Abstract
Within a strategic delegation model, this paper examines in a quantity setting oligopoly framework the determinants of the degree of strategic delegation - the latter being defined as the extent of the departure from pure profit maximization. The sub-game perfect equilibrium degree of strategic delegation is derived as a function of the two key parameters which determine market competitiveness in a homogeneous product set-up, i.e., the price-elasticity of market demand and the number of firms. With respect to both these parameters we find that their relationship with the degree of delegation is not necessarily monotone. Indeed, for an increase in elasticity or a reduction in market concentration to reduce strategic delegation, these determinants of the Lerner index of monopoly power must satisfy restrictions which guarantee that the initial market environment is sufficiently competitive.
1. Introduction

The literature on strategic delegation has shown that profit maximizing firms may strategically choose to commit to a non-profit maximizing behaviour, the latter being formalized in terms of each firm’s owner delegating market decisions to a manager, to whom an objective function is assigned in terms of a combination of profits and another variable (revenues, quantity, relative profits, etc.). The relative weight of this additional variable, strategically chosen by the owners, is a measure of the distortion from the profit-maximization procedure to which they commit, and defines the implicit structure of incentives which should support the underlying principal-agent relation.

In recent years the basic models by Sklivas (1987), Fershtman and Judd (1987) and Basu (1995) have been enriched to investigate the implications of extending the delegation (originally conceived for the choice of quantity or price) to decisions concerning, among others, quality (Ishibashi, 2001), R&D investments (Zhang and Zhang, 1997), vertical and horizontal product differentiation (Barros and Grilo, 2002; Bárcena-Ruiz and Casado-Izaga, 2005); moreover, a number of issues such as the profitability of horizontal mergers (Gonzalez-Maestre and Lopez-Cunat, 2001), the sustainability of collusive agreements (Lambertini and Trombetta, 2002), the competition between private and public firms (White, 2001) have been re-examined in a delegation framework. Less attention has been paid, however, to what determines the “degree of delegation”, i.e., the extent of the departure from profit maximization to which firms commit themselves in equilibrium.

In this paper we tackle this issue in a quantity setting framework, and by assuming that the managers’ objective function is a combination of profits and revenues. In particular, we concentrate on market competitiveness - synthesized in the Cournot equilibrium value of the Lerner index of monopoly power - as key determinant of the degree of delegation. By developing a model with constant-elasticity market demand, we parametrize the solution for the optimal degree of delegation to the elasticity itself and the number of firms, which is treated as exogenous. Our main result is that the relation between delegation and these two parameters is not necessarily monotone, allowing for a reduction of the Lerner index of monopoly power to be associated to a higher delegation.

The paper is organized as follows. In Section 2 we develop the model, discussing in Section 3 the role of demand elasticity and market concentration on the delegation decisions; Section 4 offers some conclusions.

2. The model

We consider a standard two-stage strategic delegation game in a quantity setting framework, with \( n \) oligopolistic firms producing a homogeneous product. Each firm has an owner and a manager. In this game, the quantity of each firm is set at the second stage by its manager who maximizes a linear combination of profits and revenues (Fershtman and Judd, 1987). For each firm, the weight of revenues in the objective function of the manager is the strategic decision left to the profit maximizing owner at the first stage. This decision can be thought
of as the content of a delegation contract and defines the structure of the incentives to the manager.

In order to parametrize the solution of the game to both the price elasticity of demand and the number of firms, we assume a market demand function with constant price-elasticity:

\[ P(Q) = Q^{-\frac{1}{\varepsilon}}, \text{ where } \varepsilon > 1 \text{ and } Q = \sum_{i=1}^{n} q_i. \]

The restriction on \( \varepsilon \) ensures that the reaction function of any firm is well defined for any possible choice of its rivals. All firms share the same technology, synthesized in a constant average and marginal cost \( c \).

At the quantity stage, the manager of firm \( i \) behaves consistently with the incentive structure chosen by the owner, by maximizing the following linear combination of profits \( \pi_i \) and revenues \( R_i \):

\[ M_i = \theta_i \pi_i + (1 - \theta_i) R_i = \left( \sum_{i=1}^{n} q_i \right)^{-\frac{1}{\varepsilon}} q_i - \theta_i c q_i, \quad (1) \]

where the absolute value of the weight attached to revenues (the distance of \( \theta_i \) from 1) is a measure of the distortion from pure profit maximization and therefore of the extent of delegation.

Maximization of (1) implies

\[ \varepsilon Q^{-\frac{1}{\varepsilon}} q_i - q_i Q^{-\frac{1+\varepsilon}{\varepsilon}} - \theta_i c \varepsilon = 0, \quad i = 1, \ldots, n \quad (2) \]

if (2) is satisfied for \( q_i > 0 \), or \( q_i = 0 \) otherwise. Let us start by considering quantity-stage equilibria in which all firms produce a positive quantity. In this case, by summing (2) over \( i \), we obtain the following total industry output and market price:

\[ Q = \frac{(n\varepsilon - 1)^\varepsilon}{(\varepsilon \left( \sum_{i=1}^{n} \theta_i \right))^{\varepsilon}}, \quad P = \frac{c \varepsilon \left( \sum_{i=1}^{n} \theta_i \right)}{(n\varepsilon - 1)}. \]

Substituting \( Q \) in equation (2), the latter can be solved for the Nash equilibrium in quantities:

\[ q_i^* = \frac{(n\varepsilon - 1)^\varepsilon}{c \varepsilon \left( \sum_{i=1}^{n} \theta_i \right)^{\varepsilon}} \left( \varepsilon - \frac{\theta_i (n\varepsilon - 1)}{\sum_{i=1}^{n} \theta_i} \right), \quad i = 1, \ldots, n. \]

Notice that for all \( i \), the condition \( q_i^* > 0 \) is satisfied if \( \theta_i < (\varepsilon/(\varepsilon(n - 1) - 1)) \Theta \), where \( \Theta = \sum_{j \neq i} \theta_j \). This amounts to saying that a second stage equilibrium with positive quantities is defined only for a subset of the conceivable \( \theta \) \( n \)-tuples; in particular for those \( n \)-tuples which ensure that the extent of delegation is not too different across firms. Moreover, when
\( \theta_i = 1 \) for all \( i \), i.e., in the absence of strategic delegation, the above expression clearly collapses to the symmetric Cournot-Nash solution under constant elasticity of demand.

The structure of incentives - i.e., the delegation parameters - are strategically chosen by the profit-maximizing owners at the first stage of the game. In order to identify the subgame perfect Nash equilibrium of the model, we shall proceed as follows. We first look for a constrained Nash equilibrium in \( \theta \)'s, i.e., an equilibrium over the restricted domain of the \( \theta \)'s \( n \)-tuples delivering positive quantities. Then we prove that this constrained Nash equilibrium is indeed an unconstrained sub-game perfect Nash equilibrium, i.e., an equilibrium over the \( \theta \)'s unrestricted domain.

By substituting \( P \) and \( q_i^* \) in the profit function \( \pi_i = (P - c) q_i \), and maximizing the latter with respect to \( \theta_i \), we get the following implicit reaction function:

\[
\varepsilon \left( \varepsilon - \frac{\theta_i (n\varepsilon - 1)}{(\Theta + \theta_i)} \right) - (\varepsilon (\Theta + \theta_i) - (n\varepsilon - 1)) \frac{\varepsilon^2 (\Theta + \theta_i) + (n\varepsilon - 1)(\Theta - \varepsilon \theta_i)}{(\Theta + \theta_i)^2} = 0. \tag{3}
\]

Under symmetry, \( \Theta = (n - 1)\theta_i \); therefore, under the hypothesis of positive production by all firms, we obtain the following Nash equilibrium value of the delegation parameter:\(^1\)

\[
\theta^* (\varepsilon, n) = \frac{\varepsilon^2 n (n^2 - n + 1) - (2n\varepsilon - 1)(n - 1) - \varepsilon}{\varepsilon n (\varepsilon - n + n\varepsilon (n - 1))}. \tag{4}
\]

It can be checked that \( 0 < \theta^* < 1 \). The constant elasticity hypothesis does not alter the basic feature of strategic substitutability at the quantity stage equilibrium, and this implies that at the delegation stage the owners are willing to induce strategically an aggressive behaviour of their managers.

For (4) to be an unconstrained sub-game perfect Nash equilibrium of the delegation game, it must be “deviation-proof” over the entire domain of the \( \theta \)'s. In the Appendix we show that any unilateral deviation from (4) aimed at inducing the rivals to produce a zero quantity is not advantageous, so that (4) is indeed a perfect equilibrium over the unrestricted domain of the strategy space.

### 3. Demand elasticity, concentration and managerial incentives

A nice feature of (4) is that the sub-game perfect equilibrium degree of strategic delegation is a function of the two key parameters that define market competitiveness in a homogeneous product set-up, i.e., the elasticity of market demand and the number of firms.

We consider first the role of demand elasticity. It is immediate to check that as \( \varepsilon \) approaches infinity, the incentive to strategic delegation disappears for all values of \( n \). However, the pattern of convergence to strict profit maximization is not necessarily monotone. Figure 1 shows the behaviour of \( \theta^* (\varepsilon, n) \) as a function of \( \varepsilon \), for different given values of \( n \).

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\(^1\)It can be checked that the Second Order Conditions for a maximum are satisfied at this symmetric equilibrium.
In the duopoly case the function is clearly non-monotone: for $\varepsilon \in \left(1, 1 + \frac{\sqrt{3}}{3}\right]$ the degree of strategic delegation $(1 - \theta^*)$ is increasing in $\varepsilon$, moving from zero (in the limit case $\varepsilon \to 1$) to 0.067, while it is monotonically decreasing for $\varepsilon > 1 + \frac{\sqrt{3}}{3}$. For $n \geq 3$, however, the $\theta^*$ function is monotone: any increase in the elasticity of demand leads to a reduction in the optimal delegation. In order to explain the non-monotonicity in the duopoly case, it is useful to concentrate upon the way in which the constant-elasticity hypothesis affects the reaction function at the two stages of the game. This hypothesis implies that both at the quantity stage and at the delegation stage the reaction functions exhibit first strategic complementarity and then strategic substitutability, the latter characterizing the equilibrium at both stages.\footnote{When $\varepsilon = 1$, the symmetric equilibrium of the quantity game occurs at a point where the slope of the reaction function is zero. This strategic independence implies that there is no incentive to distort from profit maximization the manager’s choice. This explains why $\lim_{\varepsilon \to 1} \theta^* (\varepsilon, 2) = 1$.} At the quantity stage, for positive quantities this shape results from the interplay of two forces: as the rival firm increases its quantity, firm $i$ experiences both a leftward shift and a flattening of its residual demand curve. While the first effect lowers the marginal revenue, the second, which dominates for high values of the firm’s quantity compared to the rival’s, tends to increase it. The behaviour of the reaction function directly derives from these changes in the marginal revenue function (Naish, 1998).

As far as the delegation stage is concerned, Figure 2 shows the reaction function of firm $i$ at this stage for different values of $\varepsilon$. The positive quantity restriction $(\theta_i < \theta_j \varepsilon / (\varepsilon - 1))$ implies that for any given $\varepsilon$, equation (3) - in the figure represented by the solid lines -
Figure 2: The reaction function at the delegation stage

applies for $\theta_j > \theta_j(\varepsilon) = (\varepsilon - 1)/\varepsilon$. Straightforward calculations show that the reaction function is first positively, then negatively sloped, reaching a maximum at the left of the symmetric equilibrium, the latter being therefore characterized by strategic substitutability. The intuition behind this slope reversal can be put as follows. As the rival moves towards profit maximization (higher $\theta_j$), firm $i$ perceives a movement along the market demand curve with a decrease in $Q$ and an increase in $P$. This exerts an effect on the delegation choice of firm $i$ which again is twofold. A lower delegation (higher $\theta_i$) and the related reduction in $q_i(a)$ has a greater upward impact on market price; $(b)$ implies a greater reduction of firm $i$’s revenues at the given price. The second effect is the only relevant one in the standard linear demand case and explains in that setup the negative slope of the reaction function over its entire domain. However, the first is peculiar of the convex shape of the demand curve in the constant-elasticity case, and induces strategic complementarity. The standard quantity effect (and thus strategic substitutability) prevails when the price is sufficiently high, and therefore when the quantity produced by the rival is low, due to a low degree of strategic delegation.

Figure 2 shows also the effects of changes in $\varepsilon$. First, notice that as $\varepsilon$ increases, the limit value of the domain of the reaction function $\theta_j$ shifts to the right: as the market environment becomes more competitive due to higher $\varepsilon$, firm $i$ finds it profitable to produce only in the presence of a progressively less aggressive behaviour by firm $j$. Moreover, since higher values of the elasticity of demand imply, ceteris paribus, a lower market price and a lower reactivity of price to quantity changes, as $\varepsilon$ rises the strength of the effects of

\footnote{For $\theta_j < \theta_j(\varepsilon)$, the best reply of firm $i$ is given by a reaction correspondence: all $\theta_i \geq \theta_j\varepsilon/(\varepsilon - 1)$ (in Figure 2, all points on the dashed lines, or above) represent the best reply of firm $i$, all entailing $q_i = 0$. Notice also that for all $\varepsilon$, along the reaction function $\theta_i$ tends to 1 as $\theta_j$ tends to $\theta_j(\varepsilon)$.}
the rival’s choices described above weakens, and therefore (a) the reaction function flattens, and (b) the dominance of strategic substitutability occurs for progressively lower values of delegation. When the elasticity is close to one both these movements are consistent with an inward shift of the part of the reaction function lying above the 45° line; on the contrary, when the elasticity is higher, as ε increases the new reaction function crosses the previous one from below, at the left of the 45° line. In the first case the equilibrium degree of strategic delegation increases, in the second it decreases. Since the optimal delegation is monotonically decreasing in ε for n > 2, the above argument suggests that an inverse relation between elasticity and delegation occurs, provided that the Lerner index of monopoly power is sufficiently low, due to either a sufficiently low market concentration (n > 2) or a sufficiently high elasticity of demand (ε > 1 + √3/3).

Finally, we consider how strategic delegation is affected by market concentration. The fact that both the monopolistic and the competitive firm do not provide to their managers any incentive to depart from profit maximization led Fersthman and Judd (1987) to suggest that “the relationship between market structure and managerial incentives will likely not be monotonic”, since “nonprofit-maximizing incentives will be given only in oligopolistic industries”. Indeed, our constant-elasticity model allows to extend their argument within the oligopolistic markets. According to (4), for any value of ε the optimal degree of delegation is not monotonically decreasing in n, but reaches its maximum (under the integer constraint) for n = 3. Again, for any given ε, in order to obtain the “intuitive” inverse relation between delegation and market concentration, the latter must be sufficiently low.

4. Conclusions

In this paper we investigated the determinants of the degree of strategic delegation in a quantity setting framework. The ideal setup to study how market fundamentals affect strategic delegation would be one which allows to parametrize the solution with respect to the elasticity of market demand, the elasticity of costs and the number of firms. However, under generic constant elasticity of demand and generic constant elasticity of costs, the two-stage game cannot be solved. Therefore, by assuming linear costs we focussed upon the role of the two factors affecting market competitiveness: the elasticity of demand and the number of firms.

While one would expect that moving towards a more competitive environment should reduce the incentive to delegation, our main result is that the relationship between the factors underlying the Lerner index of monopoly power and the degree of strategic delegation is not necessarily monotone. On the one side, in a duopoly setting there is a range of demand elasticity values for which delegation increases with elasticity; on the other side, for all values of the elasticity the highest degree of strategic delegation is not observed for n = 2, but for n = 3.

Competition weakens the incentives to commit to an overaggressive behaviour, but for this to occur the initial environment must be sufficiently competitive, either in terms of demand elasticity or in terms of market concentration.
References


Appendix

Solution (4) has been obtained under a “restricted domain” hypothesis, i.e., by concentrating upon the \( \theta \) \( n \)-tuples which entail positive production by all firms. In order to ensure that it is the unconstrained sub-game perfect Nash equilibrium of the delegation game, we have to prove that no firm has the incentive to deviate from (4) choosing a value of \( \theta \) such that rivals find it optimal to refrain from production. Here we demonstrate this deviation-proof property in the duopoly case, but the same procedure can be applied to the general oligopoly case.

Let us assume that firm \( i \) sets the value of \( \theta_i \) according to (4); for \( n = 2 \) the latter is

\[
\theta_i^* (\varepsilon, 2) = \frac{1}{2} \frac{16 \varepsilon^2 - 5 \varepsilon + 1}{\varepsilon (3 \varepsilon - 2)}
\]

Since at the quantity stage of the game

\[
q_i^* = \frac{(2 \varepsilon - 1) \varepsilon}{(c \varepsilon (\theta_i + \theta_j))^\varepsilon} \left( \varepsilon - \frac{\theta_i (2 \varepsilon - 1)}{(\theta_i + \theta_j)} \right)
\]

firm \( j \) might induce firm \( i \) to produce a zero quantity by setting

\[
\theta_j \leq \theta_j^* = \frac{\theta_i^* \varepsilon - 1}{\varepsilon} = \frac{1}{2} \frac{16 \varepsilon^3 - 11 \varepsilon^2 + 6 \varepsilon - 1}{\varepsilon^2 (3 \varepsilon - 2)}
\]

But for \( \theta_j = \theta_j^* \) and \( q_i = 0 \), the manager of firm \( j \) will produce

\[
\tilde{q}_j = \left( c \left( 2 \frac{16 \varepsilon^3 - 11 \varepsilon^2 + 6 \varepsilon - 1}{\varepsilon^2 (3 \varepsilon - 2)} \right) \frac{\varepsilon}{\varepsilon - 1} \right)^{-\varepsilon}
\]

at a market price

\[
\tilde{P} = (\tilde{q}_j)^{-\frac{1}{\varepsilon}} = \frac{1}{2} \frac{16 \varepsilon^2 - 5 \varepsilon + 1}{\varepsilon (3 \varepsilon - 2)} c,
\]

thus generating profits

\[
\tilde{\pi}_j = -\frac{1}{2} c^{1-\varepsilon} (\varepsilon - 1) (3 \varepsilon - 2)^{-\varepsilon} \varepsilon^{-1} \left( \frac{1}{2} (3 \varepsilon - 1) (2 \varepsilon - 1) \right)^{-\varepsilon}
\]

which are negative for \( \varepsilon > 1 \). The same negative profits outcome arises also in the oligopoly case.\(^4\) Solution (4) is therefore an unconstrained sub-game perfect Nash equilibrium, since no firm perceives the incentive to be unilaterally so aggressive to induce the rivals to refrain from production.

\(^4\)Detailed calculations are available upon request.