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Long-run relationship between inflation and growth in a New Keynesian framework

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Abstract

This study examines the steady-state growth effect of inflation in an endogenous growth model in which Calvo-type nominal rigidity with endogenous contract duration and monetary friction via wage-payment-in-advance constraint are assumed. On the balanced-growth path in this model, the marginal growth effect of inflation is weakly negative or even positive at low inflation rates because the effect on average markup offsets the negative marginal growth effect through the monetary friction, but the growth effect of inflation is negative and convex at higher inflation rates because the frequency of price adjustment approaches that of the flexible-price economy and the growth effect through the nominal rigidity is dominated by the growth effect through the monetary friction. With a plausible calibration of the structural parameters, this model generates a relationship between inflation and growth that is consistent with empirical evidence, particularly in industrial countries.

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1 Introduction

Recent empirical studies have found that the relationship between inflation and growth is nonlinear.¹ The stylized facts are as follows. First, there is a threshold inflation rate above which the marginal effect of inflation on growth is negative and below which it is insignificant or even positive. Second, above the threshold inflation rate, the relationship between inflation and growth is convex in the sense that the negative marginal effect is weaker when inflation is high.

On the other hand, most theoretical studies fail to generate this nonlinear relationship. For example, in flexible-price monetary endogenous growth models with cash-in-advance constraint, the marginal growth effect of inflation is always negative, as surveyed in Gillman and Kejak (2005). In monetary endogenous growth models with prototypical Calvo-type nominal rigidity, as in Funk and Kromen (2006) and Kuwahara and Sudo (2007), there is a threshold inflation rate, but above it the relationship is concave.

In this paper, we show that a monetary endogenous growth model with a Calvo-type staggered price setting with endogenous contract duration, as in Levin and Yun (2007), can generate a nonlinear relationship consistent across a wide range of inflation with the empirical evidence for industrial countries, shown by Khan and Senhadji (2001). In our calibrated model, there is a threshold inflation rate of about 0.1% a year below which the marginal effect of inflation on growth is weakly negative or even positive. Moreover, above the threshold inflation rate the marginal growth effect becomes negative and the inflation-growth relationship is convex.²

The rest of the paper is organized as follows. Section 2 describes the economy. Section 3 shows the mechanisms of the growth effect and compares the numerical results in exogenous and endogenous contract duration models. Section 4 is the conclusion.³

2 The Model

A simple two-capital endogenous growth model with wage-payment-in-advance constraint of firms is considered. There are three types of agents in this economy: the representative household, monopolistically competitive firms, and the monetary authority.

The representative household maximizes the following discounted sum of utility:

$$\sum_{t=0}^{\infty} \beta^t \{ \log C_t + \psi \log[(1 - n_t)H_t] \}, \quad \psi > 0 \text{ and } \beta \in (0, 1), \quad (1)$$

¹See Section 1 in Hung (2008).

²Bose (2002) and Hung (2008) generate a similar inflation-growth relationship by overlapping generation models with imperfect information. However, in their model quantitative analysis is difficult, because the degree of imperfect information is hard to calibrate.

³For more explanation and mathematical details see Arato (2008), of which this note is a shorter version.

where C denotes aggregate consumption, n denotes hours worked, and H denotes human capital stock, which depreciates at δ_H .⁴ The intertemporal budget constraint is as follows:

$$\begin{aligned} \frac{B_t}{P_t} + C_t + K_{t+1} - (1 - \delta_k)K_t + H_{t+1} - (1 - \delta_H)H_t \\ = \frac{i_{t-1}B_{t-1}}{P_t} + w_t n_t H_t + r_t^K K_t + \Phi_t, \end{aligned} \quad (2)$$

where B denotes the quantity of a nominal financial asset that earns the gross nominal interest rate i , K denotes physical capital stock, which depreciates at δ_K , π denotes the gross rate of inflation, w denotes the real wage rate, r^K denotes the real gross rate of return on physical capital, and Φ denotes real dividend income from firms owned by households.

Each individual firm j ($\in [0, 1]$) monopolistically supplies the variety j , using a Cobb-Douglas production technology,

$$Y_t(j) = AK_t(j)^\alpha Z_t(j)^{1-\alpha}, \quad \text{with } A > 0 \text{ and } \alpha \in (0, 1), \quad (3)$$

where $K(j)$ and $Z(j)$ denote the demand for physical capital and effective labor respectively, each of which must satisfy the resource constraints $\int_0^1 K_t(j)di = K_t$ and $\int_0^1 Z_t(j)di = n_t H_t$. It is assumed that workers must be paid their wage bills in cash in advance of production. Hence firm j borrows its nominal wage payment from a financial intermediary at the beginning of period t . Repayment occurs at the end of period t at the gross nominal interest rate i_t . Consequently, the total real production cost of firm j is $r_t^K K_t(j) + i_t w_t Z_t(j)$.

The aggregate demand index Y is assembled using the Dixit-Stiglitz aggregator, hence firm j faces a downward-sloping demand function, $Y_t(j) = (\frac{P_t(j)}{P_t})^{-\theta} Y_t$ with $\theta > 1$, where $P(j)$ denotes the price of variety j and the aggregate price level P is defined as $P_t = (\int_0^1 P_t(j)^{1-\theta} di)^{\frac{1}{1-\theta}}$. Each firm maximizes its profit by optimally setting its price subject to the demand function it faces and a sticky price assumption, the details of which will be described later.

At the beginning of period t , financial intermediaries have nominal money balances $P_{t-1}M_{t-1}$ and receive a monetary transfer $P_t M_t - P_{t-1}M_{t-1}$ from the monetary authority, where M denotes real money balances, and lend all their money to firms for their wage payments $\int_0^1 P_t w_t Z_t(j)di$. Hence the loan market clearing condition is $M_t = w_t n_t H_t$.

The aggregate demand consists of aggregate consumption, aggregate physical capital investment, aggregate human capital investment, and aggregate menu cost⁵; hence,

$$Y_t = C_t + K_{t+1} - (1 - \delta_K)K_t + H_{t+1} - (1 - \delta_H)H_t + (1 - \xi)\Omega_t. \quad (4)$$

⁴To keep the model tractable, we assume log utility and quality time of leisure. Our numerical result is robust even if the instantaneous utility function is assumed to be a more general form or to depend on raw time of leisure.

⁵The final term of RHS in (4) denotes aggregate menu cost. The details are described later.

The monetary authority sets the long-run target inflation rate π .

3 Growth Effect of Inflation

3.1 Nominal interest rate effect and markup effect

From the Euler equation on the balanced growth path $\gamma = \beta r$, the steady-state growth rate, γ , is proportional to the real rate of interest, r . Given i and average markup μ , the real rate of interest is determined by the no-arbitrage conditions between physical and human capital and financial assets:

$$r = \frac{\alpha A \left(\frac{K}{nH}\right)^{\alpha-1}}{\mu} + 1 - \delta_K, \quad (5)$$

$$r = \frac{(1 - \alpha)A \left(\frac{K}{nH}\right)^\alpha}{i\mu} + 1 - \delta_H, \quad (6)$$

hence inflation has a growth effect if inflation affects the real rate of interest through a change in the nominal rate of interest and/or average markup. We refer to these effects as *nominal interest rate effect* and *markup effect*, respectively. Substituting the Fisher equation, $i = r\pi$, into (6), it holds that:

$$r = \frac{1}{2} \left(1 - \delta_H + \sqrt{(1 - \delta_H)^2 + \frac{4}{\pi\mu} (1 - \alpha)A \left(\frac{K}{nH}\right)^\alpha} \right). \quad (7)$$

Given π , equations (5) and (7) determine the real rate of interest. We can see from Figure 1 that the marginal nominal interest rate effect is necessarily negative and from Figure 2 that a rise of average markup brings a fall of the growth rate of output.

3.2 Sticky-Price Economy with Exogenous Contract Duration

How inflation affects average markup depends on the firms' price setting behavior. First we consider the sticky-price economy with exogenous contract duration, in which each firm can reset its price with the probability $1 - \xi$ in which ξ is constant. The existence of nominal rigidity causes an inflationary effect on average markup. For a given π , the economy-wide average markup μ is determined by:

$$\mu^{1-\theta} = \xi \left(\frac{\mu}{\pi}\right)^{1-\theta} + (1 - \xi)\tilde{\mu}^{1-\theta}, \quad (\text{Price level equation}) \quad (8)$$

$$\tilde{\mu} = \frac{\theta}{\theta - 1} \frac{1 - \beta\xi\pi^{\theta-1}}{1 - \beta\xi\pi^\theta}, \quad (\text{Optimal pricing behavior}) \quad (9)$$

where $\tilde{\mu} \equiv \frac{\bar{P}}{P}\mu$ denotes the optimal markup set by firms that can reset their prices. We can show that, if β is sufficiently near 1, the relationship between inflation and average markup is U-shaped and there is an inflation rate $\pi^* \in (1, \min\{\frac{1}{\beta}, \xi^{-\frac{1}{\theta}}\})$ such that π^* attains the minimum average markup. Figure 3 shows the numerical result of the inflation-growth relationship for various values of ξ .⁶ When $\xi = 0$ (price is flexible), the inflation-growth relationship is decreasing and convex in π . As ξ becomes larger, the inverted U-shaped relationship becomes stronger. When ξ is sufficiently high, there is a threshold inflation rate below which the marginal growth effect is positive, because markup effect dominates the nominal interest rate effect. However, this relationship is inconsistent with empirical evidence. First, it is concave rather than convex at high inflation rates. Moreover, this model can analyze the growth effect only at moderate inflation, because it has an equilibrium only if $\xi\pi^\theta < 1$ and $\xi\pi^{\theta-1} < 1$, where ξ is constant.

3.3 Sticky-Price Economy with Endogenous Contract Duration

Next we consider the Calvo model with endogenous contract duration as in Levin and Yun (2007). For simplicity, we assume that the economy is on a balanced-growth path. In each period, firm j can reset the nominal price of its variety with probability $1 - \xi(j)$, where firms can choose their own $\xi(j)$; however, they must pay a fixed menu cost $\Omega_t \equiv \omega Y_t$ when they can change their prices. Restricting our analysis to a symmetric Nash equilibrium,⁷ firms change their price more frequently as inflation deviates from zero, as shown in Panel B of Figure 4. The reason is as follows. Firms face a tradeoff between more frequent price resetting and less frequent fixed menu cost payments. If inflation is near zero, the loss of profit by not changing their prices is small; hence, firms choose a high ξ to avoid paying menu costs. As inflation deviates from zero, the loss of profit by not changing their prices becomes larger; hence firms choose a higher frequency of price change even if they must pay menu costs more frequently.

Varying ξ makes the markup effect more complex. In addition to the U-shaped markup effect shown in the previous subsection, there is the effect that this U-shaped relationship becomes flatter as inflation deviates from zero. Panel A of Figure 4 indicates the growth effect of inflation. There are two threshold inflation rates, which are about 0.1%, at which the marginal growth effect changes from positive to negative, and about minus 0.1%, at which the marginal growth effect changes from negative to positive. The reason is that when inflation is near zero and below π^* , inflation has a strong positive growth effect. This is because the inflation-markup relationship is then strongly U-shaped; thus the average markup falls quickly. As inflation moves farther from zero, price becomes more

⁶The values of the structural parameters are shown in Table 1. Note that in the model the time unit is quarterly but in the Figures the time unit is annual. The Matlab programs for our numerical analysis, partly using the Compecon Toolbox developed by Miranda and Fackler (2002), are on the author's website (<http://sites.google.com/site/hirokiarato/>).

⁷That is, $\xi(j) = \xi$ for all j .

flexible, so that the markup effect becomes weaker. When inflation is sufficiently far from zero, the markup effect is dominated by the nominal interest rate effect, hence the total marginal growth effect is negative.

The inflation-growth relationship shown in Panel A of Figure 4 is consistent with the empirical evidence. First, there is a threshold inflation rate below which the marginal growth effect changes from positive to negative. Second, the threshold inflation rate is about 0.1% a year, which is in the range of the threshold rate found by empirical studies, particularly in industrial countries. In the empirical study in Khan and Senhadji (2001), the annual threshold inflation rate is below 1% in industrial countries and about 11% in developed countries for five-year averaged data. Third, above the threshold inflation rate, the relationship between inflation and growth is decreasing and convex, unlike the exogenous contract duration model. This is because the markup effect is weaker when inflation is high and the situation approaches the flexible-price economy in which only the nominal interest rate effect affects growth. Moreover, the model can analyze the growth effect at high inflation, unlike the exogenous contract duration model.⁸ The restrictions that $\xi\pi^\theta < 1$ and $\xi\pi^{\theta-1} < 1$ are not violated even at high inflation because then ξ is small.

4 Concluding Remarks

In this paper we show that the monetary endogenous growth model with Calvo-type nominal rigidity with endogenous contract duration can generate the plausible relationship between inflation and growth, particularly in industrial countries. However, there are some open questions in our analysis. First, our model suggests the existence of a lower alternative threshold inflation rate, below which the marginal growth effect becomes negative. Empirical studies have no evidence of the threshold inflation rate because we have few observations of deflation episodes. If we had more observations of deflation, we could test the existence of the alternative threshold inflation rate by dividing the low-inflation observations into two subsamples. Second, our model cannot replicate the plausible threshold inflation rate in developing countries, which is shown to be 11% a year for five-year averaged data in Khan and Senhadji (2001). This result suggests that the analysis for developing countries might need some alternative assumptions of, for example, imperfect information in credit markets as in Bose (2002) and in Hung (2008). However, the measurement of the degree of imperfect information is difficult. In order to analyze the growth effect of inflation in developing countries quantitatively, we must obtain more empirical evidence about market structure and imperfect information.

⁸Figure 4 illustrates only around zero inflation. For the numerical results at very high inflation, see Arato (2008).

References

- Arato, Hiroki (2008) “Long-run relationship between inflation and growth in a New Keynesian framework.” MPRA Paper No.15205, University Library of Munich.
- Bose, Niloy (2002) “Inflation, the Credit Market, and Economic Growth,” *Oxford Economic Papers* **54**, pp. 412–434.
- Funk, Peter and Bettina Kromen (2006) “Short-term Price Rigidity in an Endogenous Growth Model: Non-superneutrality and a Non-vertical Long-term Phillips-curve.” Working Paper Series in Economics, No.29. University of Cologne.
- Gillman, Max and Michal Kejak (2005) “Contrasting Models of The Effect of Inflation on Growth,” *Journal of Economic Surveys* **19**(1) , pp. 113–136.
- Hung, Fu-Sheng (2008) “Non-productive consumption loans and threshold effects in the inflation-growth relationship,” *Oxford Economic Papers* **60**, pp. 318–342.
- Khan, Mohsin S. and Abdelhak S. Senhadji (2001) “Threshold Effects in the Relationship between Inflation and Growth,” *IMF Staff Papers* **48**(1) , pp. 1–21.
- Kuwahara, Shiro and Nao Sudo (2007) “Inflation and Growth Revisited.” mimeo.
- Levin, Andrew and Tuck Yun (2007) “Reconsidering the natural rate hypothesis in a New Keynesian framework,” *Journal of Monetary Economics* **54**(5) , pp. 1344–1365.
- Miranda, Mario J. and Paul L. Fackler (2002) *Applied Computational Economics and Finance*, Cambridge, MA: MIT Press.

Table 1: Structural parameters

A	α	β	δ_H	δ_K	ω	ψ	θ
0.0445	0.36	$1.0045/1.03^{1/4}$	0.005	0.025	0.029	807.4	4.33

Figure 1: Nominal interest rate effect (π increases and μ is fixed)

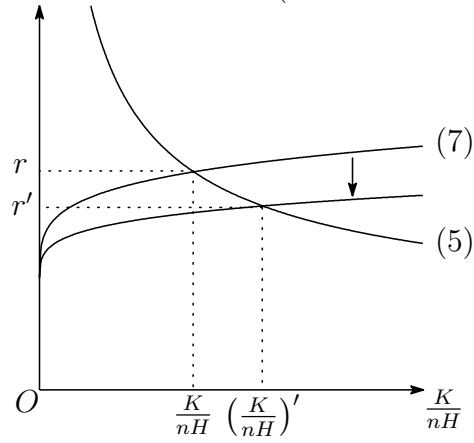


Figure 2: Markup effect (μ increases and i is fixed)

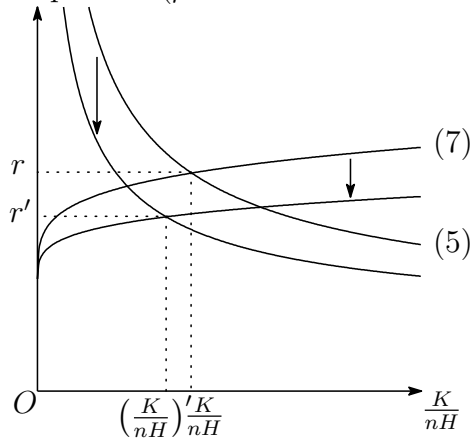
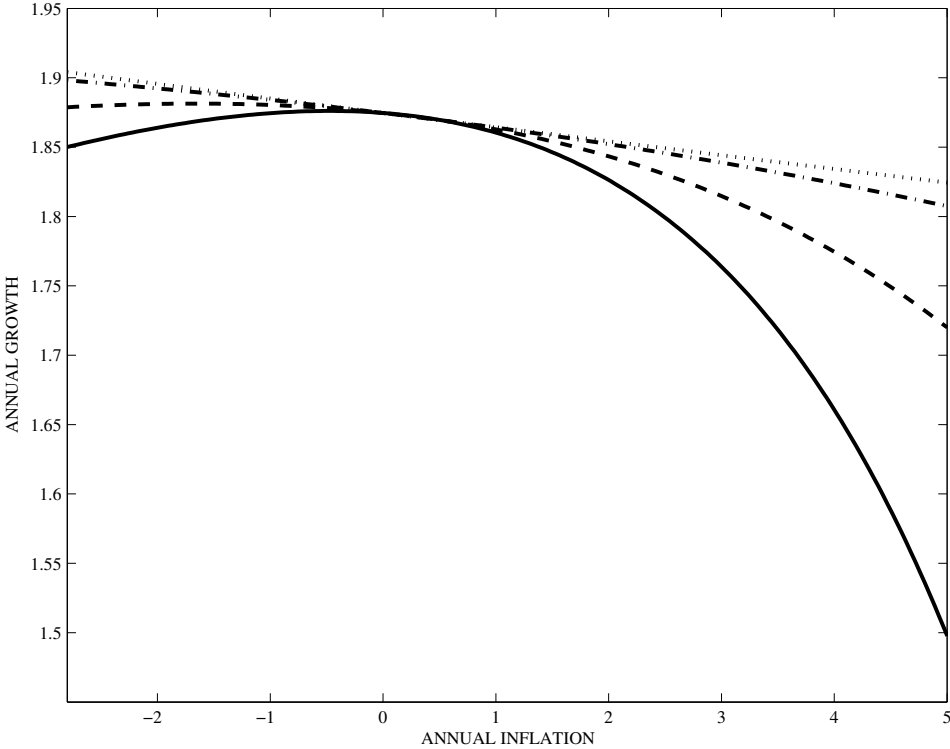


Figure 3: Effects of inflation in the exogenous contract duration model



Note: Solid line when $\xi = 0.9$, broken line when $\xi = 0.85$, dash-dotted line when $\xi = 0.7$, dotted line when $\xi = 0$ (flexible-price economy).

Figure 4: Effects of inflation in the endogenous contract duration model (around zero inflation)

