Delegation, externalities and organizational design

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**Abstract**

In a repeated interaction between a principal and two agents with inter-agents externalities and asymmetric information, we show that optimal decentralization within the organization is limited to the first period and across agents.

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1. Introduction

In multi unit organizations, with externalities (the choices made by one unit affect the profit of the others), two key factors will drive the task allocation problem: externalities and asymmetric information. In the absence of externalities, there is no need to coordinate agents’ choices and profit is maximized in a fully decentralized structure where the agents have all the power. In the absence of asymmetric information, there is no reason to delegate and profit is maximized in a centralized organization where the principal keeps all the power. In an organization where there are both externalities and asymmetric information, benefits and costs are associated with delegation.

In a single period interaction, delegation is beneficial because the decider has superior information (Dessein, 2002). With repeated interaction, delegation has an additional benefit: the principal can improve her knowledge of the agents’ information by observing their past decisions (Gautier and Paolini, 2007). Delegation is a learning process: when the agent uses his private information to make better decision, the principal revises her beliefs by observing the agent’s choice and she can improve her decisions.

We model a two-period interaction between one principal supervising two agents (units). At each period, a project must be implemented in each unit. At time zero, the principal chooses the process of decision-making for the following periods. Projects are transferable control actions (Aghion et al., 2002): projects cannot be contracted out but control is contractible. Each agent has a piece of private information and he exerts an externality on the other. In this context, the questions we raise in this paper are which decisions should be decentralized and when?

To answer the second question (‘when?’), we build up on Gautier and Paolini (2007) that have shown that, in a repeated set-up, an agent discloses his private information when he receives control over the first decision. We show that this result continues to hold true in a multi agents setting where there are multiple sources of information. Hence, optimal decentralization within the organization remains limited to the first period. The answer to the first question (‘which’?) is the main contribution of this paper. We show that delegation is limited across agents. Even though the principal acquires information, delegation remains costly because the agents do not take the externalities into account. Moreover, the benefit of an additional piece of information decreases because information is partly redundant. Hence, the number of delegated decisions depends on the quality of the signal produced
and on the cost of producing it. The quality of the signal depends on the correlation between the information of the agents. The cost of producing a signal, which is the cost of delegating decisions, depends on the externality exerted by the agent that receives the control over decisions. Following that, symmetric agents could be treated differently in the organization if one signal, obtained by the delegation of a decision to one agent, brings enough information to the principal.

2. The Model

There are three players, the principal and two agents (units) $l = A, B$, and two periods $t = 1, 2$. Before the first period, the principal decides who will have the power to choose the project ($d_l^t$) in unit $l$ at time $t$. In each period a project is implemented by both units.

Each agent $l$ has private information represented by a state parameter $\theta^l$ drawn out of a set $\Theta = \{\theta_1, \theta_2\}$ with $\theta_1 < \theta_2$ and $\Delta \theta = \theta_2 - \theta_1$.

The principal only knows the prior distribution of $(\theta^A, \theta^B)$ over $\Theta$, which is represented by a joint probability distribution $\{v_{11}, v_{12}, v_{21}, v_{22}\}$ where $v_{ij} = \text{prob}(\theta^A = \theta_i, \theta^B = \theta_j)$. The correlation between $\theta$’s can be measured by $\rho = |v_{11}v_{22} - v_{12}v_{21}|$. To simplify, we assume that: $v_{11} = v_{22} = v_{ii}$ and $v_{12} = v_{21} = v_{ij}$. Then, $\rho = |v_{ii} - \frac{1}{4}|$ and it takes a value between 0 (independence) and $\frac{1}{4}$ (perfect correlation).

The projects differ in just one dimension and there is a continuum of possible decisions: $d^t_l \in (0, +\infty)$. Each unit is a profit center. Depending on $\theta^l$, there is one project maximizing profit in unit $l$. The profit in unit $l$ when a project $d^t_l$ is implemented at time $t$ is equal to $\alpha^l d^t_l - (\theta^l - d^t_l)^2$. The unit’s profit is maximized for $d^t_l = \alpha^l + \theta^l$ and it decreases with the distance between the preferred project and the actual one.

In addition, unit $l$ exerts an externality on unit $k$. The externality is measured by a parameter $\gamma$, identical for the two units. A decision $d^t_l$ taken in unit $l$ reduces the profit in unit $k$ by $\gamma d^t_l$.

The profit in unit $l$ is thus:

$$\Pi^t_l = \alpha^l d^t_l - \frac{(\theta^l - d^t_l)^2}{2} + \alpha^l d^t_k - \frac{(\theta^l - d^t_k)^2}{2} - \gamma (d^t_k + d^t_k),$$

with $l \neq k$.

The agents maximize the profit in their unit while the principal is interested in the maximization of the total profit (maximized for $d^t_l = \alpha^l + \theta^l - \gamma$).
3. Decisions of the principal and the agents

Each decision $d_t^l$ is made by the person to whom the principal has allocated the right to decide. When the principal must decide on a project $d_t^l$, her decision depends on the information she has. Let us represent her information by a distribution of beliefs $\eta_{ij}$ over $\Theta \times \Theta$. The decision $d_t^l$ that maximizes total profit is:

$$d_t^l = \alpha^l + E(\theta^l | \eta_{ij}) - \gamma,$$

where $E(\theta^l | \eta_{ij})$ denotes the expected value of $\theta^l$ conditional on beliefs $\eta_{ij}$.

When agent $l$ must decide on a project $d_t^l$, he must take into account that the principal will revise her beliefs after observing $d_t^l$. This changes the agents’ profit if the principal can make a decision after the agent i.e. only if $d_t^1$ is delegated and $d_t^A_2$ or $d_t^B_2$ is not. If the principal cannot use the agent’s information, the agent chooses his preferred project $d_t^l = (\alpha^l + \theta^l)$. In all other cases, the agents and the principal play a signaling game and we must search for equilibria. Usually, signaling games have multiple equilibria. We use Cho and Kreps (1987) intuitive criterion (IC) to refine the set of equilibria. We can establish that:

**Proposition 1** Under delegation of $d_t^1$, the only equilibrium that survives IC is the least costly separating (LCS) equilibrium.$^1$

**Proof.** See appendix. ■

Gautier and Paolini (2007) have proved this result in the one principal - one agent case. Here, we show that the structure of preferences is such that the result extends to the one principal - two agents case. That is, whatever the principal’s knowledge of the state parameters, there always exist one state in which the agent would be better off if he manages to transmit the true information to the principal i.e. the single crossing property holds true for all possible information structure. Consequently, a separating equilibrium always exists. Moreover, the IC selects the most efficient one among the non-empty set of separating equilibria. This means that, if the principal delegates $d_t^1$ to agent $l$, she learns $\theta^l$ and improves her knowledge of $\theta^k$. Both second period decisions are thus based on a more accurate information.

To transmit information to the principal, the agents must sometimes take a suboptimal decision to make their information transfer credible i.e. the

$^1$The Riley (1979) outcome.
decisions that maximize the agent’s payoff may not be incentive compatible. Our corollary clarifies that point.

**Corollary 2** If $\Delta \theta^2 \geq \gamma^2$, the LCS equilibrium is $d^*_1(\theta) = \alpha^l + \theta^l$.

Notice that, whenever this condition does not hold true, the principal optimally retains control over all decisions (proposition 3 hereafter).

4. Optimal organization

Given that there is an externality between the two units, any form of delegation has a cost for the principal because there is no coordination in the project choices. Nevertheless, delegation benefits the principal because (i) the decisions of the agents are based on better information than those of the principal and (ii) the principal improves her knowledge of the state parameter after observing delegated decisions. In this section, we derive the optimal organization.

Starting from a fully centralized organization. The principal bases her decisions on the expected value of $\theta^l$ rather than on its true value. But the principal internalizes the externalities imposed by one agent to the other. Under **centralization**, the decisions taken by an uninformed principal are:

$$
\hat{d}_1^l = \hat{d}_2^l = \alpha^l + (v_{ii} + v_{ij})\theta^l_1 + (v_{ij} + v_{ii})\theta^l_2 - \gamma
$$

(1)

Next, consider **partial delegation** where the principal delegates $d_1^l$ to agent $l$ and retains control over the remaining decisions. If $d_1^l$ signals $\theta^l$ to the principal (and we know form proposition 1 that it is the case), the principal becomes informed about $\theta^l$ but also improves her information about $\theta^k$, $k \neq l$ if there is correlation between the information. Under the condition of corollary 2, the optimal decision under partial delegation are:

$$
\begin{align*}
  d_1^k &= \hat{d}_1^k \\
  d_1^l(\theta) &= \alpha^l + \theta^l \\
  d_2^k &= \alpha^k + E(\theta^k | \theta^l) - \gamma \\
  d_2^l(\theta) &= \alpha^l + \theta^l - \gamma
\end{align*}
$$

(2) (3) (4) (5)

With one signal obtained by delegating $d_1^l$, the principal implements the first best in period 2 for agent $l$ and bases her decision $d_2^k$ on a better information than the prior distribution of $\theta^k$.
With full delegation, the decisions implemented in unit $l$ are given by (3) and (5) and the decisions implemented in unit $k$ are given by

$$
\begin{align*}
    d^k_l(\theta) &= \alpha^k + \theta^k \\
    d^k_k(\theta) &= \alpha^k + \theta^k - \gamma
\end{align*}
$$

Let us denote by $\Pi^C$, $\Pi^{PD}$ and $\Pi^D$, the principal’s profit under centralization (C), partial delegation (PD) and full delegation (D). Starting from the profit $\Pi^C$, we can define the marginal benefit of delegation. The marginal benefit of delegating a first project to agent $l$ is equal to $Mb(1) = \Pi^{PD} - \Pi^C$.

Simple computation gives that delegating one project amounts to a change of total profit equals to $Mb(1) = -\gamma^2 + \frac{1}{2}(1 + 8\rho^2)\Delta \theta^2$.

If the principal delegates a second project to agent $k$, the principal becomes informed about $\theta^l$ and $\theta^k$. We can thus define the marginal benefit of delegating a second decision as $Mb(2) = \Pi^D - \Pi^{PD} = -\gamma^2 + \frac{1}{2}(1 - 8\rho^2)\Delta \theta^2$.

Hence, if $Mb(1) > 0$, the total profit increases if $d^l_1$ is delegated to agent $l$. Likewise, if $Mb(2) > 0$, both first period projects must be delegated to the agents. Clearly, $Mb(1) \geq Mb(2)$, that is the marginal benefit of delegating a first project is higher than the marginal benefit of delegating a second one because the information contained in a second signal is partially redundant (at least for $\rho > 0$). Therefore, if $Mb(1) > 0 > Mb(2)$, the principal optimally delegates to only one agent.

Finally, notice that under the condition of our corollary, we have $Mb(3), Mb(4) < 0$, which means that second period delegation would never be optimal. We thus have established the following:

**Proposition 3** The optimal organization is: centralization for $\Delta \theta^2 \leq \frac{2\gamma^2}{1+8\rho^2}$, delegation of $d^l_1$ to agent $l$ for $\Delta \theta^2 \in [\frac{2\gamma^2}{1+8\rho^2}, \frac{2\gamma^2}{1-8\rho^2}]$ and delegation of $d^A_1$ and $d^B_1$ to A and B for $\Delta \theta^2 \geq \frac{2\gamma^2}{1-8\rho^2}$.

The striking result of this paper is the optimality of limited delegation in a repeated context. Delegation is limited to the first period (Gautier and Paolini, 2007) and across agents. Agents can be treated asymmetrically within the organization. The principal selects one delegate that will be responsible for the production of the signal and keeps control over the decisions concerning the other agent.

The quality of the signal produced by the agent depends on the degree of correlation. $Mb(1)$ is increasing in $\rho$: when the information are correlated
the value of a unique signal is higher. Conversely, $Mb(2)$ decreases in $\rho$: the informational content of the second signal decreases with the degree of correlation. Figure 1 illustrates the optimal organizational structure as a function of the correlation parameter. Asymmetric treatment of the agents is more likely when the agents have correlated information. Note also that in a one period model, there is no learning associated with delegation and agents will be treated symmetrically.

In this model, we have analyzed a dynamic task allocation problem with externalities and asymmetric information. An alternative mechanism could be that the agents communicate to the principal and the principal chooses the actions. Crawford and Sobel (1982) show that communication is not perfectly informative. In a message game both agents reveal their private information to the principal who then takes all decisions if $4\gamma^2 \leq \Delta\theta^2$. Clearly there exists a parameter set where communication fails and delegation is optimal. This confirms Dessein (2002) who recognizes that "Delegation is typically a better instrument to use the local knowledge of the agent than communication."

\[ Mb(1) > Mb(2) > 0 \]
\[ Mb(1) > 0 > Mb(2) \]
\[ Mb(1) < 0 \]

Figure 1: Optimal organization

Appendix: Proof of proposition 1 and corollary 2

In a two-players game, IC selects the Riley outcome if there are only two states and if the payoff function satisfies the single crossing condition. To prove proposition 1, we replicate the argument (used in Gautier and Paolini, 2007) in this three-players game.

Each agent when he receives control over $d_1$ plays a signaling game with the principal. But, two versions of this game must be considered: in the
first, the other agent $k$ discloses his private information and thus, at $t = 2$, the principal knows $\theta^k$. In the second, the principal does not learn $\theta^k$ either because the agent decided not disclose information or because he did not receive control over $d^k_1$.

In this game played between $l$ and the principal, a Bayesian equilibrium is a triple $(d^*_1, d^*_2, \mu)$ where:

(BE1) $\forall \theta \in \Theta, d^*_1(\theta) \in \argmax_{d_1} \Pi^l(d_1, d^*_2, \theta)$

(BE2) $\forall d^*_1, d^*_2(\theta) \in \argmax_{d_2} \sum_{\theta} \mu(\theta|d_1)\Pi^P(d_1, d^*_2, \theta)$

(BE3) The posterior beliefs $\mu(\theta|d_1)$ are consistent with the Baye’s rule.

We apply to the set of Bayesian equilibria the Cho and Kreps (1987) intuitive criterion. A Bayesian equilibrium fails the intuitive criterion if:

(IC1) The equilibrium payoff of the agent in one state of the world ($\theta_i$) is greater with the equilibrium strategy than with any other strategy.

(IC2) It exists a strategy $\tilde{d}_1$ such that the equilibrium payoffs in the other state of the world ($\theta_j$) are smaller than those with the strategy $\tilde{d}_1$ once the principal is convinced that $\tilde{d}_1$ could not have been chosen by the agent in state $\theta_i$.

(1) Suppose that agent $k$ truly reveals his private information if he controls $d^k_1$. Then, whatever the choice of $d^*_1$ by agent $l$, he will not be able to change the principal’s belief about $\theta^k$. In this game, the standard sorting condition is satisfied; This is sufficient to kill all the pooling and the separating equilibria but the Riley outcome.

To be more explicit, consider first the set separating equilibria. In a separating equilibrium $d^*_2(\theta) = \alpha^l + \theta$; an equilibrium is separating if $d^*_1(\theta)$ satisfies the incentive constraint:

$$\Pi^l(d^*_1(\theta), d^*_2(\theta), \theta) \geq \Pi^l(\tilde{d}^*_1(\theta), d^*_2(\theta), \theta)$$

The equilibrium is supported by pessimistic out-of-equilibrium beliefs:

$\mu(\theta_1|d_1 \neq d^*_1(\theta_2)) = 1$. Then, $d^*_1(\theta_1) = \alpha^l + \theta_1$ and the set of $d^*(\theta_2)$ that satisfies the constraint (8) is the set $D$ of separating equilibria. Applied to the set $D$, the intuitive criterion refines all the out-of-equilibrium beliefs: $\mu(\theta_2|d_1 \in D) = 1$. Hence, a rational agent selects the decision $d^*_1(\theta_2)$ that maximizes his profit under constraint (8) i.e selects his preferred decision within $D$. And the only separating equilibrium that survives the intuitive criterion is the least costly separating equilibrium (Riley outcome). Last, note that $\alpha^l + \theta_2 \in D$ if $\Delta \theta^2 \geq \gamma^k)^2$.

Consider any pooling equilibrium where $d^*_1(\theta_1) = d^*_1(\theta_2)$. In state $\theta_2$, the agent is worse-off than when he can signal his type. Then, we can associate
with any pooling equilibrium a decision $\tilde{d}_1$ such that:

(IC1) In state $\theta_1$, the agent prefers the pooling equilibrium to $\tilde{d}_1$, whatever the beliefs associated with $\tilde{d}_1$.

(IC2) In state $\theta_2$, the agent prefers $\tilde{d}_1$ to the pooling equilibrium if the principal is convinced that $\mu(\theta_1|\tilde{d}_1) = 0$.

If in state $\theta_1$ the agent never deviates to $\tilde{d}_1$, the intuitive criterion imposes that the beliefs associated with $\tilde{d}_1$ change to $\mu(\theta_2|\tilde{d}_1) = 1$. Consequently, the agent will quit the pool in state $\theta_2$ and no pooling equilibria will survive the intuitive criterion.

(2) Suppose that agent $k$ does not disclose his private information at period one either because he does not control $d_k^1$ or because he plays a pooling equilibrium. In this case, disclosing the value of $\theta^l$ changes the principal’s beliefs on both $\theta^l$ and $\theta^k$. We must then identify the state $\theta_i$ in which the agent has no incentive to hide his private information (if it exists).

If agent $l$ plays a pooling equilibrium, the second period decisions are:

$$d_k^2 = \alpha^k + \frac{\theta_l + \theta^k}{2} - \gamma, \quad k = A, B.$$

If, in state $\theta_i$, the agent manages to signal his type, the second period decisions change to: $d_l^2 = \alpha^l + \theta_l - \gamma$ and $d_k^2 = \alpha^k + v_{ii}\theta_i + v_{ij}\theta_j - \gamma$, $i, j = 1, 2$. We must show that there exists a state $\theta_i \in \Theta$ in which the agent has would prefer to inform the principal. Replacing the decisions in the profit functions, we can show that such a state always exists. Then, we can use the same reasoning as above to eliminate all the pooling.

If agent $l$ plays a separating equilibrium, the second period decisions are:

$$d_l^2(\theta_i) = \alpha^l + \theta_l - \gamma \quad \text{and} \quad d_k^2 = \alpha^k + v_{ii}\theta_i + v_{ij}\theta_j - \gamma.$$ If, in state $\theta_i$, the agent deviates, he changes the second period decisions to: $d_l^2 = \alpha^l + \theta_j - \gamma$ and $d_k^2 = \alpha^k + v_{ij}\theta_i + v_{ii}\theta_j - \gamma$, $i, j = 1, 2$. We must show that there exists at most one state $\theta_i \in \Theta$ in which such a deviation is profitable. Replacing the decisions in the profit functions, we can show that indeed, deviating cannot be profitable in both states. Hence, we have our standard sorting conditions. We can thus find the set of separating equilibria and apply the same reasoning as above to select the most efficient one.

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References


