]

\[
\frac{\partial \Psi(p, \tau^j, b^j, \tilde{n}^j)}{\partial b^j} = -\frac{2(p - 1)[1 + (b^j / 2\tilde{n}^j)]}{\tilde{n}^j \{[1 + (b^j / 2\tilde{n}^j)]^2 - p^2\}^2} < 0, \quad (A.3)
\]

\(^{10}\) This paragraph was suggested by an anonymous referee, to whom we are grateful.

\(^{11}\) This point was raised by an anonymous referee, to whom we are grateful.
\begin{equation}
\frac{\partial \Psi(p, \tau^i, b^i, \pi^i)}{\partial \pi^i} = \frac{2b^i(p-1)[1+(b^i/2\pi^i)]}{(\pi^i)^2 \left[\left[1+(b^i/2\pi^i)\right]^2 - p^2\right]^2} > 0.
\end{equation}

**Appendix 2** The comparative statics analysis of the growth rate

From equation (6), we can easily obtain:

\begin{equation}
\frac{\partial g^j}{\partial b^j} = \frac{1}{2} \frac{(\pi^j)^2}{b^j} \left[\frac{b^j}{2\pi^j} - (1-p)(1+p)\right] > 0,
\end{equation}

\begin{equation}
\frac{\partial g^j}{\partial p} = -\frac{p\pi^j}{b^j} < 0,
\end{equation}

\begin{equation}
\frac{\partial g^j}{\partial \pi^j} = \frac{\pi^j}{b^j} \left[\frac{b^j}{2\pi^j} + (1-p)(1+p)\right] > 0.
\end{equation}

**References**


