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Sustainability of collusion with imperfect price discrimination and inelastic demand functions

Stefano Colombo
Catholic University of Milan

Abstract

We study the impact of product differentiation on collusion sustainability in the case of imperfect price discrimination and inelastic demand functions. We show that differentiation facilitates the sustainability of collusion. Therefore, the indifference result of Gupta and Venkatu (2002) does not hold when imperfect price discrimination is introduced.

Largo A. Gemelli 1, I-20123, Catholic University of Milan, Milan, Italy stefano.colombo@unicatt.it
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1. Introduction

The impact of product differentiation on collusion sustainability has been extensively studied in uniform pricing contexts. Chang (1991), Chang (1992) and Hackner (1995) employ the spatial competition framework of Hotelling (1929). They assume inelastic demand functions and show that collusion is easier to sustain the more differentiated are the firms. Hackner (1995) concludes that there is "*a fairly general tendency within the Hotelling framework for differentiation to facilitate collusion"* (p.293).[1]

The analysis of the relationship between product differentiation and collusion sustainability has been extended to a discriminatory pricing context by Gupta and Venkatu (2002) and Miklós-Thal (2008). Gupta and Venkatu (2002), relying on grim trigger punishment mechanisms, show that in the case of elastic demand functions perfect collusion is easier to sustain the less firms are differentiated. Instead, in the case of inelastic demand functions, sustainability of collusion is not affected by product differentiation. Miklós-Thal (2008) shows that when optimal punishment is used in sustaining the collusive agreement product differentiation tends to facilitate collusion, in line with the findings in uniform pricing literature. Both Gupta and Venkatu (2002) and Miklós-Thal (2008) assume perfect price discrimination. However, perfect price discrimination is quite unrealistic, since, as Tirole (1988) argues, "*perfect price discrimination is unlikely in practice, either because of arbitrage or because of incomplete information about individual preferences*" (Tirole, 1988, p.135). In this note, within the context of inelastic demand functions, we move from perfect price discrimination to imperfect (or third-degree) price discrimination, which refers to the case where the price offers of a firm vary across groups of consumers but not within each group of consumers (Stole, 2007). This case better describes those situations where firms have information about the preferences of *groups* of consumers, but not about *individual* consumers' preferences (Bester and Petrakis, 1996; Liu and Serfes, 2004; Armstrong, 2008). We show that product differentiation facilitates the sustainability of perfect collusion. Therefore, Gupta and Venkatu (2002) indifference result in the case of inelastic demand functions does not hold when a less-than-perfect price discrimination practice is supposed, whatever the degree of imperfectness of price discrimination. In this sense, the imperfectness of price discrimination is a second reason, in addition to the one emphasized by Miklós-Thal (2008), why Gupta and Venkatu (2002) result does not hold.

2. The model

Following Hotelling (1929), the differentiated good is represented in the unit interval $[0,1]$. Consumers are uniformly distributed over the interval. Denote by $x \in [0,1]$ the location of each consumer. For a consumer positioned at a given point, the most preferred variety is represented by the point in which he is located. Each consumer consumes no more than 1 unit of the good. Denote by v the maximum price that a consumer is willing to pay for buying his preferred variety.

There are two firms, A and B. Fixed and marginal costs are zero. Firm A produces the variety $a \in [0, 1/2]$ and firm B produces the variety $1-a$. The parameter a measures product

[1] Other papers that investigate the relationship between product differentiation and collusion on uniform prices adopting non-spatial frameworks are Deneckere (1983), Majerus (1988), Jehiel (1992) and Ross (1992).

differentiation: when $a=0$, firms are maximally differentiated; when $a=1/2$ firms are identical.

Following Liu and Serfes (2004), we suppose that there is an information technology which allows firms to partition the consumers into different groups. The technology partitions the linear market into n sub-segments indexed by m, with $m=1,...,n$. Assume $n=2,4,8,16,32...$. A higher n means a higher information precision. When $n\to\infty$ we are in the case of perfect price discrimination. Each sub-segment is of equal length, $1/n$. Sub-segment m can be expressed as the interval $[(m-1)/n, m/n]$. A firm can price discriminate between consumers belonging to different sub-segments, but not between the consumers belonging to the same sub-segment. The consumers' utility function follows D'Aspremont *et al.* (1979): if p_m^J is the price set by firm $J=A,B$ on consumers belonging to sub-segment m, the utility of the consumer x located in sub-segment m when he buys from firm A is: $u_x = v - p_m^A - t(x-a)^2$, while his utility when he buys from firm B is given by: $u_x = v - p_m^B - t(x-1+a)^2$. Moreover, denote by $m^A \in [1, n/2]$ the sub-segment in which firm A is located, and by $\overline{m} = (2m^A - 1)/2n$ the middle point of sub-segment m^A. Denote by $m^B \in [n/2+1, n]$ the sub-segment in which firm B is located, and by $\underline{m} = (2m^B - 1)/2n$ the middle point of sub-segment m^B. Given firms' symmetry, it must be: $m^B = n - m^A + 1$. Finally, assume $v > t$: this guarantees that no firm ever enjoys a natural monopoly on its side of the market.

Suppose that firms interact repeatedly in an infinite horizon setting. In supporting collusion, the firms are assumed to use the *grim trigger* strategy of Friedman (1971).[2] Denote by $\Pi^{J,C}$, $\Pi^{J,D}$ and $\Pi^{J,N}$, with $J=A,B$, respectively the one-shot collusive, deviation and punishment (or Nash) profits of each firm. The market discount factor, δ, is exogenous and common for each firm. It is well known that collusion is sustainable as a sub-game perfect equilibrium if and only if: $\delta \geq \delta^*$, where $\delta^* \equiv (\Pi^{J,D} - \Pi^{J,C})/(\Pi^{J,D} - \Pi^{J,N})$, with $J=A,B$, is the critical discount factor. Therefore, δ^* measures the cartel sustainability: the higher is δ^* the smaller is the set of market discount factors supporting collusion (i.e. collusion is *less* sustainable).

3. Sustainability of collusion

The punishment stage. The punishment profits are the Nash equilibrium profits of the one-shot stage game. We directly refer to Proposition 1 in Liu and Serfes (2004). Therefore:

$$\Pi^{J,N} = t(1-2a)(9n^2 - 18n + 40)/36n^2 \quad \text{with } J=A,B. \tag{1}$$

[2] Under the *grim trigger* strategy, firms start by charging the collusive price. The firms continue to set the collusive price until one firm has deviated from the collusive agreement in the previous period. If a firm deviates at time t, from $t+1$ onward both firms play the equilibrium price emerging in the non-cooperative stage game. Clearly, the *grim trigger* strategy is not optimal (Abreu, 1986). However, "*this is one of very realistic punishment strategies because of its simplicity*", as argued by Matsumura and Matsushima (2005, p.263). The most part of the papers which study the relationship between product differentiation and collusion sustainability adopt the *grim trigger* strategy. See for example, Deneckere (1983), Chang (1991), Chang (1992), Friedman and Thisse (1993), Hackner (1994, 1995), Lambertini *et al.* (1998), Matsumura and Matsushima (2005). Exceptions are Hackner (1996) and Miklós-Thal (2008), where optimal punishments are assumed.

The collusive stage. In the collusive stage, firms implicitly share the market evenly, and each firm serves its respective part of the market. Consider firm *A*. The collusive price schedule is set in such a way to extract the whole consumer surplus of the most distant consumer in each sub-segment. The most distant consumer from firm *A* in sub-segment *m* is located at the beginning of the sub-segment (i.e. at $(m-1)/n$) for $m < m^A$, while he is located at the end of the sub-segment (i.e. at m/n), for $m > m^A$. In the sub-segment m^A, the most distant consumer from firm *A* is located at the beginning of the sub-segment if $a \geq \overline{m}$, while he is located at the end of the sub-segment if $a \leq \overline{m}$.

Denote by $p^{A,C}(m)$ the collusive price set by firm *A* on sub-segment *m*. At the optimal collusive price the utility of the most distant consumer must be zero. Therefore:

$$p^{A,C}(m) = \begin{cases} v - t[(m-1)/n - a]^2 & \forall m \in [1, m^A) \\ v - t[(m-1)/n - a]^2 & \textit{if} \quad m = m^A \quad \textit{and} \quad a \geq \overline{m} \\ v - t(m/n - a)^2 & \textit{if} \quad m = m^A \quad \textit{and} \quad a \leq \overline{m} \\ v - t(m/n - a)^2 & \forall m \in (m^A, n/2] \end{cases}$$

Firm *A* serves all consumers in its half market. Therefore, the demand of firm *A* in each sub-segment is $1/n$. Firm *A*'s collusive profits are given by:

$$\Pi^{A,C} = \sum_{m=1}^{n/2} \frac{p^{A,C}}{n} \tag{2}$$

Consider now firm *B*. As for firm *A*, the collusive price schedule is set in such a way to extract the whole consumer surplus of the most distant consumer in each sub-segment. Note that firms' symmetry implies the following relation: $1 - a \geq (\leq)\, \underline{m} \leftrightarrow a \leq (\geq)\, \overline{m}$. We get:

$$p^{B,C}(m) = \begin{cases} v - t[(m-1)/n - 1 + a]^2 & \forall m \in [n/2+1, m^B) \\ v - t[(m-1)/n - 1 + a]^2 & \textit{if} \quad m = m^B \quad \textit{and} \quad a \leq \overline{m} \\ v - t(m/n - 1 + a)^2 & \textit{if} \quad m = m^B \quad \textit{and} \quad a \geq \overline{m} \\ v - t(m/n - 1 + a)^2 & \forall m \in (m^B, n] \end{cases}$$

The deviation stage. Suppose that firm *A* deviates from the collusive agreement. Denote by $p^{A,D}$ the deviation price schedule set by firm *A*. Clearly, firm *A* continues to set the collusive prices in the left half of the market, because they are already optimal. Instead, it sets discriminatory deviation prices in the sub-segments at the right half of the market in such a way to steal the consumers from the rival. In each sub-segment firm *A* sets the highest price which makes the most distant consumer indifferent between firm *A* and firm *B*, given that firm *B* is setting the collusive price. Note that in the right half of the market the most distant consumer from firm *A* is always the consumer located at the right endpoint of each sub-segment (i.e. the consumer located at m/n). Then, the deviation price schedule in sub-segments $m \geq n/2+1$ is obtained from: $v-p^{A,D}-t(m/n-a)^2 = v-p^{B,C}-t(m/n-1+a)^2$. Therefore, the deviation price schedule of firm A is:

$$p^{A,D} = \begin{cases} p^{A,C} & \forall m \in [1, n/2] \\ v-t[(m-1)/n-1+a]^2+t(m/n-1+a)^2-t(m/n-a)^2 & \forall m \in [n/2+1, m^B) \\ v-t[(m-1)/n-1+a]^2+t(m/n-1+a)^2-t(m/n-a)^2 & if \quad m=m^B \quad and \quad a \leq \overline{m} \\ v-t(m/n-a)^2 & if \quad m=m^B \quad and \quad a \geq \overline{m} \\ v-t(m/n-a)^2 & \forall m \in (m^B, n] \end{cases}$$

It is easy to note that, given $v>t$, the most profitable deviation involves serving the whole market. Therefore, the demand of firm A in each sub-segment is $1/n$. The deviation profits of firm A are given by:

$$\Pi^{A,D} = \Pi^{A,C} + \sum_{m=n/2+1}^{n} \frac{p^{A,D}}{n} \tag{3}$$

The critical discount factor. Denote the critical discount factor by:

$$\delta^* \equiv \begin{cases} \delta^*_{a\leq\overline{m}} & if \quad a \leq \overline{m} \\ \delta^*_{a\geq\overline{m}} & if \quad a \geq \overline{m} \end{cases} \tag{4}$$

Substituting equations (1), (2) and (3) into (4), we get:

$$\delta^*_{a\leq\overline{m}} = \frac{3[-12vn^3+t(n^2(15-36a)+n^3(7-18a+12a^2)+n(2+48a(m^A-1))-24(m^A-1)^2]}{2[-36vn^3+t(n^2(9-36a)+3n^3(7-18a+12a^2)-72(m^A-1)^2+2n(23+8a(9m^A-14)))]}$$

and

$$\delta^*_{a\geq\overline{m}} = \frac{3[3tn^2(5-12a)+n^3(t(7-18a+12a^2)-12v)-24tm^A+2tn(1+24am^A)]}{2[9tn^2(1-4a)+3n^3(t(7-18a+12a^2)-12v)-72tm^A+2tn(23+8a(9m^A-5))]}$$

Consider δ^*. Notwithstanding the complexity of (4), it can be shown that δ^* is a continuous function. In fact, substituting $a=\overline{m}$ in the upper part and in the lower part of (4), it turns out: $\delta^*_{a\leq\overline{m}} = \delta^*_{a\geq\overline{m}}$. This implies that when a moves from the left of $\overline{m}$ to the right of $\overline{m}$ there is not a "jump" in the critical discount factor. Moreover, $\delta^*_{a\leq\overline{w}}(a=\frac{w^A-1}{n}) = \delta^*_{a\geq\overline{m}}(a=\frac{m^A}{n})$, where $w^A=m^A+1$ and $\overline{w}=\overline{m}+1$. This means that the critical discount factor does not "jump" when a moves from sub-segment m^A to the successive sub-segment m^A+1. Finally, it can be shown that $\partial\delta^*/\partial a$ is always strictly positive for any finite n, while when $n\rightarrow\infty$ the critical discount factor is always equal to $1/2$.[3] Therefore, the sustainability of collusion is *positively* affected by the product differentiation degree for any finite n (imperfect price discrimination), while, as in Gupta and Venkatu (2002), it does not depend on product differentiation when $n \to \infty$ (perfect price discrimination). The intuition is the following. Recall that collusion sustainability increases with the collusive profits, while it decreases with the deviation and the punishment profits. By looking at (1), it is immediate to see that the punishment profits are decreasing in a. This depends on the fact that higher similarity between firms implies fiercer competition during the non-cooperative stage. Thereby, Nash profits are increasing in the differentiation degree. By taking the derivative of (3) with respect to a, we obtain that the deviation profits are increasing in a. This is due to the fact that when differentiation decreases, each consumer is less "loyal" to the nearer firm, because the firms are more similar. Therefore, it becomes easier for the cheating firm to steal consumers from the rival. It follows that deviation profits are decreasing in the differentiation between firms. The effect of a on the collusive profits instead is not monotone and it can be positive or negative. In our framework, the impact of the product differentiation degree on the deviation profits turns out to be the dominant effect, since it outweighs the effect of a on the punishment profits, even when the sign of the effect of a on the collusive profits is the opposite of the sign of the effect of a on the punishment profits. This determines the positive relationship between the product differentiation degree and the sustainability of collusion. Instead, in the case of perfect price discrimination, no effect dominates, since the change in the difference between the deviation profits and the collusive profits (the numerator in the critical discount factor equation) is exactly offset by the change in the difference between the deviation profits and the punishment profits (the denominator in the critical discount factor equation), and therefore collusion sustainability is not affected by product differentiation (Gupta and Venkatu, 2002, p.60). However, we showed that when we move from a perfect discriminatory framework to an imperfect discriminatory context, the impact of product differentiation on the deviation profits becomes the dominant effect and the difference between the deviation profits and the collusive profits increases faster than the difference between the deviation profits and the punishment profits when product differentiation decreases, determining a positive relationship between product differentiation and collusion sustainability.

4. Conclusions and future research

We extended the literature of the relationship between product differentiation and perfect collusion on discriminatory prices to the case of imperfect price discrimination and inelastic demand functions. We show that, contrary to Gupta and Venkatu (2002) indifference result, the more the firms are similar the less collusion is sustainable. This result is consistent with the literature on product differentiation and collusion sustainability under uniform pricing (Chang, 1991; Chang, 1992; Hackner, 1995), as well as with the literature on product differentiation and collusion sustainability under discriminatory pricing and optimal punishment (Miklós-Thal, 2008).

The model we used is quite specific and more research is needed on this issue to derive general insights and antitrust suggestions. For example, this work can be extended in the following directions. Demand can be assumed to be elastic: while there are papers which consider elastic demand functions in the Hotelling framework and in the perfect price

[3] The Mathematica file with the proof of the continuity of the critical discount factor and with the proof of the positive sign of the critical discount factor's derivative is available on request from the author.

discrimination context (Hamilton *et al*.; 1989, Hamilton and Thisse, 1992; Gupta and Venkatu, 2002) we are not aware of the existence of papers which explicitly adopt elastic demand functions in the Hotelling framework when imperfect price discrimination is assumed. Second, a more sophisticated segmentation of the market may be allowed: for example, it is not clear what happens if the sub-segments have different length. Third, asymmetry between firms may be introduced: for example, firms may have a different ability in partitioning the market.

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