Abstract

We study customization in the Hotelling model with two firms. In addition to providing ideal varieties, the perceived uniqueness of a customized product contributes independently to consumer utility. We show that only when consumer preferences for uniqueness are high customization occurs in equilibrium.
1. Introduction

Customization is a flexible technology designed to produce individually tailored products without significantly compromising cost efficiency. For example, Dell builds to order notebook and desktop computers, NikeID.com allows consumers to create their most preferred athletic pair of shoes, and LandEnd.com offers customized pants and shirts.

A number of papers have studied customization theoretically in horizontal differentiation settings, including Dewan, Jing, and Seidmann (2003), Syam and Kumar (2006), Bernhardt, Liu and Serfes (2007), Alexandrov (2008), Mendelson and Parlaktürk (2008), Loginova and Wang (2009), and Xia and Rajagopalan (2009). Customization enables firms to take advantage of consumers’ desires for ideal varieties, but reduces differentiation and intensifies price competition. An important aspect of customization that has not been modeled in this literature is the resulting product uniqueness.

Several marketing studies have explored the extent to which uniqueness plays a role in customization. Lynn and Harris (1997) and Fiore, Lee, and Kunz (2004) find that consumers with high preferences for uniqueness report significantly higher intentions to design their own products. An empirical study by Franke and Schreier (2008) explicitly suggests that the perceived uniqueness of a customized product enhances its value beyond the product “aesthetic and functional fit”.

The goal of the present paper is to incorporate consumer value for product uniqueness into customization competition. We adopt the standard Hotelling model with two firms. The firms first choose whether to customize their products, then engage in price competition. Our main result is that only when consumer preferences for uniqueness are high customization occurs in equilibrium. We also show that customization improves total welfare. The effects of customization on firms and consumers are not definitive.

2. The Basic Model

Two firms, 1 and 2 – indexed by $i$, compete in a market with heterogeneous consumers. Each firm has a standard product located on the Hotelling line of length 1, firm 1 at $x_1 = 0$ and firm 2 at $x_2 = 1$. Investing $K \geq 0$ into customization technology allows a firm to produce products that match exactly each consumer’s most preferred variety on the Hotelling line. For simplicity, we assume that both standard and customized products are produced with zero marginal costs.

Consumer preferences are two-dimensional. Each consumer is represented by a point $(x, y)$ in the unit square $[0, 1] \times [0, 1]$, where $x$ is the consumer’s most preferred variety and $y$ indexes his valuation for product uniqueness. Consumer $(x, y)$ derives utility

$$r - t|x - x_i| - p_i$$

from purchasing a standard product of firm $i$ at price $p_i$. Here, $r$ indicates the common reservation price for a standard product and $t$ measures the marginal disutility from consuming products away from $x$. When a consumer buys a customized product he gets his desired variety, which reduces the second term in (1) to zero. In addition, the consumer obtains a positive utility from product uniqueness. Specifically, consumer $(x, y)$ derives utility

$$r + vy - p_i$$

(2)
from purchasing a customized product, where \( v \) is a positive constant. Thus, consumers with \( y = 0 \) do not value uniqueness, whereas consumers with \( y = 1 \) place the highest value on uniqueness.

Each consumer has a unit demand. We will assume that \( r \) is large enough for all consumers to find a product that yields positive payoff in equilibrium. Consumers are uniformly distributed over the unit square \([0, 1] \times [0, 1]\) with a total mass equal to 1.

The game involves two stages. In the customization stage, the firms simultaneously decide whether to customize their products. These decisions become common knowledge after they are made. In the pricing stage, the firms simultaneously choose prices, consumers decide which products to purchase, and profits are realized. The equilibrium concept employed is subgame perfect Nash equilibrium. The analysis of consumer choices is straightforward. We, therefore, focus on the firms’ choices and proceed using backward induction.

3. Analysis of the Pricing Stage

In this section we investigate the firms’ pricing decisions, taking as given their choices in the customization stage. There are four subgames to consider: both firms choose not to customize (NN), only firm 1 customizes (YN), only firm 2 customizes (NY), and both firms customize (YY).

Subgames NN and YY are straightforward. When no firm customizes, the equilibrium prices and profits are as in the standard Hotelling model. That is, \( p_{1}^{NN} = p_{2}^{NN} = t \) and \( \Pi_{1}^{NN} = \Pi_{2}^{NN} = t/2 \). Horizontal differentiation disappears when both firms customize, leading to the Bertrand outcome. Thus, in subgame YY, \( p_{1}^{YY} = p_{2}^{YY} = 0 \) and \( \Pi_{1}^{YY} = \Pi_{2}^{YY} = 0 \).

Subgames NY and YN lead to symmetric results. Thus, it suffices to study one of them, subgame NY. By (1) and (2), a consumer of type \((x, y)\) purchases from firm 1 if and only if

\[
r - tx - p_{1} \geq r + vy - p_{2}.
\]

Therefore, for a given \( y \), the marginal consumer type in terms of \( x \) is

\[
\hat{x}(y) = \frac{1}{t} (-vy + p_{2} - p_{1}).
\]

It follows that the set of consumers who are indifferent between purchasing from firm 1 and firm 2 corresponds to a straight line in the unit square. The indifference line divides the unit square
into two areas representing firm 1’s and firm 2’s customers. An increase in $v$ (and/or decrease in $t$) makes the line flatter. An increase in $p_2$ (and/or decrease in $p_1$) shifts the line to the right, thereby reducing the market size of firm 2. There are four possible positions for the indifference line, as shown in Figure 1.

Let $D_1(p_1, p_2)$ and $D_2(p_1, p_2)$ denote the demand functions of the firms. The expressions for these functions depend on the position of the indifference line. Because the firms’ marginal production costs are normalized to zero, their profit functions are

$$\Pi_1(p_1, p_2) = D_1(p_1, p_2)p_1 \quad \text{and} \quad \Pi_2(p_1, p_2) = D_2(p_1, p_2)p_2.$$

Firms 1 and 2 choose their prices simultaneously to maximize $\Pi_1(p_1, p_2)$ and $\Pi_2(p_1, p_2)$.

**Proposition 1.** Suppose firm 1 does not customize and firm 2 customizes in the customization stage. Then the equilibrium prices and profits in the pricing stage are as follows.

(i) If $v \leq t/2$,

$$\begin{align*}
p_{1NY}^N &= \frac{1}{3}t - \frac{1}{6}v \\
p_{2NY}^N &= \frac{2}{3}t + \frac{1}{6}v
\end{align*}$$

$$\begin{align*}
\Pi_{1NY}^N &= \frac{1}{7}(\frac{1}{3}t - \frac{1}{6}v)^2 \\
\Pi_{2NY}^N &= \frac{1}{7}(\frac{2}{3}t + \frac{1}{6}v)^2
\end{align*}$$

(ii) If $v \in (t/2, 2t]$,

$$\begin{align*}
p_{1NY}^N &= \frac{1}{4}\sqrt{2tv} \\
p_{2NY}^N &= \frac{3}{4}\sqrt{2tv}
\end{align*}$$

$$\begin{align*}
\Pi_{1NY}^N &= \frac{1}{16}\sqrt{2tv} \\
\Pi_{2NY}^N &= \frac{9}{16}\sqrt{2tv}
\end{align*}$$

(iii) If $v > 2t$,

$$\begin{align*}
p_{1NY}^N &= \frac{1}{3}v - \frac{1}{6}t \\
p_{2NY}^N &= \frac{2}{3}v + \frac{1}{6}t
\end{align*}$$

$$\begin{align*}
\Pi_{1NY}^N &= \frac{1}{v}(\frac{1}{3}v - \frac{1}{6}t)^2 \\
\Pi_{2NY}^N &= \frac{1}{v}(\frac{2}{3}v + \frac{1}{6}t)^2
\end{align*}$$

Proposition 1 deserves a discussion. For this purpose we contrast each of the three cases with subgame NN. Customization by one of the firms makes their products less differentiated in the horizontal dimension. On the other hand, it creates vertical differentiation as consumers value product uniqueness. Both effects are reflected in the equilibrium prices $p_{1NY}^N$ and $p_{2NY}^N$.

Consider case (i), in which $v$ is small relative to $t$. The decrease in horizontal differentiation is more important than the increase in vertical differentiation, resulting in intensified price competition. Algebraically, we have

$$p_{1NY}^N + p_{2NY}^N = t < p_{1NN}^N + p_{2NN}^N = 2t.$$

Note that $v$ affects the price of the non-customizing firm negatively and the price of the customizing firm positively. In equilibrium, both firms serve consumers with all $y$'s, and each firm attracts consumers closer to its position on the variety interval. This case corresponds to Figure 1(a).

Consider next case (iii), in which $v$ is large relative to $t$. Here, the increase in vertical differentiation is more important than the decrease in horizontal differentiation, reducing the intensity of price competition. Specifically,

$$p_{1NY}^N + p_{2NY}^N = v > p_{1NN}^N + p_{2NN}^N = 2t.$$
Note that $t$ affects $p_{NY}^1$ negatively and $p_{NY}^2$ positively. In equilibrium, firm 1 serves consumers of all variety preferences, and so does firm 2. Firm 1 attracts consumers with low $y$’s and firm 2 attracts consumers with high $y$’s. This case corresponds to Figure 1(c).

In the intermediate case (ii), $p_{NY}^1 + p_{NY}^2 = \sqrt{2tv}$. In equilibrium, firm 1 attracts only consumers who are close to its variety position and have small $y$’s. This case corresponds to Figure 1(b). Note that Figure 1(d) does not arise in equilibrium.

4. Equilibrium Customization Choices

In the customization stage of the game the firms simultaneously choose between not customizing (N) and customizing (Y). This stage is represented by the following matrix.

\[
\begin{array}{c|cc}
 & \text{Firm 2} \\
\hline
\text{Firm 1} & N & Y \\
N & \frac{1}{2}t, \frac{1}{2}t & \Pi_{NY}^1, \Pi_{NY}^2 - K \\
Y & \Pi_1^{YN} - K, \Pi_2^{YN} & -K, -K \\
\end{array}
\]

Let $\Delta$ denote a firm’s gain in gross profit from customization given that the other firm does not customize,

$$\Delta \equiv \Pi_1^{YN} - \frac{1}{2}t = \Pi_{2}^{YN} - \frac{1}{2}t.$$  

Obviously, if $\Delta$ is negative no firm will customize. For positive $\Delta$, whether a firm has an incentive to customize depends on the value of $\Delta$ relative to $K$.

**Proposition 2.** The following hold for the firms’ equilibrium choices in the customization stage.

(i) If $v < (3\sqrt{2} - 4)t \approx 0.24t$, then $(N,N)$ is the unique Nash equilibrium for any value of $K$.

(ii) If $v \geq (3\sqrt{2} - 4)t$, then $(N,N)$ is the unique Nash equilibrium for $K > \Delta$, $(N,Y)$ and $(Y,N)$ are the two pure-strategy Nash equilibria for $K < \Delta$.

Customization by both firms makes their products perfect substitutes for each consumer, resulting in the Bertrand outcome. Hence, $(Y,Y)$ is never a Nash equilibrium. Customization by one of the firms makes the rivals closer to each other in the variety dimension and creates vertical differentiation. The former intensifies price competition, whereas the latter softens it. When $v$ is small relative to $t$, no firm has an incentive to customize because the cost of increased price competition overwhelms the gains by consumers from customization that the customizing firm can appropriate. This intuition is reversed when $v$ is large relative to $t$.

5. Effects of Customization

In order to investigate how customization affects firms, consumers, and total welfare, we contrast the customization model studied above with the benchmark model in which customization is not feasible. Obviously, there are no effects of customization if both firms choose not to customize.
in the equilibrium of the customization model. Accordingly, we focus on situations in which customization occurs, i.e., \( K < \Delta \) (Proposition 2).

In the benchmark model, total welfare is \( r \) minus disutility from consuming less preferred varieties,

\[
W = r - \frac{1}{4}t,
\]

and consumer surplus is

\[
CS = W - t = r - \frac{5}{4}t.
\]

In the customization model, total welfare can be calculated using subgame NY. It is \( r \) plus utility derived from product uniqueness, minus disutility from consuming less preferred varieties, minus the fixed cost of customization. Algebraically,

\[
W = W^{NY} = r + v \int \int_{2's \ mkt} y \ dx \ dy - t \int \int_{1's \ mkt} x \ dx \ dy - K. \tag{4}
\]

Consumer surplus can be obtained from (4) by eliminating the term \(-K\) and subtracting the total payments by consumers. That is,

\[
CS = r + v \int \int_{2's \ mkt} y \ dx \ dy - t \int \int_{1's \ mkt} x \ dx \ dy - \Pi^{NY}_1 - \Pi^{NY}_2. \tag{5}
\]

**Proposition 3.** Compared to the benchmark model, the following hold.

(i) The customizing firm is always better off, the non-customizing firm is worse off if \( v < t(11 + 3\sqrt{13})/4 \approx 5.45t \) and better off if \( v > t(11 + 3\sqrt{13})/4 \).

(ii) Customization increases consumer surplus if \( v < t(37 + \sqrt{1401})/8 \approx 9.30t \) and decreases consumer surplus if \( v > t(37 + \sqrt{1401})/8 \).

(iii) Customization improves total welfare.

Obviously, the customizing firm is always better off compared to the benchmark. As mentioned earlier, a higher value of \( v \) softens price competition. Hence, when \( v \) is sufficiently large the non-customizing firm is also better off when reduced price competition offsets its disadvantage caused by customization of the other firm. Reduced price competition may hurt the buyers so as to offset their gains from consuming customized products. Part (iii) of Proposition 3 indicates that consumers and producers together always benefit from customization.

6. Concluding Remarks

The novelty of our paper is the incorporation of product uniqueness into customization competition. While customization makes the firms less differentiated in the horizontal dimension, it creates vertical differentiation through uniqueness. The former intensifies price competition, the latter softens it. Customization by one of the firms occurs in equilibrium only if consumers have high preferences for uniqueness.
Appendix

Proof of Proposition 1. Each part is proven in turn.

(i) Consider \( v \leq t/2 \) and suppose the indifference line (3) intersects the unit square as shown in Figure 1(a). The firms’ demand functions are
\[
D_1(p_1, p_2) = \frac{1}{t} \left( -\frac{1}{2} v + p_2 - p_1 \right) \quad \text{and} \quad D_2(p_1, p_2) = \frac{1}{t} \left( t + \frac{1}{2} v + p_1 - p_2 \right).
\]

The first-order conditions yield the prices and profits as in part (i) of the proposition. It is easily checked that \( \hat{x}(1) \geq 0 \) and \( \hat{x}(0) < 1 \), confirming the pattern in Figure 1(a).

(ii) Consider \( v \in (t/2, 2t] \) and suppose (3) intersects the unit square as shown in Figure 1(b). The firms’ demand functions are
\[
D_1(p_1, p_2) = \frac{1}{2tv} (p_2 - p_1)^2 \quad \text{and} \quad D_2(p_1, p_2) = 1 - \frac{1}{2tv} (p_2 - p_1)^2.
\]

The first-order conditions yield the prices and profits as in part (ii) of the proposition. It is easily checked that \( \hat{x}(1) < 0 \) and \( \hat{x}(0) \in (0, 1] \).

(iii) Consider \( v > 2t \) and suppose (3) intersects the unit square as shown in Figure 1(c). The firms’ demand functions are
\[
D_1(p_1, p_2) = \frac{1}{v} \left( -\frac{1}{2} t + p_2 - p_1 \right) \quad \text{and} \quad D_2(p_1, p_2) = \frac{1}{v} \left( v + \frac{1}{2} t + p_1 - p_2 \right).
\]

The first-order conditions yield the prices and profits as in part (iii) of the proposition. It is easily checked that \( \hat{x}(1) < 0 \) and \( \hat{x}(0) > 1 \).

Proof of Proposition 2. Simple algebra reveals that \( \Delta > 0 \) in cases (ii) and (iii) of Proposition 1. Hence, consider \( v \leq t/2 \). In this case
\[
\Delta = \frac{1}{t} \left( \frac{2}{3} t + \frac{1}{6} v \right)^2 - \frac{1}{2} t.
\]

It is negative iff \( v < (3\sqrt{2} - 4)t \approx 0.24t \). The results of Proposition 2 follow immediately.

Proof of Proposition 3. Each part is proven in turn.

(i) It is straightforward to verify that \( \Pi_1^{NY} < t/2 \) in cases (i) and (ii) of Proposition 1. If \( v > 2t \),
\[
\Pi_1^{NY} = \frac{1}{v} \left( \frac{1}{3} v - \frac{1}{6} t \right)^2 > \frac{1}{2} t
\]
iff \( v > t(11 + 3\sqrt{13})/4 \).
(ii) Consider $v \leq t/2$. The indifference line (3) intersects the unit square as in Figure 1(a). By (4),

$$W = r - t \int_0^1 \int_0^{\hat{x}(y)} x \, dx \, dy + v \int_0^1 \int_0^{\hat{x}(y)} y \, dx \, dy - K = r + \frac{7}{18} v - \frac{1}{18} t + \frac{v^2}{9t} - K,$$

where $\hat{x}(y) = (-vy + t/3 + v/3)/t$. If $v \in (t/2, 2t]$, (3) intersects the unit square as in Figure 1(b), in which case

$$W = r - t \int_0^1 \int_0^{\sqrt{\frac{t}{2\pi}} \hat{x}(y)} x \, dx \, dy + v \int_0^1 \int_0^{\sqrt{\frac{t}{2\pi}} \hat{x}(y)} y \, dx \, dy - K = r + \frac{1}{2} v - \frac{1}{12} \sqrt{2tv} - K,$$

where $\hat{x}(y) = (-vy + \sqrt{tv}/2)/t$. If $v > 2t$, (3) intersects the unit square as in Figure 1(c), in which case

$$W = r - t \int_0^1 \int_0^{\frac{\hat{y}(x)}{t}} x \, dy \, dx + v \int_0^1 \int_0^{\frac{\hat{y}(x)}{t}} y \, dy \, dx - K = r + \frac{4}{9} v - \frac{1}{9} t + \frac{t^2}{9v} - K,$$

where $\hat{y}(x) = (-tx + v/3 + t/3)/v$. By (5),

$$CS = \begin{cases} 
  r + \frac{5}{18} v - \frac{11}{18} t + \frac{v^2}{18t}, & \text{if } v \leq t/2 \\
  r + \frac{1}{2} v - \frac{11}{24} \sqrt{2tv}, & \text{if } v \in (t/2, 2t] \\
  r - \frac{1}{3} v - \frac{7}{9} t + \frac{t^2}{18e}, & \text{if } v > 2t 
\end{cases}$$

Comparing it with $\overline{CS}$ implies $CS > \overline{CS}$ iff $v > t(37 + \sqrt{1401})/8$.

(iii) Evaluating $W$ at $K = \Delta$ and comparing it with $\overline{W}$ implies $W > \overline{W}$.

\[\square\]

References


