Economics Bulletin

Volume 29, Issue 3

External Debt, Informal Economy and Growth

Mahmoud Sami Nabi
LEGI-Polytechnic School of Tunisia and University of Sousse, IHEC

Imed Drine
UNU- WIDER

Abstract

We develop an endogenous growth model with overlapping generations taking into account important characteristics of the developing countries: high public external debt and large informal sector. We show that an increasing of the public external debt has two opposite effects. On the one hand, it enhances growth through a positive externality affecting the productivity of private firms. On the other hand, it inhibits growth by ousting the external financing of private firms and enlarging the less efficient informal sector. These two effects generate a non-linear effect of the public external debt on growth and an optimal share of the public external indebtedness. We also show that, under a certain condition, the enlargement of the informal sector could be accompanied by higher growth.

Citation: Mahmoud Sami Nabi and Imed Drine, (2009) ''External Debt, Informal Economy and Growth'', *Economics Bulletin*, Vol. 29 no.3 pp. 1695-1707.
Submitted: May 20 2009. **Published:** July 17, 2009.

1. Introduction

There is general acceptance amongst economists that good policies and good institutions are required for the design of a viable strategy of economic development. For Acemoglu et al., (2002, 2004) the degree of institutional development is a fundamental factor of economic development and poor macroeconomic policies are the symptoms of poor institutions. The effectiveness of external aid/debt in financing the economic development process is one of the active research areas where the role of institutions is emphasized. Burnside and Dollar (2000, 2004) show that the impact of aid on economic growth depends on the quality of institutions and policies. Imbs and Ranciere (2005) found that countries with good policies and good institutions have lower debt overhang. The international financial institutions are designing technical assistance and capacity-building programs to enhance the management of the external aid/debt in the developing countries (Bangura et al., 2000).

The management of the external debt is particularly important for developing countries accumulating large stock of debt since the 1970s. Indeed, their debt burden is so severe that they carry costly macroeconomic reforms in order to pay the debt principal and services. In order for the external debt management to be efficient we need a deep understanding of the of the external debt effect on economic growth. It is insufficient to draw policy recommendation focusing only on the external debt/GDP ratio. In this paper we take into consideration two fundamental aspects of the developing countries not enough stressed in the existing literature. The first aspect is the high public external indebtedness. The second aspect is the informal economy[1].

From the theoretical models we know that the external debt has a negative effect on growth when the debt stock exceeds the reasonable thresholds. The most well known explanation of this effect was established by the theory of excessive debt (debt overhang) through the studies of Krugman (1988), Sachs (1989) and Cohen (1992). This theory establishes that beyond a certain threshold, external debt affects growth negatively through decelerating the dynamic of factors' accumulation and declining the total factor productivity. The first explanation is that when external debt is excessive, investors anticipating a gradual tax increase for future debt repayment reduce their investment which in turn slows down capital accumulation. The second explanation is related to the governments' decision not to carry out costly economic reforms, considering that future higher domestic production will serve only foreign creditors. This weakness in the economic environment affects capital allocation and investment quality and hence slows down total factor productivity. Relatively to the above studies, our paper presents a theoretical model stressing the importance of a neglected channel in the literature of debt and growth: the sensitivity of the informal economy to the external debt management.

The proposed model is an endogenous growth one with overlapping generations where investors have to choose between formal projects and less efficient informal ones. It shows that a misallocation of the external debt between the government and the private formal sector could reduce the economic growth through two channels. The first channel is a reduction of private sector productivity. The second channel is a reduction of capital accumulation associated with an enlarging of the informal sector. The fact that a large

[1] Chickering and Salahdine (1991) argue that for the majority of developing countries, the informal sector contributes for 35% to 65% to the total employment and produces between 20% and 40% of GDP. According to Friedman and al. (2000) the size of the informal sector is approximately 68% in Egypt, 39% in Malaysia, 76% in Nigeria, 71% in Thailand, 45% in Tunisia, etc.

informal sector is associated with lower growth rates is widely accepted in the literature (Loayza, 1997; Johnson et al., 1999 and Schneider and Enste, 2000). We show that this is not always the case and we identify a case where an enlargement of the informal sector is accompanied by higher growth.

The rest of the paper is organized in three main sections. Section 2 presents the theoretical framework. Section 3 analyzes the effect of the external public debt on the size of the formal/informal sector. Section 4 investigates the effect of the external public debt on growth through different channels. Finally section 5 concludes and gives some policy implications.

2. An endogenous growth model

2.1 Economic environment

We consider an economy with an infinite, discrete time horizon, $t = 0, 1, 2, ...$ Date t corresponds to the beginning of period $t+1$ and the end of period t. The economy is endowed with two production sectors with different technologies. The first sector produces a final (or consumption) good using capital and labour. The second sector produces an investment (or capital) good with the flow of capitals (wages) generated by the production of the final good. At each date a new generation of two-periods living agents of mass 1 is born. An initial generation of old agents coexists with young agents at date $t = 0$. The old of the first generation are endowed at $t = 0$ with a stock k_0 of capital good. All agents are endowed with one unit of labour which they supply during their first-period of live inelastically at no disutility cost. In compensation for their work (when young) in the final good sector, they earn a wage which is invested during the second period in order to maximize the final wealth which finances their consumption. Two investment opportunities are available for each young agent after receiving its wage: undertaking a formal or an informal investment project (producing the investment good). A formal project is eligible for a complementary external financing but is taxable. However, the informal project is self-financed, non-taxable and supports a cost of tax evasion.

Final good sector

This sector is composed of competitive firms producing the final good instantaneously from the combination of two substitutable factors: capital (good) K and labour L. The technology which is assumed to be of Cobb-Douglas type exhibits constant factors' return but includes an aggregate level of "knowledge" denoted A which is common to all firms and is considered as a free public good: $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$. We associate (à la Romer, 1986) A_t to the aggregate stock of capital [2] : $A_t = \overline{k}_t^{1-\alpha}$. This choice enables the endogenous growth of the aggregate production. Hence, the per capita output is given by $y_t = k_t$. The output is entirely distributed to the workers and to the entrepreneurs producing the capital good. Finally, capital depreciates fully after production and the factor's prices are equal to their marginal productivities:

$$\begin{aligned} \rho_t &= \alpha A_t (k_t)^{\alpha-1} \\ w_t &= (1-\alpha) A_t (k_t)^{\alpha} \end{aligned}$$

[2] The choice of this technology is common in the literature(Bose and Cohtern (1996))

In the equilibrium the aggregate stock of capital is also the capital stock per capita: $\overline{k}_t = k_t$. Hence, we obtain

$$\begin{aligned} \rho_t &= \alpha \\ w_t &= (1-\alpha)k_t \end{aligned} \tag{1}$$

Agents' investment decisions

The agent supplies, inelastically at no disutility cost, a unit of labour during its first-period of live. Hence, the total labor supply in each period is $L = 1$. In return, he earns a wage w_t which he invests during the second period in order to maximize its final consumption. Indeed, to simplify the model we assume consumption occurs only at the end of the second period. Under this assumption there is no trade-off between consumption and saving at the end of the first period. The only trade-off we consider in this model is between investing in a formal project or in informal one. It can be a formal project or an informal one. Whatever the project's type is, investment good is produced using a linear technology transforming any quantity q of the final good in $(ag_t)q$ investment good with $a > 1$. The term g_t denotes the amount of public expenditures per capita which increases the productivity of the two types of projects[3].

Undertaking a formal project: When undertaking a formal project, an agent can obtain an external financing of d_t^f in terms of the final good. This amount is lent by international investors (through a domestic financial intermediary) in return of a gross interest rate denoted r. Therefore, the total amount invested in the formal project is $w_t + d_t^f$ and the quantity of the investment good produced is

$$\kappa_{t+1}^f = ag_t \left(w_t + d_t^f \right) \tag{2}$$

This quantity is sold to the final sector at the price α which provides the agent an income $\alpha\kappa_{t+1}^f$ in terms of the final good. Hence, his gross profit after repaying his debt[4] is $\pi_{t+1}^f = \alpha\kappa_{t+1}^f - rd_t^f$ and his net profit after paying the tax τ_t is $(1-\tau_t)\,\pi_{t+1}^f$.

Undertaking an informal project: An agent who undertakes an informal project has no access to the external financing and don't pay the tax on profit. He produces a quantity of the investment good given by

$$\kappa_{t+1}^j = ag_t w_t \tag{3}$$

His gross profit is $\pi_{t+1}^j = \alpha\left(\kappa_{t+1}^j - c_{t+1}^j\right)$ where c_{t+1}^j represents the cost of informality. This cost can be related to the masking of the activity through paying bribes or localization far from urban area which exposes the agent to more risks and high transport costs. The agents are heterogenous relatively to this cost which is assumed to vary proportionally to the production $c_{t+1}^j = (1-\theta_j)\kappa_{t+1}^j$. The parameter θ_j is specific to each agent and is distributed uniformly on $[0,1]$. This signifies that agents who support very low cost of informality have a high value of θ_j (at the extreme no such cost if $\theta_j = 1$). Therefore, the profit derived from the informal project is given by $\pi_{t+1}^j = \alpha\left(\kappa_{t+1}^j - c_{t+1}^j\right) = \alpha\theta_j ag_t w_t$.

[3] One can think about the quality of public services, infrastructure, etc.

[4] Note that the agent has no incentive to borrow if the cost of capital is superior to its project return or equivalently $\alpha ag_t < r$.

Agents' decisions: Each agent chooses the type of his project maximizing profit. Hence, at date t the informal projects are realized by the agents characterized by θ_j such that $\pi^j_{t+1} \geq (1-\tau_t)\,\pi^f_{t+1}$. Using the above expression of the profits we obtain the set of informal entrepreneurs $\Theta = \{j \text{ such that } \theta_j \in [\theta_t, 1]\}$ where θ_t is defined by

$$\begin{aligned}\theta_t &= \frac{(1-\tau_t)\pi^f_{t+1}}{\alpha a g_t w_t} &(4)\\ &= (1-\tau_t)[1+(1-\frac{r}{\alpha a g_t})\frac{d^f_t}{w_t}] &(5)\end{aligned}$$

The set $\overline{\Theta}$ of formal entrepreneurs includes agents who support sufficiently high cost of informality and for who more interesting to undertake a formal projects $\overline{\Theta} = \{j \text{ such that } \theta_j \in [0, \theta_t]\}$. Hence, if the threshold θ_t is equal to one[5], there is no informal projects in the economy during period $t+1$. Note that we can interpret θ_t as the size of the formal sector and $1-\theta_t$ as the size of the informal one.

Government

At the beginning of period $t+1$, we denote D_t the stock of (inherited) external debt (in terms of the tradable good). The economy raises a new line of external debt of an amount d. The government controls the allocation of the external debt in the economy. It allocates a proportion λ_t to finance its expenditures g_t and a proportion $1-\lambda_t$ to the financing of the private sector (formal investment projects) so we have

$$\begin{aligned} g_t = d^g_t = \lambda_t d \\ d^f_t = (1-\lambda_t)\, d \end{aligned} \tag{6}$$

3. External debt and Informality

The size of the formal sector defined by (5) can be written equivalently using (6)

$$\theta_t = (1-\tau_t)\left[1+\left(1-\frac{r}{\alpha a \lambda_t d}\right)\frac{(1-\lambda_t)\,d}{w_t}\right] \tag{7}$$

This expression shows that an increase of the external public debt share (λ_t) to the detriment of the private (formal) sector has an ambiguous effect on the size of the formal sector. In one hand, it increases the public expenditures which induces positive externality and increases the project productivity. By this channel, its affects positively the size of the formal sector, since more agents will have incentive to quit the informal sector seeing that the return of the formal project increases relatively to the cost of external borrowing. This effect is captured through the term $\left(1-\frac{r}{\alpha a \lambda_t d}\right)$. In the other hand, it reduces the amount of external financing to formal projects which tend to diminish the size of the formal sector. This effect is captured through the term $(1-\lambda_t)$.

[5] The case $\theta_t = 1$: there is no taxation $\tau_t = 0$ and the external financing is very costly so that $d^f_t = 0$.

Proposition 1

i) The size of the formal sector is a concave function of the public external debt share.

ii) The size of the formal sector is maximal $\overline{\theta}_t$ for a share of the public external debt given by $\overline{\lambda}_t = \min(1, \sqrt{\frac{r}{\alpha ad}})$ and we have $\dfrac{\partial \theta_t}{\partial \lambda_t}\left(\lambda_t - \overline{\lambda}_t\right) \leq 0.$

iii) The size of the formal sector decreases with the tax rate.

Proof: It is straightforward using (7) and differentiating θ_t relatively to τ_t to obtain $\frac{\partial \theta_t}{\partial \tau_t} < 0$. Then, differentiating θ_t relatively to λ_t we obtain

$$\begin{cases} \frac{\partial^2 \theta_t}{\partial \lambda_t^2} < 0 \\ \frac{\partial \theta_t}{\partial \lambda_t} = \frac{(1-\tau_t)d}{w_t}\left[\frac{r}{\alpha ad}\frac{1}{\lambda_t^2} - 1\right] \end{cases} \tag{8}$$

Figure 1 illustrates the case $\overline{\lambda}_t < 1$ and shows how the size of the formal sector θ_t varies when the external public debt share λ_t and the tax rate τ_t vary. As it can be noted, the effect of external public debt share (λ_t) on the size of the formal sector is non linear and depends on the taxation rate. An increase of λ_t improves the size of the formal sector when the positive effect of an increase in the government expenditures exceeds the negative effect due to a decrease in the external financing to formal projects. The turning point after which any increase in the external public debt share induces a smaller formal sector is $\overline{\lambda}_t$.

4. External debt and Growth

In this section we will explain how the allocation of the external debt between private formal sector and government expenditure affects economic growth. For period $t+1$, the growth factor G_{t+1} is defined by y_{t+1}/y_t or equivalently using (1) k_{t+1}/k_t.The quantity of the capital (investment) good available at $t+1$ is the sum of the output of the formal projects $\theta_t \kappa_{t+1}^f$ and that of the informal projects $\int_{\theta_t}^1 \kappa_{t+1}^j d\theta_i$. Using (2) and (3) we obtain

$$\begin{aligned} k_{t+1} &= \theta_t \left(ag_t\left(w_t + d_t^f\right)\right) + \int_{\theta_t}^1 \left(\theta_j ag_t w_t\right) d\theta_i \\ &= aw_t\left(\lambda_t d\right) h(\theta_t, \lambda_t) \end{aligned}$$

where

$$h(\theta_t, \lambda_t) = \theta_t\left(1 + \frac{(1-\lambda_t)\,d}{w_t}\right) + \frac{1-(\theta_t)^2}{2} \tag{9}$$

Hence, using the expression of (3) of w_t, the growth factor is given by

$$G_{t+1} = a(1-\alpha)d\lambda_t h(\theta_t, \lambda_t) \tag{10}$$

The relationship between the external debt and growth is summarized by the following proposition.

Proposition 2

1) For a given tax rate τ_t, there exists λ_t^* in $\left[\overline{\lambda}_t, 1\right]$ maximizing growth.

2) The growth is a concave function of the external public debt share in the following cases

i) $w_t \geq d$

ii) $w_t < d$ *and* $\lambda_t < \overline{\lambda}_t$

iii) $w_t < d$ *and* $\lambda_t \geq \overline{\lambda}_t$ *if* $\tau_t < \tau_t^*$ *where* τ_t^**depending only on* $\frac{r}{\alpha a}$*,* d *and* w*.*

Proof: See the appendix.

Figure 2 illustrates different configurations of the effect of the external public debt share on economic growth. This configurations are derived from proposition 2 and others characteristics detailed in its proof. Among these configurations, note that (c) (corresponding to the cases ii and iii of proposition 2) and (d) (corresponding to the cases i and ii) are the most economically acceptables. As we showed in proposition 1, when $\lambda_t > \overline{\lambda}_t$, an increase in the external public debt share λ_t diminishes the size of the formal sector. However, as long as the share λ_t remains in $\left[\overline{\lambda}_t, \lambda_t^*\right]$, this negative effect is dominated by the positive effect of higher externalities generated by the increase of public expenditures. Therefore, the resulting effect is positive and there is an increase in growth although the informal sector widens. Graphs (a) and (b) illustrate two possible configurations where the growth function is concave only for $\lambda_t < \overline{\lambda}_t$ (case ii) and in a neighbourhood of unity (denoted N(1)). The configurations (a) and (b) are economically difficult to interpret since they show that the resulting effect becomes positive again in a third region. The configuration (d) is a particular case of (c) since it corresponds to $\lambda_t^* = 1$.

5. Conclusion

We proposed an endogenous growth model with nested generations taking into account an important characteristics of the developing countries: the high public indebtedness and the informal sector. We show that an increasing of the public external debt has two opposite effects. On the one hand, it enhances growth through a positive externality affecting the productivity of private firms. On the other hand, it inhibits growth by reducing the capital accumulation dynamic. Indeed, higher public debt ousts the external financing of private projects and makes the informel sector more attractive for entrepreneurs with lower cost of tax evasion. The enlargement of the less efficient informal sector reduces the capital accumulation dynamic. These two effects generate a non-linear effect of the public external debt on growth and an optimal share of the public external indebtness. Interestingly, it is also shown that, under certain condition, the enlargement of the informal sector could be accompanied by higher growth. This is the case when the reduction of the formal sector size is more than compensated by the productivity increase of its remaining firms. By means of the results outlined in this paper we argue that the external debt management in developing countries should take in account not only the classic debt/GDP ratio but has to be designed according to the optimal allocation between public sector versus private sector. This optimal allocation depends on the structural characteristics of each economy. Chiefly, policy makers have to consider the sensitivity of the informal sector size when implementing a strategy to reduce their external indebtedness.

Appendix

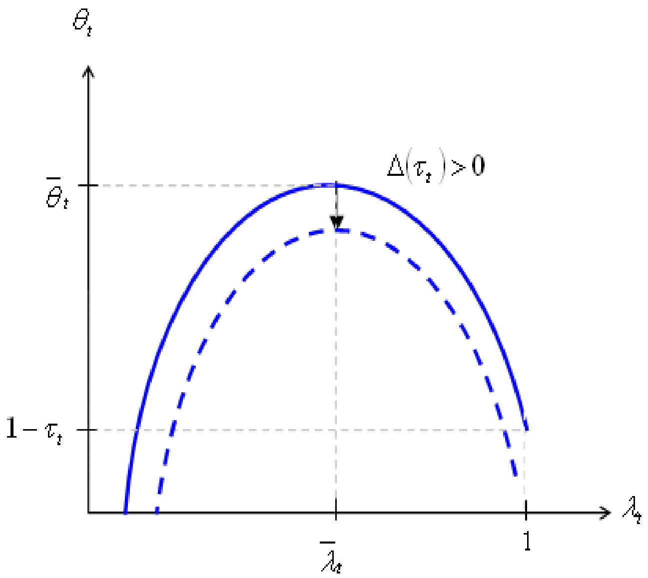

Figure 1: The effect of the external public debt share on the formal sector size
An illustration of proposition 1

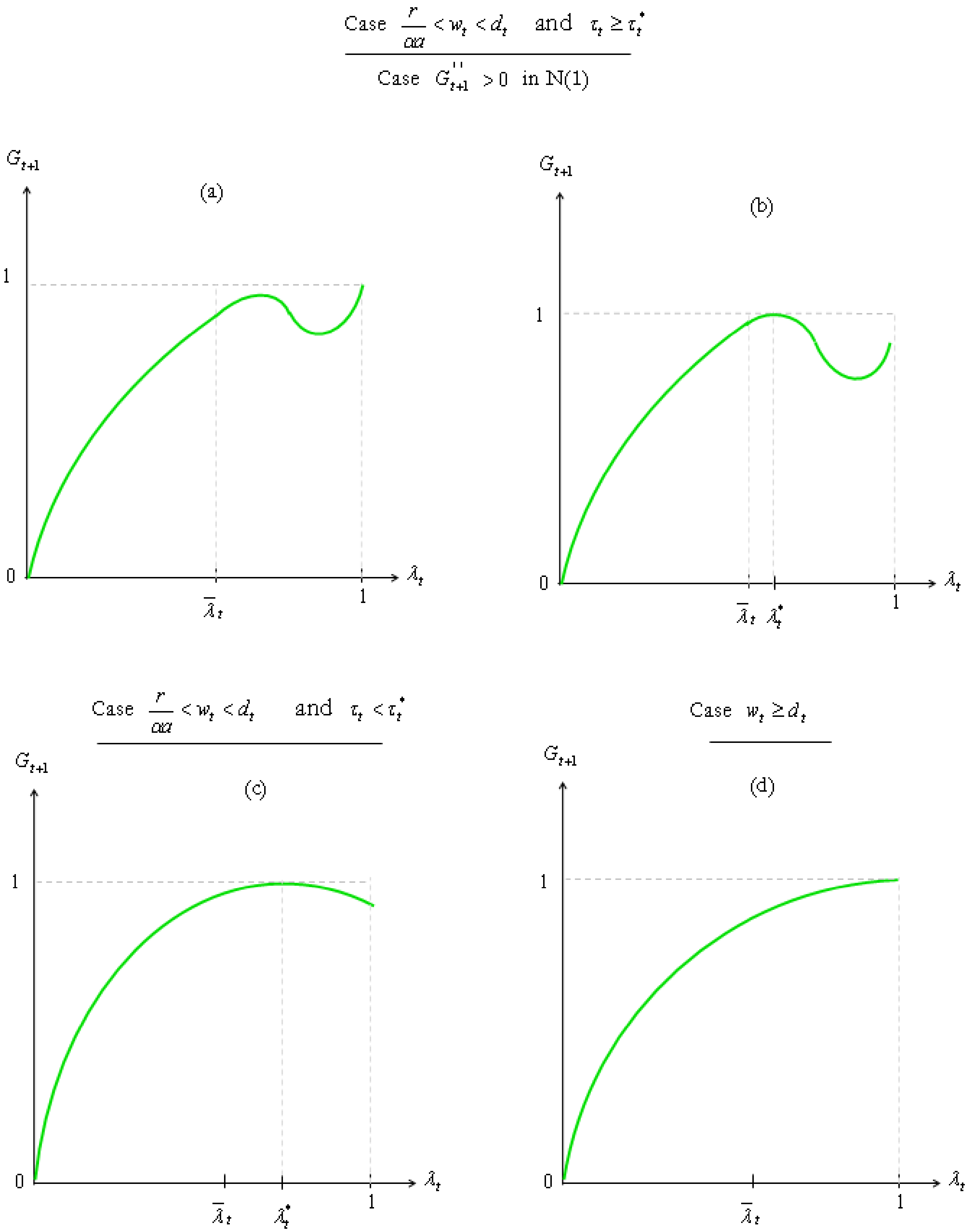

Figure 2: The effect of the external public debt share on growth
An illustration of proposition 2.

Proof of proposition 2

From (10) the maximal growth is obtained for λ_t^* maximizing $f(\lambda) = \lambda h(\theta(\lambda), \lambda)$ which is not necessarily $\overline{\lambda}$.We have $\frac{\partial G}{\partial \lambda} = a(1-\alpha)d\frac{df}{d\lambda}$ with

$$\frac{df}{d\lambda} = h + \lambda \frac{\partial h}{\partial \theta}\frac{\partial \theta}{\partial \lambda} + \lambda \frac{\partial h}{\partial \lambda}$$

and

$$\begin{aligned}
\frac{d^2 f}{d\lambda^2} &= \left(\frac{\partial h}{\partial \theta}\frac{\partial \theta}{\partial \lambda} + \frac{\partial h}{\partial \lambda}\right) + \left(\frac{\partial h}{\partial \theta}\frac{\partial \theta}{\partial \lambda} + \lambda\frac{\partial^2 h}{\partial \theta^2}\left(\frac{\partial \theta}{\partial \lambda}\right)^2 + \lambda\frac{\partial^2 h}{\partial \theta \partial \lambda}\left(\frac{\partial \theta}{\partial \lambda}\right) + \lambda\frac{\partial h}{\partial \theta}\frac{\partial^2 \theta}{\partial \lambda^2}\right) \\
&\quad + \left(\frac{\partial h}{\partial \lambda} + \lambda\frac{\partial^2 h}{\partial \lambda \partial \theta}\left(\frac{\partial \theta}{\partial \lambda}\right)\right) \\
&= 2\left(\frac{\partial h}{\partial \lambda} + \left(\frac{\partial h}{\partial \theta} + \lambda\frac{\partial^2 h}{\partial \theta \partial \lambda}\right)\left(\frac{\partial \theta}{\partial \lambda}\right)\right) + \lambda\frac{\partial^2 h}{\partial \theta^2}\left(\frac{\partial \theta}{\partial \lambda}\right)^2 + \lambda\frac{\partial h}{\partial \theta}\frac{\partial^2 \theta}{\partial \lambda^2} \\
&= 2\left[-\frac{\theta d}{w} + \left(\frac{\partial h}{\partial \theta} - \frac{d\lambda}{w}\right)\left(\frac{\partial \theta}{\partial \lambda}\right)\right] - \lambda\left(\frac{\partial \theta}{\partial \lambda}\right)^2 + \lambda\frac{\partial h}{\partial \theta}\frac{\partial^2 \theta}{\partial \lambda^2}
\end{aligned}$$

and

$$\frac{\partial^2 \theta}{\partial \lambda^2} = -\frac{2}{\lambda}\frac{\partial \theta}{\partial \lambda} - \frac{2(1-\tau)d}{\lambda w}$$

Hence, we have

$$\begin{aligned}
\frac{d^2 f}{d\lambda^2} &= \frac{-2d}{w}\left[\theta + \lambda\left(\frac{\partial \theta}{\partial \lambda}\right)\right] - \lambda\left(\frac{\partial \theta}{\partial \lambda}\right)^2 - \frac{2(1-\tau)d}{\lambda w}\frac{\partial h}{\partial \theta} \\
&= \frac{-2d}{w}\frac{\partial(\lambda\theta)}{\partial \lambda} \underbrace{-\lambda\left(\frac{\partial \theta}{\partial \lambda}\right)^2 - \frac{2(1-\tau)d}{\lambda w}\underbrace{\frac{\partial h}{\partial \theta}}_{>0}}_{<0}
\end{aligned} \tag{11}$$

* Case $d \leq w$

$$\frac{\partial(\lambda\theta)}{\partial \lambda} = \frac{(1-\tau)d}{w}\left(1 + \frac{w}{d} - 2\lambda + \frac{r}{\alpha a d}\right) \geq 0 \tag{12}$$

Therefore, $\frac{d^2 f}{d\lambda^2} < 0$ and the function $\frac{df}{d\lambda}$ is strictly decreasing. Meanwhile, we have

$$\begin{aligned}
\left.\frac{df}{d\lambda}\right)_{\lambda=1} &= h(\theta(1), 1) + \left.\frac{\partial h}{\partial \theta}\right)_{\lambda=1}\left.\frac{\partial \theta}{\partial \lambda}\right)_{\lambda=1} + \left.\frac{\partial h}{\partial \lambda}\right)_{\lambda=1} \\
&= \theta(1)\left(1 - \frac{d}{w}\right) + \frac{1-(\theta(1))^2}{2} + \left(\theta(1) - (\theta(1))^2\right)\left(\frac{d}{w}\left[\frac{r}{\alpha a d} - 1\right]\right) \\
&= \theta(1)\left(1 + \left[\frac{r}{\alpha a d} - 2\right]\frac{d}{w}\right) - \frac{1}{2}(\theta(1))^2\left[1 + \frac{d}{w}\left[\frac{2r}{\alpha a d} - 2\right]\right] + \frac{1}{2}
\end{aligned} \tag{13}$$

since $\theta(1) = 1-\tau$. Let's denote $g(\tau) = \left.\frac{df}{d\lambda}\right)_{\lambda=1}$. We have $g(1) = \frac{1}{2}$ and $g(0) = 1 - \frac{d}{w} \geq 0$. Varying τ in $[0, 1]$ it is easy to show that $g(\tau) = \left.\frac{df}{d\lambda}\right)_{\lambda=1} > 0$. Hence, $\frac{df}{d\lambda} > \left.\frac{df}{d\lambda}\right)_{\lambda=1} > 0$ and we obtain $f(\lambda) < f(1)$ for every λ. We conclude that growth is maximal for $\lambda^* = 1 > \overline{\lambda}$.

** Case $d > w$

We show using (13) that it exists $\tau^* \in]0,1[$ depending only on $\frac{r}{\alpha a}$, d and w verifying $g(\tau^*) = 0$ such that

$$\left.\frac{df}{d\lambda}\right)_{\lambda=1} \begin{cases} > 0 & \text{if} \quad \tau > \tau^* \\ \leq 0 & \text{if} \quad \tau \leq \tau^* \end{cases} \tag{14}$$

We have $\frac{df}{d\lambda} = h + \frac{\partial\theta}{\partial\lambda}\frac{\partial h}{\partial\theta} + \lambda\frac{\partial h}{\partial\lambda}$ and since $\left.\frac{\partial\theta}{\partial\lambda}\right)_{\lambda=\overline{\lambda}} = 0$ we obtain

$$\begin{aligned} \left.\frac{df}{d\lambda}\right)_{\lambda=\overline{\lambda}} &= h(\theta(\overline{\lambda}),\overline{\lambda}) + \overline{\lambda}\left.\frac{\partial h}{\partial\lambda}\right)_{\lambda=\overline{\lambda}} \\ &= \theta(\overline{\lambda})\left(1 + (1-\overline{\lambda})\frac{d}{w}\right) + \frac{1-\left(\theta(\overline{\lambda})\right)^2}{2} - \overline{\lambda}\frac{d}{w}\theta(\overline{\lambda}) \\ &= \theta(\overline{\lambda})\left(1 + (1-2\overline{\lambda})\frac{d}{w}\right) + \frac{1-\left(\theta(\overline{\lambda})\right)^2}{2} \end{aligned}$$

It is straightforward to show that $\left.\frac{df}{d\lambda}\right)_{\lambda=\overline{\lambda}} < 0$ if and only if $w < \dfrac{r}{\alpha a}$ and

$$(u-v)^2 < \frac{d}{w} < (u+v)^2$$

with $u = \sqrt{\frac{r}{\alpha a w}}$ and $v = \sqrt{u^2-1}$.Therefore, if $w \geq \dfrac{r}{\alpha a}$ we have $\left.\frac{df}{d\lambda}\right)_{\lambda=\overline{\lambda}} \geq 0$ which combined with (14) enables us to announce it exists $\lambda^* \in \left[\overline{\lambda}, 1\right]$ which maximises the value of f and therefore the growth factor. The concavity/convexity of f determines the position of λ^* relatively to the two limits of $\left[\overline{\lambda}, 1\right]$.Besides, we have $\left.\frac{\partial(\lambda\theta)}{\partial\lambda}\right)_{\lambda<\overline{\lambda}} = \overline{\lambda}\left.\frac{\partial\theta}{\partial\lambda}\right)_{\lambda<\overline{\lambda}} + \theta)_{\lambda<\overline{\lambda}} > 0$ since we have from proposition 1 $\left.\frac{\partial\theta}{\partial\lambda}\right)_{\lambda<\overline{\lambda}} > \left.\frac{\partial\theta}{\partial\lambda}\right)_{\overline{\lambda}} = 0$. Therefore, we conclude using (11) that $\left.\frac{d^2f}{d\lambda^2}\right)_{\lambda<\overline{\lambda}} < 0$ or equivalently $\left.\frac{d^2G}{d\lambda^2}\right)_{\lambda<\overline{\lambda}} < 0$.

References

Acemoglu, D., S., Johnson, and A. Robinson (2005) "Institutions as the Fundamental Cause of Long-Run Growth" in *Handbook of Economic Growth*, by P. Aghion and S. Durlauf, Eds. Elsevier: Amsterdam, 385-472.

Acemoglu, D., S., Johnson, A., Robinson and Y. Thaicharoen (2002) "Institutional Causes, Macroeconomic Symptoms: Volatility, Crises and Growth" NBER Working Papers 9124.

Bangura, S.A.F., D., Kitabire, and P., Robert. (2000) "External Debt Management in Low-Income Countries" IMF Working Paper 00/196, 1-35.

Bose, N., and R., Cothern (1996) "Equilibrium Loan Contracts and Endogen-ous Growth in the Presence of Asymmetric Information" *Journal of Monetary Economics*, 38, 423-439.

Burnside, C., and D., Dollar (2000) "Aid, Policies, and Growth" *American Economic Review* **90**, 847–68.

Burnside, C., and D., Dollar (2004) "Aid, policies, and growth : revisiting the evidence" Policy Research Working Paper Series 3251, The World Bank.

Chickering, L.A., and M., Salahdine (1991) *The Silent Revolution: The Informal Sector in Five Asian and Near Eastern Countries*, ICS Press: San Francisco.

Clements, B., R.Bhattacharya, and T. Q. Nguyen (2003) "External Debt, Public Investment, and Growth in Low-Income Countries" IMF working paper, 249.

Cohen, D. (1992) "Large External Debt and (Slow) Domestic Growth: A Theoretical Analysis" *Journal of Economic Dynamics and Control* **19**, 1141-63.

Cordella, T., L.A., Ricci, and M. Ruiz-Arranz. (2005) "Debt Overhang or Debt Irrelevance? Revisiting the Debt-Growth Link" IMF Working Paper 05/223.

Imbs, J., and R., Ranciere (2005) "The Overhang Hangover" Economics Working Papers 878, Department of Economics and Business, Universitat Pompeu Fabra.

Johnson, S., D., Kaufmann, and A., Shleifer (1997) "The Unofficial Economy in Transition" *Brookings Papers on Economic Activity* **2**, 159-239.

Kamin, S.B. , Kahn, R.B., and R., Levine (1989) "External Debt and Developing Country Growth" International Finance Discussion Papers, 352, Board of Governors of the Federal Reserve System.

Kaufmann, D., and A., Kraay (2003) "Governance and Growth. Causality which way? Evidence from the World, in brief", mimeo, The World Bank.

Krugman, P. (1988) "Financing vs. Forgiving a Debt Overhang" *Journal of Development Economics*, **29**, 253-268.

Sachs J. (1989) "The Debt Overhang of Developing Countries" in *Debt, Stabilization and Development: Essays in Memory of Carlos Díaz Alejandro*, by G. Calvo, R. Finlay, P. Kouri and J. Braga de Macedo, Basil Blackwell: Oxford.