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Aggregating Performance Measures in Multi-Task Agencies

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Abstract

It has been argued in the multi-task agency literature that effort distortion can be mitigated by applying several performance measures in incentive contracts. This paper analyzes the efficient aggregation of multiple performance measures aimed at motivating non-distorted effort. It demonstrates that non-distorted effort can be induced by combining a sufficient quantity of informative performance measures. However, this is only optimal if the required aggregation concurrently maximizes the precision of the agent`s performance evaluation. This paper further illustrates how the optimal performance evaluation is affected by the ability of individual agents to perform relevant tasks.

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1. Introduction

In many employment relationships, firms utilize objective performance measures to provide their employees with incentives. Since effort is usually multidimensional, firms must not only induce a sufficient effort intensity, they must also motivate an efficient effort allocation across tasks. However, if available performance measures do not reflect employees' true contributions to firm value, the inclusion in incentive contracts will motivate employees to choose inefficient effort allocations across relevant tasks (Feltham and Xie, 1994).

It has been argued in previous multi-task agency literature that effort distortion can be mitigated by applying several performance measures in incentive contracts. In particular, Feltham and Xie (1994), Datar, Kulp, and Lambert (2001), and Thiele (2007) have demonstrated that utilizing multiple measures for evaluating an agent's performance can improve the efficiency of his effort allocation. Nevertheless, these papers stopped short of identifying the requirements of information systems for inducing non-distorted effort. To close the existing knowledge gap, this paper analyzes the efficient combination of multiple performance measures aimed at motivating non-distorted effort.

This paper provides two important implications. First, inducing non-distorted effort necessitates access to, at the very least, the same quantity of informative performance measures as the number of tasks the agent has to perform. Motivating non-distorted effort however, is only optimal if the required combination of performance measures concurrently maximizes the precision of the agent's performance evaluation. Second, the optimal aggregation of multiple performance measures depends on individual agent's ability to perform relevant tasks. Accordingly, mitigating potential effort distortion provokes different performance evaluations for heterogenous agents even if their jobs are identical.

This paper proceeds as follows. In section 2, I give an overview of the model and derive the first-best effort allocation as a benchmark in section 3. The required aggregation of multiple performance measures to induce non-distorted effort is derived in section 4 and analyzed in section 5. Section 6 concludes.

2. The Model

Consider a single-period agency relationship between a risk-neutral principal and a risk-averse agent. The agent is employed to perform $n \geq 2$ tasks which cannot be split and allocated to different agents. Thus, the agent is in charge of implementing an effort vector $\boldsymbol{e} = (e_1, ..., e_n)^T$, $\boldsymbol{e} \in \mathbb{R}^{n+}$, where e_i denotes the agent's non-verifiable effort allocated to task i.[1] Implementing effort $\boldsymbol{e}$ imposes costs $C(\boldsymbol{e}) = \boldsymbol{e}^T \boldsymbol{\Psi} \boldsymbol{e}/2$, where $\boldsymbol{\Psi}$ is a symmetric and positive definite $n \times n$ matrix representing the agent's marginal effort costs. The agent's preferences are represented by the negative exponential utility function

$$U(w, \boldsymbol{e}) = -\exp\left[-\rho\left(w - C(\boldsymbol{e})\right)\right], \tag{1}$$

where ρ denotes the measure of absolute risk-aversion and w his wage. His reservation utility is $\bar{U}$.

By implementing effort $\boldsymbol{e}$, the agent contributes to the principal's non-verifiable gross payoff $V(\boldsymbol{e}) = \boldsymbol{\mu}^T \boldsymbol{e}$, where $\boldsymbol{\mu} = (\mu_1, ..., \mu_n)^T$, $\boldsymbol{\mu} \in \mathbb{R}^{n+}$, characterizes the marginal effect of $\boldsymbol{e}$ on $V(\boldsymbol{e})$. Since $V(\boldsymbol{e})$ is non-verifiable, it cannot be part of an explicit incentive contract. However, the principal receives an m-dimensional vector of verifiable and additively separable performance measures $\boldsymbol{P}(\boldsymbol{e}) = (P_1(\boldsymbol{e}), ..., P_m(\boldsymbol{e}))^T$, $\boldsymbol{P}(\boldsymbol{e}) \in \mathbb{R}^m$. Let $\boldsymbol{\Xi} = (\boldsymbol{\omega}_1^T, ..., \boldsymbol{\omega}_m^T)^T$ denote the $m \times n$ matrix of the respective performance measure sensitivities $\boldsymbol{\omega}_i = (\omega_{i1}, ..., \omega_{in})^T \in \mathbb{R}^{n+}$, $i \in \{1, m\}$. Thus,

$$\boldsymbol{P}(\boldsymbol{e}) = \boldsymbol{\Xi} \boldsymbol{e} + \boldsymbol{\varepsilon}, \tag{2}$$

[1] All vectors are column vectors where 'T' denotes the transpose.

where $\boldsymbol{\varepsilon} = (\varepsilon_1, ..., \varepsilon_m)^T$, $\boldsymbol{\varepsilon} \in \mathbb{R}^m$, is a normally distributed m-dimensional vector of random variables with zero mean and covariance matrix $\boldsymbol{\Sigma}$. A performance measure $P_i(\boldsymbol{e})$ is referred to be *incongruent*, if there exists no constant $\lambda \neq 0$ satisfying $\boldsymbol{\mu} = \lambda \boldsymbol{\omega}$. Then, its exclusive application in an incentive contract would motivate the agent to implement an inefficient effort allocation across the relevant tasks (Feltham and Xie, 1994; Baker, 2002).

In line with previous multi-task agency literature, I restrict my analysis to a linear compensation scheme w:

$$w(\boldsymbol{e}) = \alpha + \boldsymbol{\beta} P(\boldsymbol{e}), \tag{3}$$

where α denotes the fixed payment. Moreover, $\boldsymbol{\beta} = (\beta_1, ..., \beta_m)^T$, $\boldsymbol{\beta} \in \mathbb{R}^m$, is the vector of incentive parameters representing the weight of each performance measure in the linear aggregation.[2]

3. The First-Best Effort Allocation

As a benchmark for the subsequent analysis, let us first identify the first-best (i.e. non-distorted) effort allocation. Suppose the principal can contract over $\boldsymbol{e}$. In this case, she would choose $\boldsymbol{e}$ aimed at maximizing the difference between the gross payoff $V(\boldsymbol{e})$ and costs $C(\boldsymbol{e})$:

$$\max_{\boldsymbol{e}} \ \Pi(\boldsymbol{e}) = \boldsymbol{\mu}^T \boldsymbol{e} - \frac{1}{2} \boldsymbol{e}^T \boldsymbol{\Psi} \boldsymbol{e}. \tag{4}$$

Accordingly, the first-best effort vector is characterized by

$$\boldsymbol{e}^{fb} = \boldsymbol{\Psi}^{-1} \boldsymbol{\mu}. \tag{5}$$

For the remainder of this paper keep in mind that any implemented (second-best) effort vector $\boldsymbol{e}^*$ characterizes a distorted effort allocation, if there exists no constant $\lambda \neq 0$ satisfying $\boldsymbol{e}^{fb} = \lambda \boldsymbol{e}^*$.

4. Aggregating Performance Measures

If the principal cannot directly contract over $\boldsymbol{e}$, she faces an incentive problem for motivating an appropriate effort intensity and effort allocation across the relevant tasks. Hence, the principal's problem is to design a contract $(\alpha^*, \boldsymbol{\beta}^*)$ that maximizes her expected profit $\Pi = \mathrm{E}[V(\boldsymbol{e}) - w(\boldsymbol{e})]$ while ensuring the agent's participation. The optimal linear contract thus solves

$$\max_{\alpha, \boldsymbol{\beta}, \boldsymbol{e}} \ \Pi \equiv \mathrm{E}[V(\boldsymbol{e}) - w(\boldsymbol{e})] \tag{6}$$

s.t.

$$\boldsymbol{e} = \arg\max_{\tilde{\boldsymbol{e}}} \mathrm{E}\left[U(w, \tilde{\boldsymbol{e}})\right] \tag{7}$$

$$\mathrm{E}\left[U(w, \boldsymbol{e})\right] \geq \bar{U}, \tag{8}$$

where (7) is the agent's incentive, and (8) his participation constraint. Recall that $w(\boldsymbol{e})$ is linear, $U(w, \boldsymbol{e})$ is exponential, and the error term $\boldsymbol{\varepsilon}$ is normally distributed. Consequently, maximizing $\mathrm{E}\left[U(w, \boldsymbol{e})\right]$ is analogous to maximizing the agent's certainty equivalent

$$CE(\boldsymbol{e}) = \alpha + \boldsymbol{\beta}^T \boldsymbol{\Xi} \boldsymbol{e} - \frac{1}{2} \boldsymbol{e}^T \boldsymbol{\Psi} \boldsymbol{e} - \frac{\rho}{2} \boldsymbol{\beta}^T \boldsymbol{\Sigma} \boldsymbol{\beta}, \tag{9}$$

[2] As shown by Banker and Datar (1989), a linear aggregation of performance measures is optimal whenever the noise term is normally distributed.

where $\rho\boldsymbol{\beta}^T\boldsymbol{\Sigma}\boldsymbol{\beta}/2$ describes the agent's risk premium. To maximize his expected utility, the agent chooses $\boldsymbol{e}^* = \boldsymbol{\Psi}^{-1}\boldsymbol{\Xi}^T\boldsymbol{\beta}$. Apparently, if the principal receives at least two performance measures, she can influence the relative effort allocation by adjusting the weights β_i, $i = 1, ..., m$, in the agent's performance evaluation.

Cost minimization requires setting α such that (8) binds. Solving $CE(\boldsymbol{e}) = \bar{U}$ for α and substituting this expression with $\boldsymbol{e}^* = \boldsymbol{\Psi}^{-1}\boldsymbol{\Xi}^T\boldsymbol{\beta}$ in the principal's objective function yield an unconstrained maximization problem:

$$\max_{\boldsymbol{\beta}} \Pi \equiv \boldsymbol{\mu}^T\boldsymbol{\Psi}^{-1}\boldsymbol{\Xi}^T\boldsymbol{\beta} - \frac{1}{2}\boldsymbol{\beta}^T\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\Xi}^T\boldsymbol{\beta} - \frac{\rho}{2}\boldsymbol{\beta}^T\boldsymbol{\Sigma}\boldsymbol{\beta} - \bar{U}. \tag{10}$$

The first-order condition with respect to $\boldsymbol{\beta}$ leads to

$$\boldsymbol{\beta}^* = \left[\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\Xi}^T + \rho\boldsymbol{\Sigma}\right]^{-1}\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\mu}, \tag{11}$$

where $\left[\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\Xi}^T + \rho\boldsymbol{\Sigma}\right]^{-1}$ is the inverse of an $m \times m$ matrix.

We can infer from $\boldsymbol{\beta}^*$ that the objective of aggregating performance measures is to balance three effects: (i) the effort distortion characterized by $\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\mu}$, (ii) the measure-cost efficiency described by $\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\Xi}^T$; and (iii), the precision of the aggregated performance evaluation with the agent's risk aversion, characterized by $\rho\boldsymbol{\Sigma}$. Since these three effects are determined by $\boldsymbol{\Psi}$ and ρ, we can conclude that the optimal aggregation of performance measures is tailored to the agent's specific characteristics. Thus, the employment of heterogenous agents calls for different performance evaluations, inducing diverse effort allocations across the relevant tasks.

5. Inducing the Efficient Effort Allocation

As noted earlier, the principal can influence the agent's effort allocation if she receives at least two performance measures. The next proposition identifies conditions which allow the principal to induce the first-best (i.e. non-distorted) effort allocation.

Proposition 1. *If* $\operatorname{rank}\boldsymbol{\Xi}^T \geq n$*, the principal can aggregate the available performance measures to induce* $\boldsymbol{e}^* = \lambda\boldsymbol{e}^{fb}$, $0 < \lambda \leq 1$*. However, this is only optimal, if and only if,*

$$\hat{\lambda}\,\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\Xi}^T = \rho\boldsymbol{\Sigma}, \qquad \hat{\lambda} = \frac{1-\lambda}{\lambda}. \tag{12}$$

Proof See appendix.

The first condition emphasizes that the principal needs access to an information system generating at least the same quantity of performance measures as number of tasks the agent has to perform.[3] Moreover, their sensitivity vectors $\boldsymbol{\omega}_i$ are required to be linearly independent, i.e. performance measures must differ in their information content with respect to the implemented effort allocation. If these two requirements are satisfied, the principal can combine the performance measures appropriately to induce the first-best effort allocation. However, as the second condition in Proposition 1 highlights, the aggregation of performance measures with the purpose of motivating non-distorted effort is only optimal if the covariance matrix $\boldsymbol{\Sigma}$ is a transformation of the measure-cost efficiency $\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\Xi}^T$. Intuitively, aggregating performance measures to motivate non-distorted effort can only be optimal, if this concurrently maximizes the

[3]Note that this condition is sufficient. For instance, the principal can also induce a first-best effort allocation if one measure is perfectly congruent.

precision of the agent's performance evaluation, and consequently, minimizes his risk premium. To see this, consider the following example with two tasks and two performance measures:

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \quad \boldsymbol{\Psi} = \begin{pmatrix} \psi_1 & 0 \\ 0 & \psi_2 \end{pmatrix}, \quad \boldsymbol{\Xi} = \begin{pmatrix} \omega_{11} & 0 \\ 0 & \omega_{22} \end{pmatrix}, \quad \boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix}.$$

In this example, performance measure $P_1(e_1)$ captures only task 1, whereas task 2 is only measured by $P_2(e_2)$. For simplicity, assume that $\mu_i = \psi_i$, $i = 1, 2$, which implies $\boldsymbol{e}^{fb} = (1, 1)^T$, i.e., the agent would implement the same effort intensity for each task under first-best. Using this example, condition (12) from Proposition 1 simplifies to

$$\hat{\lambda} \begin{pmatrix} \frac{\omega_{11}^2}{\psi_1} & 0 \\ 0 & \frac{\omega_{22}^2}{\psi_2} \end{pmatrix} = \rho \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix}. \tag{13}$$

Let $\sigma_1^2 > \sigma_2^2$, i.e., performance measure $P_2(e_2)$ is more precise than $P_1(e_1)$. Then, for arbitrary $\hat{\lambda}$ and ρ, motivating the first-best effort allocation can only be optimal, if the less precise performance measure $P_1(e_1)$ is associated with a higher measure-cost ratio ω_{11}^2/ψ_1 (i.e., $\omega_{11}^2/\psi_1 > \omega_{22}^2/\psi_2$). Clearly, to induce the first-best effort allocation, the principal can put a lower weight on the less precise performance measure $P_1(e_1)$ (i.e., $\beta_1 < \beta_2$), which in turn maximizes the precision of the aggregated performance measure $\boldsymbol{\beta}^T \boldsymbol{P}(\boldsymbol{e})$, and hence, curbs the risk imposed on the agent.

Finally observe that condition (12) (and for the above example, the simplified condition (13)) is tied to the agent's marginal effort costs parameterized by $\boldsymbol{\Psi}$.[4] Hence, depending on the characteristics of the information system, inducing non-distorted effort can be optimal for a certain type of agent, but inefficient for other types. Consider again the above example with $\mu_i = \psi_i$, $i = 1, 2$. Clearly, for arbitrary performance measure sensitivities ω_{11} and ω_{22}, it can only be optimal to induce the first-best effort allocation, if the parameters ψ_i, $i = 1, 2$, of the agent's marginal effort cost are such that the less precise performance measure $P_1(e_1)$ is associated with a higher measure-cost ratio ω_{11}^2/ψ_1. Otherwise, balancing effort incentives and insurance for the risk-averse agent requires the principal to combine both performance measures differently, which in turn motivates the agent to implement distorted effort.

Put differently, for a given set of available performance measures satisfying $\operatorname{rank} \boldsymbol{\Xi}^T \geq n$ (see Proposition 1), personal characteristics of agents determine whether it is optimal for the principal to motivate the efficient (i.e., non-distorted) effort allocation by combining these measures appropriately. Moreover, not only the respective informativeness of available performance measures, but also individual characteristics of agents, dictate the relative importance of these measures for evaluating agents' individual contributions to firm value as basis for incentive payments.

The previous observations have two important implications. First, the principal has some latitude to improve the efficiency of the induced effort allocation by employing 'suitable' agents for the relevant jobs. The selection criteria, however, are not only determined by the potential contributions of tasks to firm performance (captured by $\boldsymbol{\mu}$), but also by the characteristics of the available information system $\boldsymbol{P}(\boldsymbol{e})$. Second, instead of using standardized contracts, profit maximization requires to tailor incentive contracts to agents' individual characteristics, even if their jobs are identical.

Even though it might be optimal to motivate the first-best effort allocation for a certain type of agent, it is not necessarily optimal to concurrently induce the first-best effort intensity. To see this, recall that non-distortion requires $\boldsymbol{e}^* = \lambda \boldsymbol{e}^{fb}$. Moreover, the agent implements the first-best effort intensity only if $\lambda = 1$. This leads to the next Corollary to Proposition 1.

[4] In contrast, a change of the scalar ρ affects only the parameter $\hat{\lambda}$ without violating (12).

Corollary 1. *Suppose that* $\operatorname{rank} \Xi^T \geq n$*. Then, it is optimal to induce* $\boldsymbol{e}^{fb}$*, if and only if* $\rho = 0$ *or* $\boldsymbol{\Sigma} = [0]_{ij}$*,* $i, j = 1, ..., m$*.*

Besides the conditions emphasized by Proposition 1, inducing the first-best effort allocation and intensity requires that either all performance measures are perfectly precise or the agent is risk-neutral. For single-task agency relationships, it is well known that the latter criteria are sufficient to achieve first-best if the agent is not financially constrained. Multi-task principal-agent relationships however, impose additional requirements on the information system with respect to the quantity and characteristics of contractible performance measures. In particular, only if available measures can be combined such that the agent's performance evaluation reflects his true contribution to firm value, non-distorted effort can be induced.

6. Conclusion

The application of performance measures in incentive contracts can motivate employees to implement inefficient effort allocations if their performance evaluations do not perfectly reflect their true contributions to firm value. This paper analyzes the aggregation of multiple performance measures as a means of motivating a non-distorted effort allocation across relevant tasks. Two important observations are noted. First, to induce non-distorted effort, the principal depends on a sufficient quantity of informative performance measures. However, motivating non-distorted effort is only optimal if the required aggregation of performance measures concurrently maximizes the precision of agents' performance evaluations. Second, the optimal aggregation of multiple performance measures is tied to individual agent's ability to carry out relevant tasks. Therefore, the intention to mitigate effort distortion can explain why the performance of heterogeneous agents are evaluated differently even if their jobs are identical.

Appendix

Proof of Proposition 1.
The agent implements the first-best effort allocation, if $\boldsymbol{e}^* = \lambda \boldsymbol{e}^{fb}$. Note that $0 < \lambda \leq 1$ since it cannot be optimal to induce a higher effort intensity under second-best than under first-best. Therefore, $\boldsymbol{\beta}$ needs to solve $\boldsymbol{\Psi}^{-1}\Xi^T\boldsymbol{\beta} = \lambda\boldsymbol{\Psi}^{-1}\boldsymbol{\mu}$, which is equivalent to $\Xi^T\boldsymbol{\beta} = \lambda\boldsymbol{\mu}$. If $\operatorname{rank} \Xi^T \geq n$, there exists at least one solution to this equation system. Particularly, h columns in Ξ^T, $n \leq h \leq m$, must be linearly independent.
Inducing the first-best effort allocation is only optimal if $\boldsymbol{e}(\boldsymbol{\beta}^*) = \lambda\boldsymbol{e}^{fb}$. This requires that $\Xi^T\boldsymbol{\beta}^* = \lambda\boldsymbol{\mu}$, or equivalently, $\boldsymbol{\beta}^* = \lambda \left[\Xi^T\right]^{-1}\boldsymbol{\mu}$. Thus, $\boldsymbol{e}(\boldsymbol{\beta}^*) = \lambda\boldsymbol{e}^{fb}$ is equivalent to

$$\left[\Xi\boldsymbol{\Psi}^{-1}\Xi^T + \rho\boldsymbol{\Sigma}\right]^{-1}\Xi\boldsymbol{\Psi}^{-1}\boldsymbol{\mu} = \lambda\left[\Xi^T\right]^{-1}\boldsymbol{\mu}, \tag{14}$$

which can be transformed to

$$\frac{1-\lambda}{\lambda}\ \Xi\boldsymbol{\Psi}^{-1}\Xi^T = \rho\boldsymbol{\Sigma}. \tag{15}$$

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