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Long-Run Impacts of Inflation Tax in the Presence of Multiple Capital Goods

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Abstract

This paper examines the long-run impact of inflation tax in the context of a generalized Ak growth model in which the production technology uses two types of capital stocks under a constant-returns-to-scale technology. We find that unless investment expenditure for each type of capital is subject to the same degree of cash-in-advance constraint, a change in the money growth rate affects the steady-state level of factor intensity. It is shown that if the balanced-growth path is uniquely given, we still have a negative longrun relationship between money growth and the growth rate of real income. However, due to the endogenous determination of the factor intensity, the negative relation between the velocity of money and the rate of inflation may not be established.

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1 Introduction

Several authors have analyzed monetary Ak growth models in which the cash-in-advance constraint applies to investment as well as to consumption spending: see, for example, Chen and Guo (2008a and 2008b), Jha, Yip and Wang (2002), Li and Yip (2004) and Suen and Yip (2005). This literature reveals that unless the elasticity of intertemporal substitutability in consumption is implausibly high, a higher inflation depresses the growth rate of real income and lowers the velocity of money on the balanced-growth path. This paper re-examines those findings in the context of a generalized Ak growth model with two types of capital.

We assume that the production technology uses two types of capital stocks under a constant-returns-to-scale technology. We show that either if there is no cash-in-advance constraint or if investment spending for both capital are subject to the same degree of liquidity constraint, then the factor intensity between the two capitals is fixed and the model reduces to the standard Ak model with a single capital. In contrast, if different degrees of cash-in-advance constraint applies to each type of investment, a change in money growth affects the steady-state level of factor intensity. It is shown that if the balanced-growth path is uniquely given, we still have a negative long-run relationship between money growth and the growth rate of real income. However, due to the endogenous determination of the factor intensity, the negative relation between the velocity of money and the rate of inflation may not be established.

2 The Model

A homogeneous output is produced by using the production technology such that

$$y = F(k_1, k_2), \tag{1}$$

where y is final good and k_i $(i = 1, 2)$ are stocks of capital. According to the standard implication, we may consider that one of inputs is physical capital and the other is human capital. The production function satisfies constant returns to scale so that it is written as $y = k_1 f(x)$, where $x = k_2/k_1$. We assume that $f(x)$ is strictly concave and monotonically increasing function of x and it satisfies the Inada conditions. Given this specification, the competitive rate of return to each type of capital is given by

$$r_1 = \frac{\partial y}{\partial k_1} = f(x) - xf'(x), \tag{2}$$

$$r_2 = \frac{\partial y}{\partial k_2} = f'(x). \tag{3}$$

The final goods is used for consumption as well as for investment spendings on both types of capital. Hence, the equilibrium condition of commodity market is

$$y = c + v_1 + v_2, \tag{4}$$

where c is aggregate consumption and v_i is gross investment for type i $(= 1, 2)$ capital.

There is a continuum of identical households with a unit mass. The representative household solves the following optimization problem:

$$\max \int_0^\infty e^{-\rho t} \frac{c^{1-\sigma} - 1}{1 - \sigma} dt, \quad \rho > 0, \quad \sigma > 0, \quad \sigma \neq 1$$

subject to

$$\dot{m} = r_1 k_1 + r_2 k_2 - c - v_1 - v_2 - \pi m + \tau, \tag{5}$$

$$\dot{k}_i = v - \delta k_i, \quad \delta \in [0, 1), \quad i = 1, 2, \tag{6}$$

$$c + \phi_1 v_1 + \phi_2 v_2 \leq m, \quad \phi_i \in [0, 1], \quad i = 1, 2, \tag{7}$$

together with the initial holdings of k_i and m. In the above, m denotes real money balances, π is the rate of inflation, τ is a lump-sum transfer from the government and δ denotes the capital depreciation rate. The household's choice variables are consumption, c, and investment expenditures, v_1 and v_2. Equation (5) is the flow budget constraint for the household and equations (6) display capital formation. Condition (7) shows the cash-in-advance constraint that applies to entire consumption as well as to parts of investment spending. Note that we assume that each type of investment expenditure may be subject to a different degree of cash-in-advance constraint.

In order to derive the household's optimization conditions, we set up the following Hamiltonian function:

$$\begin{aligned} H &= \frac{c^{1-\sigma} - 1}{1 - \sigma} + \eta \left(\sum_{i=1}^{2} r_i k_i - \sum_{i=1}^{2} v_i - c - \pi m + \tau \right) + \sum_{i=1}^{2} q_i (v_i - \delta k_i) \\ &\quad + \lambda \left(m - c - \sum_{i=1}^{2} \phi_i v_i \right), \end{aligned}$$

where η and q_i $(i = 1, 2)$ are shadow values of real money balances and capital stocks, respectively. The necessary conditions for an optimum are:

$$c^{-\sigma} = \eta + \lambda, \tag{8}$$

$$-\eta + q_i - \phi_i \lambda = 0, \quad i = 1, 2 \tag{9}$$

$$\dot{\eta} = (\rho + \pi)\,\eta - \lambda, \tag{10}$$

$$\dot{q}_i = (\rho + \delta)\,q_i - \eta r_i, \quad i = 1, 2, \tag{11}$$

$$\lambda\left(m - c - \sum_{i=1}^{2}\phi_i v_i\right) = 0, \quad \lambda \geq 0, \quad m - c - \sum_{i=1}^{2}\phi_i v_i \geq 0, \tag{12}$$

$$\lim_{t\to 0} e^{-\rho t} q_i k_i = 0 \quad (i = 1, 2)\,; \quad \lim_{t\to 0} e^{-\rho t}\eta m = 0, \tag{13}$$

together with (5), (6) and the initial conditions on k_i and m. Here, (12) displays the Kuhn-Tucker conditions and (13) gives the transversality conditions.

As for the monetary policy rule, we use the traditional assumption under which the central bank keeps the growth rate of nominal money supply constant over time. Hence, letting μ (≥ 0) be the growth rate of nominal money stock, we see that the real money stock changes according to

$$\dot{m} = m\,(\mu - \pi)\,. \tag{14}$$

We also assume that newly created money is distributed back to each household as a lump-sum transfer and, hence, the government budget constraint is $\tau = \mu m$.

3 Balanced-Growth Equilibrium

In what follows we focus on the balanced growth path on which the following conditions hold:

$$\frac{\dot{c}}{c} = \frac{\dot{k}_i}{k_i} = \frac{\dot{m}}{m} = g, \tag{15}$$

$$\frac{\dot{q}_i}{q_i} = \frac{\dot{\eta}}{\eta} = \gamma, \tag{16}$$

where g and γ are constant growth rates.

Assuming that $\phi_1 \neq \phi_2$, from (9) we obtain $\lambda = (q_1 - q_2)\,/\,(\phi_1 - \phi_2)\,.$ We express this condition as

$$\frac{\lambda}{\eta} = \frac{z_1 - z_2}{\phi_1 - \phi_2},$$

where $z_i \equiv q_i/\eta$. Additionally, (9) also yields

$$\phi_2\,(z_1 - 1) = \phi_1\,(z_2 - 1)\,, \tag{17}$$

implying that

$$\text{sign}\ (z_1 - z_2) = \text{sign}\ (\phi_1 - \phi_2)\,.$$

Since z_i represents the value of capital in terms of money, the above relation means that the capital subject to a heavier cash constraint has a higher monetary value. Additionally, from (2) , (3) , (11) and (16) we obtain:

$$\frac{z_2}{z_1} = \frac{f'(x)}{f(x) - xf'(x)}. \tag{18}$$

This equation means that on the balanced-growth path the relative implicit price of two types of capitals equals the factor-price ratio. Equations (17) and (18) present

$$z_1 = \frac{(\phi_2 - \phi_1)[f(x) - xf'(x)]}{\phi_2[f(x) - xf'(x)] - \phi_1 f'(x)}. \tag{19}$$

Notice that (8) is rewritten as

$$c^{-\sigma} = q_1 + (1 - \phi_1)\frac{q_1 - q_2}{\phi_1 - \phi_2}.$$

This equations reveals that on the balanced growth path where (15) and (16) are satisfied, it holds that

$$g = -\frac{1}{\sigma}\gamma. \tag{20}$$

As a result, in view of (11) and (19) , Āit holds that

$$\begin{aligned} g &= \frac{1}{\sigma}\left\{\frac{1}{z_1}[f(x) - xf'(x)] - \rho - \delta\right\} \\ &= \frac{1}{\sigma}\left\{\frac{\phi_2[f(x) - xf'(x)] - \phi_1 f'(x)}{\phi_2 - \phi_1} - \rho - \delta\right\}. \end{aligned} \tag{21}$$

This equation gives a relation between the factor intensity, x, and the balanced-growth rate, g. It is to be noted that, in view of (17) and (18) , if $\phi_1 > \phi_2$,then $\phi_2[f(x) - xf'(x)] < \phi_1 f'(x)$, while if $\phi_1 < \phi_2$, then $\phi_2[f(x) - xf'(x)] > \phi_1 f'(x)$. Hence the 'after tax' rate of return, $\frac{\phi_2[f(x)-xf'(x)]-\phi_1 f'(x)}{\phi_2-\phi_1}$, has a positive value.

To obtain an additional relation between g and x, we use (10) , (16) and (20) to derive

$$-\sigma g = \rho + \mu - g - \frac{1}{\phi_1}(z_1 - 1)$$

Since $\dot{m}/m = g$ in the balanced-growth equilibrium, (14) means that $\pi = \mu - g$. Substituting this and (19) into the above equation yields

$$g = \frac{1}{1-\sigma}\left[\rho + \mu + \frac{1}{\phi_1} - \left(\frac{1}{\phi_1}\right)\frac{(\phi_2 - \phi_1)[f(x) - xf'(x)]}{\phi_2[f(x) - xf'(x)] - \phi_1 f'(x)}\right]. \tag{22}$$

We now find that equations (21) and (22) may determine the steady-state level of factor intensity, x, and the long-run growth rate of the economy, g.

4 Long-Run Effects of Inflation Tax

We first consider a special case where $\phi_1 = \phi_2$. In this case (17) indicates that $z_1 = z_2$. This means that the rate of return to each type of capital is always identical, that is,

$$f'(x) = f(x) - xf'(x).$$

Since the left and right-hand side of the above respectively decreases and increases as x rises, there is a unique level of x^* that satisfies above. Therefore, it is satisfied that $k_2 = x^* k_1$ for all $t \geq 0$. The production function (1) is thus written as $y = f(x^*) k_1$, where $f(x^*)$ is a positive constant: the economy has an Ak technology. This shows that when $\phi_1 = \phi_2$, the model reduces to one with a single capital stock.[1]

In the general case where $\phi_1 \neq \phi_2$, we have four cases. For notational simplicity, we respectively denote (21) and (22) as $g = \Lambda(x)$ and $g = \Gamma(x)$, where

$$\Lambda(x) \equiv \frac{1}{\sigma}\left\{\frac{\phi_2 [f(x) - xf'(x)] - \phi_1 f'(x)}{\phi_2 - \phi_1} - \rho - \delta\right\},$$

$$\Gamma(x) \equiv \frac{1}{1-\sigma}\left[\rho + \mu + \frac{1}{\phi_1} - \left(\frac{1}{\phi_1}\right)\frac{(\phi_2 - \phi_1)[f(x) - xf'(x)]}{\phi_2 [f(x) - xf'(x)] - \phi_1 f'(x)}\right].$$

The steady state value of x is determined by $\Lambda(x) - \Gamma(x) = 0$.

Case (i): $\sigma > 1$ and $\phi_1 > \phi_2 > 0$.

If this is the case, $\Lambda(x)$ is a monotonically decreasing function and $\Gamma(x)$ is a monotonically increasing function of x. Hence, the graphs of those two functions may have a unique intersection, which gives a unique balanced growth path. We see that a rise in the money growth rate, μ, shifts down the graph of $g = \Gamma(x)$, so it rises the steady-state level of x and lowers the balanced growth rate, g. This is an intuitively plausible outcome: as a result of a rise in inflation tax, the firms relatively increase investment for k_2 with a less cash constraint, which raises the steady-state level of $x \, (= k_2/k_1)$. In addition, a higher inflation tax raises the investment costs and thus capital formation is decelerated.

Case (ii) : $\sigma > 1$ and $\phi_2 > \phi_1 > 0$.

Under these conditions, $\Lambda(x)$ monotonically increase with x, while $\Gamma(x)$ monotonically decreases with x. Again, the balanced-growth equilibrium may be uniquely determined. We also find that a higher μ depresses both g and x.

Case (iii) : $\sigma < 1$, $\phi_1 > \phi_2 > 0$

In this case both graphs have negative slopes, implying that there may exist multiple balances growth paths. It is easy to confirm that a rise in μ depresses

[1]See Chapter 4 in Barro and Sala-i-Martin (2005) and Chapter 11 in Acemoglu (2009) for detailed expositions of the one-sector Ak growth model with two types of capital.

g and increases x in the balanced growth equilibrium if the graph of $g = \Gamma(x)$ is steeper than that of $g = \Lambda(x)$ around the intersection. In the opposite case, a higher μ raises g and lowers x.

Case (iv) : $\sigma < 1$ and $\phi_2 > \phi_1 > 0$

Given these conditions, both graphs have positive slopes. In this case if $g = \Lambda(x)$ is steeper than $g = \Gamma(x)$, then a rise in μ increase both g and x. In the opposite case it is shown that a higher μ depresses both g and x.

Consequently, as long as $\sigma > 1$, we obtain intuitively plausible results: an increase in inflation tax lowers the growth rate of income and substitutes the capital stock with a tighter cash-in-advance constraint with one with a weaker constraint. If $\sigma < 1$, a rise in μ may accelerate long-run growth. It is, however, noted that the positive relation between monetary growth and growth of real income is established only when a higher inflation tax makes the firms substitute production factor that is less constrained by cash holding with the one with a tighter cash-in-advance restraint.

Finally, let us examine the velocity of money in the long-run equilibrium. When the cash-in-advance constraint is effective on the balanced growth path, it holds that $m = c + \phi_1 (g + \delta) k_1 + \phi_2 (g + \delta) k_2$. Thus, using $y = c + v_1 + v_2$, we can express the steady-state level of velocity of money as

$$V = \frac{y}{m} = \frac{f(x)}{f(x) - [(1 - \phi_1) + (1 - \phi_2) x] (g + \delta)}. \tag{23}$$

As was shown, if $\phi_1 = \phi_2$, then x is fixed. Hence, if $\sigma > 1$, a rise in μ depresses g alone, so that the velocity, V, becomes smaller on the balanced growth path. Such a negative relationship may not be always held when $\phi_1 \neq \phi_2$. In our setting, a change in μ affects both x and g in the right hand side of (23). Hence, we may not obtain unambiguous conclusion about the long-run relation between inflation and velocity, unless we specify the functional forms and the parameter values involved in the model.

To be more concrete, we present some numerical examples. We specify the production function as $y = Ax^{\alpha} \quad (0 < \alpha < 1)$ and set the baseline parameter values in the following manner:

$$A = 0.20, \quad \alpha = 0.3, \quad \rho = 0.04, \quad \sigma = 1.5, \quad \delta = 0.04$$

Given those magnitudes, we calculate the steady state levels of factor intensity, x, the balanced growth rate, g, and the velocity of money, V under alternative values of μ, ϕ_1 and ϕ_2. Here, we consider Case (ii) where $\sigma > 1$ and $\phi_2 > \phi_1 > 0$. Table (a) sets $\phi_1 = 0.2$ and $\phi_2 = 0.3$. Similarly, in Table 1 (b) we set $\phi_1 = 0.2$ and $\phi_2 = 0.8$, while $\phi_1 = 0.6$ and $\phi_2 = 0.8$ in Table 1 (c). In every example, we change the growth rate of money, μ, from 0.02 up to 0.2.

μ	x	g	V
0.02	0.4259	0.0245	1.5842
0.04	0.4251	0.0243	1.5817
0.10	0.4227	0.0234	1.5700
0.15	0.4208	0.0226	1.5596
0.20	0.4189	0.0219	1.5507

Table 1 (a): $\phi_1 = 0.2, \quad \phi_2 = 0.3$

μ	x	g	V
0.02	0.4133	0.0240	1.5827
0.04	0.4088	0.0235	1.5782
0.10	0.3959	0.0221	1.5632
0.15	0.3861	0.0209	1.5513
0.20	0.3768	0.0197	1.5411

Table 1 (b), $\phi_1 = 0.2, \quad \phi_2 = 0.8$

μ	x	g	V
0.02	0.4235	0.0228	1.5612
0.04	0.4221	0.0207	1.5331
0.10	0.4178	0.0191	1.5134
0.15	0.4145	0.0177	1.4964
0.20	0.4114	0.0158	1.4734

Table 1 (c): $\phi_1 = 0.6, \quad \phi_2 = 0.8$

In the above examples, a rise in the money growth rate depresses all of x, g and V in the balanced-growth equilibrium. The figures in Panel (a) show that if ϕ_1 and ϕ_2 are relatively small, the long-run impact of monetary expansion on the steady-state levels of factor intensity, the velocity of money as well as on the growth rate of real income are considerably small: even when the money growth rate increases from 0.02 to 0.2, the growth rate decreases only by 0.0026. In Panel (b), ϕ_2 takes 0.8. Since type 2 investment is subject to a much tighter cash constraint than type 1 investment, a higher money growth yields a larger change in factor intensity relative to the example shown in Panel (a). Panel (c) sets $\phi_1 = 0.6$ and $\phi_2 = 0.8$. Since relatively strict cash-in-advance constraints apply to both types of investment, the negative growth effect of inflation is larger than in the examples in Panels (a) and (b). It is to be noted that in Panel (c) a change in money growth alters the long-run level of factor intensity by a small amount, because the difference in the degrees of cash-in-advance constraint on each type of investment is relatively small in this example.

5 Conclusion

This paper examines a monetary, one-sector Ak growth model with two types of capital. We show that if a different degree of cash-in-advance constraint applies to each type of investment, then a change in money growth alters the long-run level of factor intensity in production. This additional impact of inflation tax yield quantitative effects on the balanced-growth rate of the economy as well as the steady-state level of velocity of money. Our numerical examples reveal that those quantitative effects heavily depends on the absolute as well as relative strength of cash-in-advance constraint on each type of investment.[2]

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[2]Itaya and Mino (2008) examine a monetary Ak growth model with variable labor supply. Fujisaki and Mino (2007) discuss a monetary Ak growth model with a Taylor-type interest control rule. It would be useful to consider the topic of this paper in those more general settings.