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A Unidirectional Hotelling Model

Mohammed Kharbach
HEC Montreal

Abstract

The standard hotelling model with linear transportation costs predicts an aggregation of the two competing firms in the middle of the customers support interval (Minimum Differentiation Principle). Using quadratic transportation costs, the two firms would locate in the opposite extremities of the interval (Maximum Differentiation Principle). Both cases assume bidirectional purchasing ability : a consumer can buy from a firm whether it is located on her right or on her left. In a situation where a consumer can buy only from firms located on her right (left), we show that one firm would locate at the right (left) end of the hotelling line while the other would locate at 3/5 (2/5) from the left end.

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1. Introduction

Hotelling competition model (1929) is one of the pionnering work in the theory of location. In a linear city with linear transportation costs, the model predicts an aggregation of the two competing firms in the middle of the customers support interval (Minimum Differentiation Principle).

On the other hand, D'Aspremont, Gabszewicz, and Thisse (1979) have shown that the Minimum Differentiation Principle does not hold with quadratic transportation costs; to reduce price competition, firms have a tendency to differentiate themselves by choosing locations at the ends of the customers' line.

Other studies show that the minimum differentiation may arise in equilibrium. For example, Jehiel (1992) and Friedman and Thisse (1993) consider price collusion after firms have made location choices while De Palma et al. (1985) use unobservable attributes in brand choice.

All such studies assume bidirectional purchasing ability: a consumer can buy from a firm whether it is located on her right or on her left. In unidirectional hotelling lines, location equilibria could be different from the standard hotelling results. Examples of unidirectional cases are highway and one way roads and non revertible flows in oil and gas pipelines: a consumer can buy only from a firm located on her right (one way road) or on her left (firms pumping oil and gas on her left).

The remainder of this note is organized as follows: Section 2 describes the setting. Section 3 derives the equilibrium and Section 4 concludes.

2. A hotelling unidirectional model

We consider a variant of the linear hotelling model . Two firms A *and* B with zero marginal costs are located within the segment $[0,1]$ which represents a continuous set of consumers distributed with a density equal to 1. Firm $A(B)$ is located in position a (b), where

$$\begin{aligned} a+b &\leq 1 \\ and,\ a &\leq 1-b \end{aligned}$$

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As in the standard Hotelling model, a consumer pays a price P for the product and a quadratic transportation cost. A consumer can buy only from a firm located on her right hand side. All consumers derive the same intrinsic utility (U) from consumption. U is assumed large so that all consumers want to consume the product as long as they can do it. If the price charged by firm $A(B)$ is $P_A(P_B)$ and the unit transportation cost is t then a consumer located in x derives a disutility equal to:

$$\begin{array}{rcl} P_B + t(1-b-x)^2 \;\; if \;\; a & \leq & x \leq 1-b \\ P_A + t(a-x)^2 \;\; if \;\; 0 & \leq & x \leq a \end{array}$$

If $x > 1-b$ and $b \neq 0$, then the consumer located in x cannot buy any product.

The two firms compete in a two stage game; in the first period they choose their locations then compete in prices in the second stage.

3. Equilibrium

In the second stage, the two firms compete in prices. A consumer located in $[0, a]$ is indifferent in buying from A or B if:

$$P_A + t(a - x_I)^2 = P_B + t(1 - b - x_I)^2$$

Therefore,

$$x_I = \frac{P_B - P_A}{2t(1-a-b)} + \frac{1+a-b}{2}$$

The profit of firm $A(B)$ is:

$$\begin{array}{rcl} \Pi_A & = & P_A x_I \\ \Pi_B & = & P_B(1 - b - x_I) \end{array}$$

The FOCs yield,

$$
\begin{aligned}
P_A &= \frac{P_B}{2} + \frac{t(1+a-b)(1-a-b)}{2} \\
P_B &= \frac{P_A}{2} + \frac{t(1-a-b)(1-a-b)}{2}
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
P_A &= \frac{t}{3}(1-a-b)(3-3b+a) \\
P_B &= \frac{t}{3}(1-a-b)(3-3b-a)
\end{aligned}
$$

The indifferent customer and the market shares of the firms are defined by:

$$
\begin{aligned}
x_I &= \frac{3-3b+a}{6} \\
1-b-x_I &= \frac{3-3b-a}{6}
\end{aligned}
$$

In the first stage, the two firms choose their locations:

$$
\begin{aligned}
\Pi_A(a) &= \frac{t}{18}(1-a-b)(3-3b+a)^2 \\
\Pi_B(b) &= \frac{t}{18}(1-a-b)(3-3b-a)^2
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\frac{\partial \Pi_A(a)}{\partial a} &= -\frac{t}{18}(1+3a-b)(3-3b+a) < 0 \\
\frac{\partial \Pi_B(b)}{\partial b} &= -\frac{t}{18}(9-7a-9b)(3-3b-a) < 0
\end{aligned}
$$

Firm B is therefore better off located in location $1(b=0)$.Firm A is better off located in the lowest possible value of a. Since $x_I = \frac{3-3b+a}{6} \leq a$, we have,

$$\frac{3-3b+a}{6}=a,$$

$$\begin{aligned} for \;\; b &= 0, \\ a &= \frac{3}{5} \end{aligned}$$

The prices, markets shares and profits for the two firms are:

$$\begin{aligned} P_A &= \frac{12t}{25}, x_I = \frac{3}{5}; \Pi_A = \frac{36t}{125} \\ P_B &= \frac{8t}{25}; 1 - x_I - b = \frac{2}{5}; \Pi_B = \frac{16t}{125} \end{aligned}$$

Notice that both firms are serving their hinterlands but because of competition, they are not pricing their products in a way to extract consumers rent.

4. Conclusion

In the standard hotelling model, customers can buy from firms regardless of their positions on the Hotelling line. The linear transportation costs imply the principle of Minimum Differentiation while a quadratic transportation cost leads to the Maximum Differentiation Principle. In cases where the customer can buy only from firms located on her right, we show that one company would locate in position 3/5 from the left end while the other would locate in the right end. By symmetry, if the customer is able to buy only from firms located on her let, then one firm would locate in the left end while the other could locate at 3/5 from the right end.

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