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Mixed Oligopoly with Distortions: First Best with Budget-balance and the Irrelevance Principle

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Abstract

This paper extends the literature on mixed oligopoly in two directions. First, it introduces distortions in the working of the public firm, an issue that is of some concern, especially in transitional economies. Thus the classical model of mixed oligopoly emerges as a special case of our formulation. Second, we examine the implementation of the first best outcome in the presence of budget balancing. We show that as long as such distortions are not too severe, these do not prevent the implementation of the first best outcome with budget balancing, with the first best policy involving a tax on the public firm, coupled with subsidies to the private firms. Further, in the absence of any distor tions, implementing the first best with budget balancing necessarily involves complete socialization. This shows the importance of budget balancing to the applicability of the irrelevance principle.

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1. Introduction

In India, a decision to raise tram fares by one paisa by Calcutta Tramways sparked off violent protests in the 1960s. Very recently, a decision to raise petrol prices so as to cut down on losses by public sector oil companies created a furore in the Indian parliament.[1] Equally recently, Indian railways reversed its decision to discontinue certain trains when faced with violent protests in the affected region. Public sector banks are often not allowed to close down loss making loan schemes on the grounds that doing so may adversely affect the common man. Such distortions may also arise internally, if such ideals are entrenched in the mindsets of the bureaucrats, as well as in the standard operating procedures in such public firms. All of these goes to make the point that public firms are often subject to diverse pressures and distortions in their operations that seek to make these more "consumer friendly".

While our motivating examples are from transitional economies, many of which have had (and in some cases continue to have), political parties and electoral groups with socialist sympathies,[2] such pulls and pressures may exist in other economies as well. This is because the very existence of public firms can perhaps be traced to a perception that protection of consumer interests in certain industries may require intervention.

This paper adopts an approach that can accommodate both these kinds of distortions, political as well as bureaucratic, building in such distortions in the utility function of the public firm itself. Thus formally the utility function of the public firm is taken to be a weighted average of share-holder utilities and consumers' surplus. This provides a generalization of the classical mixed oligopoly model which emerges as a special case when the weight on consumers' surplus is zero. The focus is on implementing the first best outcome in such a setup in the presence of budget-balancing.

We consider a mixed oligopoly with one public, and n private firms. We begin by analyzing the case where the degree of privatization is exogenously given, and then use this as a building block to examine the case where the extent of privatization is endogenously determined.

[1] In fact, the government has agreed to re-visit this decision in case international oil prices continue to remain low.

[2] Even though some of these economies are moving away from their socialist past, sympathies for socialist ideals remain. In India, for example, privatization of public sector units is facing rough weather. One of the main reasons for such opposition is that doing so may hurt the consumers.

For any exogenously given level of privatization, we demonstrate that a pure strategy equilibrium exists and moreover any equilibrium involves the public firm producing more compared to the private firms. Further, as the public firm becomes more socialistic, in any interior and stable equilibrium, the output of the public firm increases, and that of the private firms decrease. We also briefly discuss the issue of uniqueness.

The welfare comparison between mixed oligopoly and Cournot competition however turns out to be ambiguous. Interestingly, the mixed oligopoly outcome welfare dominates complete privatization whenever the cost functions are not too convex.

We then turn to the question of implementing the first best when the government can decide on the extent of socialization of the public firm, say μ, and the tax/subsidies on all firms, private as well as public. We demonstrate that, as long as the distortions are not too extreme, the first best outcome can always be implemented using a policy that involves providing subsidies to the private firms and a tax on the public firm. Further, this policy turns out to be budget balancing. Interestingly, in the absence of any distortions, implementing the first best with budget balance necessarily involves complete socialization.

We then briefly relate our work to the literature. The early work of Merrill and Schneider (1966) has been followed by, among others, Cremer *et al.* (1989, 1991), De Fraja and Delbono (1989), White (1996), Anderson *et al.* (1997), Poyago-Theotoky (2001), Myles (2002), Fjell and Heywood (2002, 2004) and Kato and Tomaru (2007). One of the central issues in the literature concerns the effect of privatization on welfare. While Cremer at. al. (1989, 1990) examine the role of public firms as regulatory agents, De Fraja and Delbono (1989) finds that under certain conditions privatization of the public firm may be welfare improving. Anderson *et al.* (1997) demonstrate the importance of entry decisions and the consumers' love for variety in making this comparison.

Further, while many of these papers deal with domestic firms, Fjell and Pal (1996), Fjell and Heywood (2002), Matsumura (2003) and Matsushima and Matsumura (2006), among others, examine mixed oligopoly with foreign firms.[3] In the present paper though we restrict attention to domestic firms. We refer the readers to De Fraja and Delbono (1990) for a succinct survey of the early literature.

[3] Pal and White (1998) examine strategic trade policy in such a mixed oligopoly context.

Turning to the literature on mixed oligopoly under partial privatization, Matsumura (1998) and Matsumura and Kanda (2003) demonstrate that partial privatization is optimal in the short-run, while full nationalisation would be optimal in the long-run *if* there is also free entry. Fujiwara (2007) extends the analysis to the case where there is product differentiation. Finally, Chao and Yu (2006) examines partial privatization in the context of optimal trade policies. None of these papers, however, examine the issue of first best implementation with budget balancing. Our analysis shows that when the public firm can also choose tax/subsidy policies, the first best can be implemented even without free entry and with budget balance.

Our analysis also contributes to another branch of the literature, the so called *irrelevance principle*. In the presence of welfare maximizing public firms, the irrelevance principle states that the *same* non-discriminating subsidy implements the first best *irrespective of whether there is privatization or not* (see e.g. White (1996), Poyago-Theotoky (2001), Myles (2002), Fjell and Heywood (2004) and Kato and Tomaru (2007)). In fact, Tomaru (2006) demonstrates that the irrelevance principle holds even under partial privatization. For the case where there are no distortions, we find that the first best can be implemented with budget balancing if and only if the public firm is welfare maximizing. Thus whether the irrelevance principle holds or not depends on whether one insists on budget balance or not.

The paper is organized as follows. Section 2 sets up the basic model, while the case with exogenously given μ is analyzed in Section 3. Section 4 solves for the problem of implementing the first best, as well as relates our results to the irrelevance principle. Finally, Section 5 concludes.

2. The Model

There are n profit-maximizing private firms and one public firm, all producing a homogeneous good. The output of the i-th private firm is denoted q_i and that of the public firm is q_0, so that aggregate output $Q = q_0 + \sum_1^n q_i$. The inverse demand function is $f(Q)$. Firms are symmetric with the cost function of all firms, including the public firm, being $c(q)$.

Assumption 1. (i) The inverse demand $f : (0, \infty) \to [0, \infty)$ and $\exists \hat{Q}$, $0 < \hat{Q} < \infty$, such that $f(Q) > 0$ if $0 \leq Q < \hat{Q}$, and $f(Q) = 0$ if $Q \geq \hat{Q}$. Further, $f(Q)$ is twice differentiable, decreasing and (weakly) concave, i.e. $f'(Q) < 0$ and $f''(Q) \leq 0$, for all Q such that $\hat{Q} > Q > 0$.

(ii) The cost function $c : [0, \infty) \to [0, \infty)$ is twice differentiable, increasing and convex, i.e. $c'(q_i) > 0$ and $c''(q_i) > 0$, $\forall q_i \geq 0$. Also, $c(0) = c'(0) = 0$.

The assumption that $f''(Q) \leq 0$ implies that the second order conditions for the private firms are satisfied. Assumption 2 below ensures that the corresponding condition holds for the public firm.

Assumption 2. $f'(Q) - (Q - q)f''(Q) - c''(q) < 0$, $\forall Q \geq q > 0$.

One sufficient condition (given that $f''(Q) \leq 0$), for A2 to hold is that the marginal demand function, $f'(Q)$, is elastic, i.e. $\frac{f'(Q)/Q}{f''(Q)} \geq 1$. A2 also holds if the demand function is linear.

The profit function of the i-th firm is given by

$$\pi_i = q_i f(Q) - c(q_i),\ i = 0, 1, \cdots, n. \tag{1}$$

Note that aggregate welfare

$$W = \int_0^Q f(q)dq - \sum_0^n c(q_i), \tag{2}$$

and consumers' surplus

$$CS = \int_0^Q f(q)dq - Qf(Q). \tag{3}$$

We next turn to the task of formulating the utility function of the public firm where the government holds a share of μ in the public firm, with $1 - \mu$ shares being held privately. Thus, with a welfare maximizing government, the average utility of the two groups of shareholders, the government and the private agents, is

$$\mu W + (1 - \mu)\pi_0. \tag{4}$$

Following our earlier discussion we then add another layer of complexity to allow for bureaucratic and political distortions in the workings of public firms. This is formalized by a utility function, P, that is a weighted average of share-holder utilities (as expressed in (4)) and consumers' surplus

$$P = [\mu W + (1 - \mu)\pi_0] + D(\mu)CS, \tag{5}$$

where $D(\mu)$, denotes the weight on consumers' surplus. $D(\mu)$ satisfies the following reasonable restrictions.

Assumption 3.
(i) $D : [0, 1] \to [0, 1]$ is once differentiable with $D'(\mu) \leq 0$.
(ii) $D(\mu) \leq 1 - \mu$ and $D'(\mu) \geq -1$.

A3(i) captures the idea that as the extent of privatization increases, this may unleash countervailing forces seeking to correct for an anticipated loss of consumers' surplus. In order to concentrate on the case of interest, we assume that such distortions cannot be too large. This is formalized in A3(ii). Note that the case where there is no distortion, i.e. $D(\mu) = 0 \ \forall \mu$, emerges as a special case.

This formulation of P, placing an weight on consumers' surplus, has some similarities to Matsumura (1998), who also allows for an additional weight of β (possibly zero), on consumers' surplus. The difference with this paper is twofold. First, in Matsumura (1998), this additional weight enters the government's objective function, whereas in the present paper it appears as a distortion of the shareholders' objective. Second, unlike in Matsumura (1998) this paper allows for the possibility that weight on consumers' surplus could depend on the extent of privatization.

We consider a mixed oligopoly game where the firms simultaneously decide on their output levels. In the next section we solve for the pure strategy Nash equilibrium of the game where μ is exogenously given. In Section 4, we solve for the game where the government can select μ so as to maximize welfare.

3. The Mixed Oligopoly Game

For expositional purposes, we adopt a specific functional form for $D(\mu)$, namely $1 - \mu$ in this section. Note that this satisfies A3. Further we shall see in Section 4 that this functional form has an interesting interpretation. With this simplification, the utility of the public firm is

$$P = q_0 f(Q) - c(q_0) + \int_0^Q f(z)dz - f(Q)Q + \mu \sum_{i=1}^{n} \pi_i, \tag{6}$$

where $\mu \leq 1$. It is thus a weighted sum of its own profits and consumers surplus, and the aggregate profit of the private firms.

We first define the first best outcome, i.e. the output vector that maximizes social welfare. Clearly, it involves every firm producing q^*, where q^* solves

$$f((n+1)q^*) = c'(q^*). \tag{7}$$

At this output vector, price equals marginal cost for every firm. Given A1, q^* is well defined. Let $Q^* = (n+1)q^*$.

Before proceeding further let us solve for the benchmark case of Cournot competition with private firms. This is of particular interest since Cournot competition can be interpreted as arising out of privatizing the public firm.

Cournot competition: There are $n+1$ profit-maximizing firms who compete over quantity so that the first order conditions (FOCs) involve

$$f(Q) + q_i f'(Q) = c'(q_i). \tag{8}$$

It is standard to show that there is a unique Cournot equilibrium[4] where all firms produce an output level of q_C (> 0) where q_C is the unique solution to $f((n+1)q) + qf'((n+1)q) = c'(q)$. Let $Q_c = (n+1)q_c$. Clearly, $Q_c < Q^*$.

Mixed oligopoly: Let the equilibrium output vector be denoted by $(q'_0, q'_1, \cdots, q'_n)$, with the aggregate output being denoted by Q'.

First note that in any equilibrium all private firms have the same output, q'.[5] Further, in equilibrium $q' > 0$.[6] Consequently, the first order condition of the private firms is given by

$$f(q_0 + nq) + qf'(q_0 + nq) = c'(q). \tag{9}$$

Let $q_0 = g(q)$, where $g(q)$ solves (9) when a solution exists,[7] otherwise let $g(q) = 0$. From A1, $g(q)$ is decreasing in q. Further, $g(0) = \hat{Q} > 0$ (since $c'(0) = 0$) and there exists q'' which is the smallest q such that $g(q) = 0$.[8]

[4]See, e.g. Kolstad and Mathiesen (1987).

[5]Suppose to the contrary, $q'_i > q'_j \geq 0$ for some $i \neq j$. Since $q'_i > 0$, $q'_i f'(Q') + f(Q') - c'(q'_i) = 0$. Hence $q'_j f'(Q') + f(Q') - c'(q'_j) > q'_i f'(Q') + f(Q') - c'(q'_i) = 0$, where the first inequality follows from the fact that $q'_i > q'_j$, $f'(Q) < 0$ and $c''(q) > 0$. Thus the j-th firm has an incentive to increase its output, contradiction. Hence $q'_i = q'$ for all $i \geq 1$.

[6]Suppose to the contrary $q' = 0$, so that $Q' = q'_0$. Then, from the public firm's first order condition, the equilibrium q'_0 solves $f(q'_0) = c'(q'_0) > 0$. Since $c'(0) = 0$, for q_i small, $q_i f'(q'_0) + f(q'_0) - c'(q_i) > 0$, so that the i-th private firm has an incentive to increase its output.

[7]From A1, if (9) has a solution for q_0, it is unique.

[8]Such a q'' exists since, for $q = \hat{Q}/n$, the LHS of (9) is less than the RHS, $\forall q_0$.

Next consider the first order condition of the public firm:

$$f(Q) + q_0 f'(Q) - c'(q_0) - Qf'(Q) + \mu \sum_i q_i f'(Q) = 0. \tag{10}$$

Using symmetry, the above condition simplifies to

$$f(q_0 + nq) - c'(q_0) - nqf'(q_0 + nq)(1 - \mu) = 0. \tag{11}$$

From (11) one can write $q_0 = h(q)$. From A1, $h(q)$ is well defined for all q. Further, $h(0)$ solves $f(q_0) = c'(q_0)$, so that $h(0) < g(0)$.

First note that since $h(q)$ is well defined, $\forall q$, $g(q)$ and $h(q)$ necessarily intersects so that an equilibrium exists. Further, if $h(q'') > 0$, then an interior equilibrium exists. Next using the first order conditions of the private firms, (11) can be written as:[9]

$$c'(q) - c'(q_0) - qf'(Q)[1 + n(1 - \mu)] = 0. \tag{12}$$

From (12), in any equilibrium $q_0 > q$. Finally, note that as the public firm becomes more socialistic, i.e. μ decreases, $h(q)$ shifts upwards. Thus, in any *stable* equilibrium the output of the public firm $q_0(\mu)$ increases, and that of the private firms $q(\mu)$ decreases.

Summarizing the above discussion we have

Proposition 1 *Suppose A1-A2 hold and $D(\mu) = 1 - \mu$.*
(i) An equilibrium exists. Further, an interior equilibrium exists whenever $h(q'') > 0$.
(ii) In any equilibrium, all private firms produce the same output, say q'. Further, the output level of the public firm exceeds that of the private firms, i.e. q'.
(iii) As the public firm becomes more socialistic, i.e. μ decreases, in any stable interior equilibrium, the output level of the public firm increases, and that of the private firms decrease.[10]

Note that Proposition 1 says nothing about uniqueness. In what follows we impose the following assumption (or an appropriately modifed version thereof), which ensures that a unique interior equilibrium exists.

[9]Note that for q_0 small, the LHS of (10) is strictly positive. Thus any equilibrium must involve a positive output for the public firm.

[10]It is easy to check that Proposition 1 goes through for any $D(\mu)$ satisfying A3.

Assumption 4.
(i) For any $\tilde{q}$ satisfying $g(q) = h(q)$, $h'(\tilde{q}) > g'(\tilde{q})$.
(ii) $h(q'') > 0$, where q'' is the smallest q such that $g(q) = 0$.[11]

Remark 1. We then argue that for linear demand and quadratic costs the equilibrium is necessarily unique. Suppose that there is one public firm (firm 0) and one private firm (firm 1). We assume that the demand function is $a - q_0 - q_1$ and the cost function is $cq_i^2/2, i = 0, 1$. Under mixed oligopoly, it is easy to check that $h(q)$ and $g(q)$ are both negatively sloped, and there is an interior equilibrium if and only if $2+c > \mu$. Solving we obtain $q_0' = \frac{a(2+c-\mu)}{(1+c)(2+c)-\mu}$ and $q' = \frac{ac}{(1+c)(2+c)-\mu}$. It is straightforward to generalize the argument to the case with $n \geq 2$ private firms.

3.1. Welfare Analysis

In this sub-section we compare the welfare level under mixed oligopoly with that under Cournot competition.

To begin with note that while the first best outcome is symmetric, the mixed oligopoly one is not. This immediately implies that the mixed oligopoly outcome is sub-optimal.

As to comparing welfare under mixed oligopoly with that under Cournot competition, there are two opposing effects at work here. First, because of the presence of the public firm, aggregate output, and consequently consumers' surplus, tends to be larger under a mixed oligopoly. On the other hand, there is an efficiency loss arising out of the fact that the output levels are asymmetric, so that aggregate profit under mixed oligopoly is likely to be less than that under Cournot competition. Thus, depending on parameter values, either effect may dominate, hence the ambiguity.

We start by comparing the consumers' surplus (i.e. the aggregate output) under the two regimes. Clearly, the Cournot equilibrium is defined by the intersection of $g(q)$ with the 45 degree line. Next recall that the mixed oligopoly involves $q_0' > q'$, so that it lies above the 45 degree line. Hence, $q_0' > q_C > q'$. Next, totally differentiating (9)

$$\frac{dQ}{dq} = \frac{dq_0}{dq} + n = \frac{c''(q) - f'(Q)}{qf''(Q) + f'(Q)} < 0. \tag{13}$$

[11]Uniqueness follows from the continuity of $g(q)$ and $h(q)$, and the fact that in case of multiple intersections, $h'(q) \leq g'(q)$ at one of these intersections.

Since $q' < q_C$, from (13) it follows that the aggregate output level is higher under a mixed oligopoly. This immediately implies that the consumers' surplus is higher compared to that under Cournot competition.

Proposition 2 *Suppose A1, A2 and A4 hold and $D(\mu) = 1 - \mu$. Aggregate output, and consequently consumers' surplus, under mixed oligopoly exceeds that under Cournot competition. Moreover, if the marginal cost is (weakly) convex, then aggregate output under mixed oligopoly is less than that under the first best.*

Proof. Consider the second part of the proposition. Multiplying (9) by n and adding (10), we have that

$$(n+1)f(Q') - nc'(q') - c'(q_0') + \mu \sum_i q_i f'(Q) = 0. \tag{14}$$

If $c'(q)$ is (weakly) convex, then $nc'(q') + c'(q_0') \geq (n+1)c'(\frac{Q'}{n+1})$. Hence, from (14),

$$(n+1)f(Q') - (n+1)c'(\frac{Q'}{n+1}) > 0. \tag{15}$$

Next recall that the first best solves

$$f(Q^*) - c'(\frac{Q^*}{n+1}) = 0. \tag{16}$$

Given that $f(Q) - c'(\frac{Q}{n+1})$ is decreasing in Q, from (15) and (16), $Q' < Q^*$.■

Remark 2. In case the public firm is fully privatized, it is straightforward to show that a converse result holds: If $\mu = 0$ and the marginal cost is concave, then aggregate output under mixed oligopoly is *greater* than that under the first best.

We then compare the welfare level under mixed oligopoly with that under Cournot competition. For sufficiently convex cost functions, it is well known from De Fraja and Delbono (1989) that the efficiency loss effect is large and dominates the output effect, so that welfare levels under a mixed oligopoly tends to be lower than that under privatization. We next provide an example to show that the converse is also true, in that if the cost functions are not too convex then welfare under a mixed oligopoly may be higher than that under complete privatization.

Remark 3. Consider the example discussed in Remark 1 earlier with one public and one private firm, demand function $a - q_0 - q_1$ and cost function $cq_i^2/2, i = 0, 1$. Recall that nder a mixed oligopoly $q_0' = \frac{a(2+c-\mu)}{(1+c)(2+c)-\mu}$ and $q' = \frac{ac}{(1+c)(2+c)-\mu}$. It is straightforward to check that W', the equilibrium welfare level under mixed oligopoly, goes to $a^2/2$ as c goes to zero. Whereas under Cournot oligopoly, we obtain $q_i^C = \frac{a}{3+c}$ and $W^C = \frac{(4+c)a^2}{(3+c)^2}$, where q_i^C is the equilibrium output of firm i and W^C is the equilibrium welfare level. As c goes to zero, W^C goes to $4a^2/9 < a^2/2$. For c small, the cost functions are not too convex, hence the efficiency loss arising out of asymmetric output is not too large, so that the consumers' surplus effect dominates.

4. Implementing the First Best with Budget-balancing

In examining the issue of first best implementation with budget-balancing, we shall argue that the exact form of $D(\mu)$ plays a crucial role. We thus revert to a general $D(\mu)$ satisfying A3.

The government maximizes aggregate welfare, i.e. the sum of consumers' surplus and profits, using a policy-tuple $< \mu, s_0, s >$, where $\mu \in [0, 1]$ denotes the extent of socialization, s_0 denotes the tax/subsidy on the public firm and s denotes the tax/subsidy on the private firms. Note that we restrict attention to simple linear schedules. For one, this simplifies implementation of such policies. Further, this makes tax/subsidy policies transparent, reducing the scope of bureaucratic corruption and harassment.

We consider a two stage model.

Stage 1. The government selects some mechanism $< \mu, s_0, s >$, so as to maximize welfare.

Stage 2. The firms play a simultaneous move quantity-setting game where every firm maximizes its own objective, with the firm objectives clearly depending on the selected mechanism.

From (5), the utility function of the public firm is

$$\mu W + (1-\mu)[q_0 f(Q) - c(q_0) + t.q_0] + D(\mu)[\int_0^Q f(q)dq - Qf(Q)]. \quad (17)$$

Consequently, the FOC for the public firm can be written as follows:

$$\begin{aligned}(\mu + D(\mu))\frac{\partial W}{\partial q_0} \quad &+ \quad (1-\mu-D(\mu))[q_0 f'(Q) + f(Q) - c'(q_0)] \\ &- \quad D(\mu)\sum_1^n q_i f'(Q) + (1-\mu)s_0 = 0. \qquad (18)\end{aligned}$$

We then characterizes conditions under which the first best may be implementable with budget balancing.

Proposition 3 *Let A1-A3 and an appropriately modified version of A4 hold. For any μ, the first best outcome with budget-balancing is implementable if and only if $D(\mu) = 1-\mu$. Further, the first best policy involves a per unit **tax** on the public firm, so that $s_0 = nq^* f'(Q^*)$, and a per unit **subsidy** of $s = -q^* f'(Q^*)$ to the private firms.*

Proof. Fix $\mu = \mu'$.

Sufficiency. Let $D(\mu') = 1-\mu'$ and further let the tax/subsidy policy be as prescribed. To show that given that all other firms are producing q^*, it is optimal for every firm to do so. The FOC of the i-th private firm involves

$$f(q_i + nq^*) + q_i f'(q_i + nq^*) - c'(q_i) + s = 0. \qquad (19)$$

Clearly, given that $s = -q^* f'(Q^*)$ and (7), $q_i = q^*$ solves the above equation.

Next consider the public firm. Consider (18). Given that all other firms are producing at q^*, $q_0 = q^*$ ensures that $\frac{\partial W}{\partial q_0} = 0$. Thus the first order condition is satisfied if and only if

$$\begin{aligned}& (1-\mu'-D(\mu'))[q^* f'(Q^*) + f(Q^*) - c'(q^*)] - D(\mu')nq^* f'(Q^*) + (1-\mu)s_0 \\ = \; & (1-\mu'-D(\mu'))[Q^* f'(Q^*) + f(Q^*) - c'(q^*)] \\ = \; & (1-\mu'-D(\mu'))Q^* f'(Q^*) = 0. \qquad (20)\end{aligned}$$

Given $D(\mu') = 1-\mu'$, this is indeed true.

Necessity. The FOC for the private firm imply that implementing the first best must involve, $s = -q^* f'(Q^*)$. Consequently, budget balancing must involve $s_0 = nq^* f'(Q^*)$. Thus the FOC for the public firm implies that (20) must hold. Given that $Q^* f'(Q^*) < 0$, this implies that $D(\mu') = 1-\mu'$.■

Given Proposition 3, it is easy to see that the first best can necessarily be implemented. This follows since from the continuity of $D(\mu)$, there is some

μ' such that $D(\mu') = 1 - \mu'$. This of course is subject to the qualification that these distortions are not too large, as formalized in A3.

Summarizing we have

Corollary 1. *A mechanism implementing the first best with budget balance exists.*

We then use Proposition 3 to analyze some special cases of interest.

First consider the case where the distortions arise out of political pressures. Assuming that political parties respond to high visibility indicators like the extent of privatization, such political opposition is likely to be small whenever the public firm is completely socialized. This can be approximated as $D(1) = 0$, so that the distortions vanish for $\mu = 1$. In that case the optimal policy involves complete socialization, i.e. $\mu = 1$. It is interesting that with a welfare maximizing public firm the first best can be implemented with budget balancing, especially in view of the fact that the literature emphasizes that in this case a uniform subsidy implements the first best.

Next consider the case where there is no distortion so that $D(\mu) = 0 \ \forall \mu$, which is the case that the literature has largely focused on. From Proposition 3, the optimal policy involves $\mu = 1$, i.e. a welfare maximizing public firm. This shows that if one insists on budget balancing, then implementing the first best, even in the absence of any distortions, involves complete socialization. To reiterate, the first best is *not* implementable under privatization, either complete or partial, whenever one insists on budget balancing. This demonstrates the importance of budget-balancing to the applicability of the irrelevance principle.

Summarizing the preceding discussion we have the following corollary.

Corollary 2.

(i) Even in the presence of distortions, there exists an optimal level of socialization μ^*, satisfying $D(\mu^*) = 1 - \mu^*$, such that the first best can be implemented with budget balance.

(ii) With a welfare maximizing public firm, the first best can be implemented with budget balance whenever $D(1) = 0$, which is likely if, for example, the distortions are political in nature.

(iii) In the absence of any distortions, i.e. $D(\mu) = 0 \ \forall \mu$, implementing the first best with budget balance necessarily involves complete socialization.

5. Conclusion

This paper extends the literature on mixed oligopoly in two directions. First, it introduces distortions in the working of the public firm, an issue that is of some concern, especially in transitional economies. This constitutes a generalization of the classical model of mixed oligopoly, with the classical model emerging as a special case of our formulation. Second, we examine the implementation of the first best outcome in the presence of budget balancing, an issue that his been relatively unexplored in the literature. We show that as long as such distortions are not too severe, these do not prevent the implementation of the first best outcome with budget balancing, with the first best policy involving a tax on the public firm, coupled with subsidies to the private firms. Further, in the absence of any distortions, implementing the first best with budget balancing necessarily involves complete socialization. This shows the importance of budget balancing to the applicability of the irrelevance principle.

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