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Tax enforcement may decrease government revenue

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Abstract

We analyze the relation between tax enforcement, aggregate output and government revenue when imperfectly competitive firms evade a specific output tax. We show that aggregate output decreases with tax enforcement. Government revenue increases with enforcement when the tax is low. When the tax is high, government revenue is either inversely U-shaped or decreasing with enforcement.

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1. Introduction

This note contributes to the (small but growing) literature on tax compliance by firms.[1] Cremer and Gahvari (1999) assert that "it is widely believed that the presence of tax evasion reduces tax revenues." In this note, we show that this need not be the case. We study the impact of tax enforcement on aggregate output and government revenue when imperfectly competitive firms evade a specific output tax.[2] We obtain that aggregate output decreases with tax enforcement. For the family of linear demands, government revenue increases with enforcement when the tax is low. When the tax is high, government revenue is either inversely U-shaped or decreasing with enforcement. In the latter case, we obtain the counter-intuitive result that government revenue is larger with evasion than without evasion.

2. The model

We model a two-stage game.[3] In the first stage, n identical, risk neutral, firms simultaneously decide how much to produce of a homogenous good. Firms have constant returns to scale technologies, with the same marginal cost c. Given output decisions $(q_1, ..., q_n)$, the price adjusts to the level that clears the market. We denote by $P(Q)$ the inverse market demand, where $Q = \sum_i q_i$ is aggregate output. The function $P(Q)$ is twice-continuously differentiable, with $P'(Q) < 0$ at all Q. In the second stage, taxation, evasion and tax enforcement occur. Each firm i has to pay a specific tax of $t > 0$ per unit sold.[4] We assume that firm's output is private information and that firms decide the fraction $e_i \in [0, 1]$ of output that they report to the tax authority. We follow Cremer and Gahvari (1993) by assuming that concealment of the fraction e_i entails a cost[5] of $g(e_i)$ per unit sold. The function g is strictly increasing, convex, and verifies $g(0) = 0$ and $g'(0) = 0$. The government audits each firm with the same probability $\alpha \in (0, 1)$. Audits are costless[6] and perfect (i.e., they reveal the amount evaded with certainty). When a firm is not audited,

[1] Virmani (1989) and Cremer and Gahvari (1999) study the impact of tax evasion in a perfectly competitive market whereas Marelli and Martina (1988), Bayer and Cowell (2006) and Goerke and Runkel (2006, 2007) focus on settings with imperfect competition.

[2] Our model is similar to Goerke and Runkel's (2007). But, as the goal of their paper is to analyze the impact of competition on tax evasion, they do not study how enforcement affects aggregate output and tax revenue.

[3] This assumption is made for expository convenience, as our results would carry through if production and evasion decisions were simultaneous.

[4] Our qualitative results hold with proportional taxation.

[5] These costs arise from the necessity to buy specialist advice or avoidance schemes, or to reorganize transactions, so that a casual inspection by the tax authority does not reveal the value of output sold.

[6] This assumption biases our results against finding a negative relation between enforcement and tax revenue.

it pays taxes based on the amount reported: $t(1-e_i)q_i$. If audited, an evading firm has to pay the tax that it legally owes, tq_i, plus a fine which is a fraction λ of the amount of taxes evaded.

3. Equilibrium production and evasion

We solve the model, starting with the second stage.

3.1 Evasion

In the second stage, each firm i chooses e_i^* to maximize its expected profit

$$\mathbb{E}\Pi_i = \alpha\Pi_i^A + (1-\alpha)\Pi_i^{NA},$$

where Π_i^A and Π_i^{NA} denote ex-post profits when firm i is (respectively, is not) audited. If firm i is audited, its ex-post profit is

$$\Pi_i^A = [p\left(Q\right) - (1+\lambda e_i)t - g(e_i) - c]q_i,$$

whereas, if it is not audited,

$$\Pi_i^{NA} = [p\left(Q\right) - (1-e_i)t - g(e_i) - c]q_i.$$

Rearranging, the expected profit is

$$\mathbb{E}\Pi_i = [p(Q) - (1-e_i(1-\xi))t - g(e_i) - c]q_i$$

where $\xi = \alpha(1+\lambda)$ denotes the expected payment rate on undeclared tax, as a fraction of t. From now on, we take ξ as our measure of tax enforcement. The following first-order condition

$$\frac{\partial \mathbb{E}\Pi_i}{\partial e_i} = [(1-\xi)t - g'(e_i^*)]q_i = 0 \tag{1}$$

characterizes the interior optimal fraction e_i^*, which equalizes the marginal expected net benefit from evading with the marginal cost of concealing. Observe that e_i^* is independent of any production variable chosen by the firm or determined in the market (as in Cremer and Gahvari (1993)). Moreover, as firms are identical and are audited with the same probability, they all evade the same amount. We gather these results in the following proposition.

Proposition 1 *Each firm fails to report a fraction of output e^* when $\xi < 1$. Otherwise, no firm evades.*

In order to evade, a firm has to face an expected rate of payment on undeclared tax that is lower than unity. If this were not the case, evasion would not be optimal. Applying the Implicit Function Theorem to (1), it is straightforward to show that the fraction e^* decreases with enforcement ξ, as expected.

3.2 Equilibrium production

Given the production decisions of the other firms (q_{-i}) and anticipating that it will evade a fraction e^*, each firm i chooses its output q_i to maximize its expected profit

$$\mathbb{E}\Pi_i = [p(Q) - c - \widetilde{t} - g(e^*)]q_i$$

where $Q = q_i + \sum_{-i} q_{-i}$ and $\widetilde{t} = t(1 - e^*(1 - \xi))$ is the expected "effective" unit tax (as opposed to the "legislated" tax t). Using the convexity of g and the first-order condition (1), we obtain that

$$\widetilde{t} + g(e^*) < t \text{ when } \xi < 1,$$

so that evasion attenuates the impact of taxation, provided that $\xi < 1$. Straightforward differentiation shows that

$$\frac{\partial \widetilde{t}}{\partial \xi} = t\left[e^* - \frac{\partial e^*}{\partial \xi}(1 - \xi)\right] > 0. \tag{2}$$

The effective tax rate increases with enforcement through two channels: a direct "enforcement effect" (first term in brackets in (2), which increases the expected payment rate on undeclared sales) and an indirect "evasion effect" (second term in brackets in (2), which decreases the fraction of sales undeclared).

The first-order condition for firm i is

$$\frac{\partial \mathbb{E}\Pi_i}{\partial q_i} = p\left(Q\right) + p'\left(Q\right) q_i^* - c - \widetilde{t} - g(e^*) = 0,$$

from which we see that $p(Q)+p'(Q)q_i^* > 0$ in order to obtain an interior solution. Existence and uniqueness of the Cournot equilibrium are ensured if we also assume

$$\frac{\partial^2 \mathbb{E}\Pi_i}{\partial q_i \partial q_j} = p'(Q) + q_i p''(Q) < 0,\ i \neq j, \tag{3}$$

(see Vives 1999).[7]

As firms are identical, production decisions q_i^* are the same, the equilibrium is symmetric and we sum the n first-order conditions to obtain

$$\left[np\left(Q^*\right) + p'\left(Q^*\right)Q^*\right] = n\left[c + \widetilde{t} + g(e^*)\right]. \tag{4}$$

By the Maximum Theorem, Q^* is a continuous function of the enforcement parameter ξ. The Implicit Function Theorem allows us to obtain the following result.

[7] With this assumption, the second-order condition $\partial^2 \mathbb{E}\Pi_i/\partial q_i^2 = [2p'(Q)+p''(Q)q_i^*] \leq 0$ automatically holds.

Proposition 2 *When there is evasion, Q^* decreases with enforcement ξ. Otherwise, Q^* is independent of ξ.*

Proof. See the Appendix ■

When ξ increases, the "effective" marginal cost $c + \widetilde{t} + g(e^*)$ that firms face increases. So, as shown by Seade (1985), for any market structure (i.e., number of firms n), each firm produces less. Therefore, in equilibrium, aggregate output decreases.

4. The relation between tax enforcement and expected government revenue

Expected government revenue (including both taxes and fines) is defined as

$$R^* = \widetilde{t}Q^*,$$

and is a continuous function of the enforcement parameter ξ. The total effect of an increase in ξ upon R^* can be decomposed as follows

$$\frac{\partial R^*}{\partial \xi} = \frac{\partial \widetilde{t}}{\partial \xi} Q^* + \widetilde{t}\, \frac{\partial Q^*}{\partial \xi}. \tag{5}$$

The first term on the right hand side of (5), which we dub the "tax effect", measures the positive impact of enforcement on fiscal revenues due to the increase in the effective tax $\widetilde{t}$. The second term, called the "base effect", is negative since more enforcement decreases total quantity (see Proposition 2). Therefore, the sign of $\partial R^*/\partial \xi$ is a priori ambiguous. This has been noted by Cremer and Gahvari (1993) in the context of a perfectly competitive market. But they only point out this ambiguity, without exploring the possible forms of the curve R^*. This is precisely what we do. The next proposition shows that, when the inverse market demand is linear and concealment costs are quadratic, the curve R^* has at most three forms, one for each parameter configuration of the model.

Proposition 3 *Assume that $P(Q) = a - bQ$, where $a > 0$, $b > 0$ and $g(e) = e^2/2$. Assume further that $t < a - c$.*[8] *There exist threshold values of t, denoted by $\widehat{t} = 2(a-c)/3$ and $t_1 = \left(3 - \sqrt{9 - 16(a-c)}\right)/4$ such that:*

(i) if $t \leq \widehat{t}$, then R^* is increasing in ξ.

(ii) if $\widehat{t} < t \leq \min\{t_1, a-c\}$, then R^* is inversely U-shaped in ξ.

(iii) if $\min\{t_1, a-c\} \leq t < a - c$, then R^* is decreasing in ξ.

[8] This assumption ensures that equilibrium production and profits are both positive for any value of enforcement.

Proof. See the Appendix ■

Observation of (5) suggests that the tax effect dominates for small values of the effective tax, while the base effect is more important for large values of $\widetilde{t}$. Proposition 3 confirms this intuition. There exists a threshold $\widehat{t} = 2(a-c)/3$ that separates when R^* is increasing from the two other cases of figure. This threshold is above $t^* = (a-c)/2$, the tax that maximizes fiscal revenues under full compliance.[9]

Then, when t is large enough ($t > \widehat{t}$), two different cases emerge, depending upon the value of the maximal mark-up $a-c$. When $a - c \geq 1/2$, the tax effect dominates for low values of ξ (and thus of $\widetilde{t}$) while the base effect dominates for larger values of ξ: tax proceeds are first increasing and then decreasing in the enforcement level. As $t_1 \geq a-c$, (iii) is not pertinent. But, when $a - c < 1/2$, $t_1 < a - c$. Thus, for even larger values of $t \geq t_1$, tax proceeds monotonically decrease with ξ. We obtain the counter-intuitive result that tax proceeds are always larger with evasion than without evasion. The reason for this result is the following. As the maximal mark-up $a-c$ is relatively small, Q^* may be too low. Thus, when t is sufficiently high, an increase in ξ causes a percent increase in $\widetilde{t}$ lower than the percent decrease in Q^*.

Figure 1 illustrates Proposition 3 when $P(Q) = 6 - Q, c = 5.6$, and $n = 10$. With this parameter configuration, $\widehat{t} = 0.267$ and $t_1 = 0.347 < a - c = 0.4$. Hence, R^* adopts, depending upon the value of the tax t, the three possible forms described in Proposition 3.

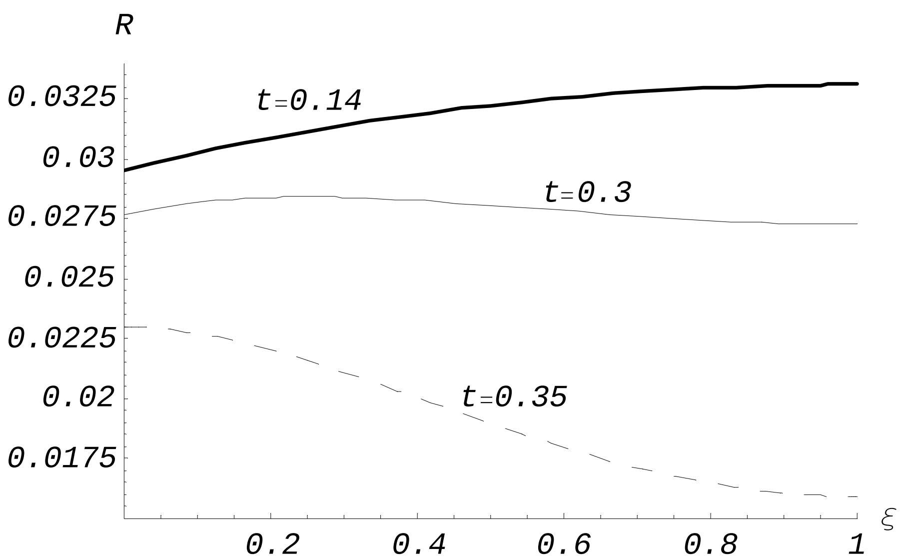

Figure 1: Expected government revenue as a function of enforcement ξ

[9] This result generalizes Cremer and Gahvari (1999), as they only find a negative relation between tax evasion and tax revenue when $t < t^*$.

Finally, observe that our results hold true for any value of n – i.e., that they do not depend on market structure.

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Appendix

Proof of Proposition 2

Applying the Implicit Function Theorem, we differentiate (4) and we obtain, using an envelope argument

$$\frac{\partial Q^*}{\partial \xi} = \frac{ne^*t}{(n+1)P'(Q^*) + Q^*P''(Q^*)}. \tag{6}$$

As

$$(n^*+1)P'(Q^*) + Q^*P''(Q^*) = P'(Q^*) + n^*\left[P'(Q^*) + q^*P''(Q^*)\right]$$

and, from (3),

$$P'(Q)+q^{*}P''(Q)<0,$$

then[10]

$$(n^{*}+1)P'(Q^{*})+Q^{*}P''(Q^{*})<0.$$

This implies that $\partial Q^{*}/\partial\xi<0$ ■

Proof of Proposition 3

When $g(e)=e^{2}/2$, $e^{*}=(1-\xi)t$. In what follows, we assume $t<1/(1-\xi)$, so interior solutions for e^{*} obtain. When $P(Q)=a-bQ$, first and second derivatives of $P(Q)$ are

$$P'(Q)=-b \text{ and } P''(Q)=0,$$

which verify (3). Using (4) and e^{*}, the equilibrium production is thus given by:

$$Q^{*}=\frac{n}{b\,(n+1)}\left[(a-c)-t\left(1-\frac{t\,(1-\xi)^{2}}{2}\right)\right].$$

Assuming that $t<a-c$, equilibrium quantities and profits are non negative for any enforcement level ξ. After some manipulation of (5), we obtain

$$\frac{\partial R^{*}}{\partial\xi}=\frac{ne^{*}t}{-b(n+1)}[M-2(e^{*})^{2}] \tag{7}$$

where $M=3t-2(a-c)$. The sign of this derivative is the opposite of the sign of the expression in brackets. On the one hand, when $t\leq\widehat{t}=2(a-c)/3$, $\partial R^{*}/\partial\xi$ is positive for all ξ. On the other hand, when $\widehat{t}<t<(a-c)$, $\partial R^{*}/\partial\xi$ is positive (negative) when $\xi\leq(>)\hat{\xi}=1-(1/t)\sqrt{M/2}$. So R^{*} is inverse U-shaped in ξ if $\hat{\xi}>0$ and decreasing, if $\hat{\xi}<0$.

The conditions for $\hat{\xi}>0\,(<0)$ is that the polynomial defined by

$$\Upsilon\left(t\right)=2t^{2}-3t+2\left(a-c\right)$$

is greater (lower) than 0. When $a-c\leq1/2$, $\Upsilon(t)\geq0$ if $2(a-c)/3<t\leq t_{1}$, where

$$t_{1}=\frac{3-\sqrt{9-16(a-c)}}{4}$$

and $\Upsilon(t)<0$ if $t_{1}<t\leq(a-c)$. When $a-c>1/2$, $\Upsilon(t)\geq0$ for any $2(a-c)/3<t<a-c$. This leads to Proposition 3 ■

[10] The condition (3) holds for any q_{i},q_{j}, so, *a fortiori,* for the equilibrium value q^{*}.