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Partial vaccination programs and the eradication of infectious diseases

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Abstract

We incorporate the possibility of eradicating infectious diseases through partial vaccinations into a static model where each individual incurs a cost, which differs among the individuals, when he/she is administered a vaccine. We show that if an infectious disease is eradicated by vaccinating up to the individual whose vaccination cost is lower than a threshold, then the partial compulsory vaccination program is strictly preferable to the free choice vaccination program in terms of social welfare otherwise, the free choice vaccination program is strictly preferable to the partial compulsory vaccination program. Our result is a generalization of Brito, Sheshinski, and Intriligator (1991).

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1 Introduction

Following the eradication of smallpox by mass vaccination in 1980, public health agencies are implementing global vaccination programs against other infectious diseases such as polio, measles, and tuberculosis. Brito, Sheshinski, and Intriligator (1991) modeled the problem faced by the public health agencies as a static model in which each individual incurs a cost, which differs among individuals, when he/she is administered a vaccine, that provides perfect immunity against the disease. They obtained a result to the effect that the full compulsory vaccination, which requires that all individuals should be vaccinated, is strictly dominated by free choice vaccination in terms of social welfare.[1] Ever since Brito, Sheshinski, and Intriligator (1991), a large number of detailed studies have been conducted on the economic analysis of vaccination against infectious diseases (see, for example, Francis (1997), Barrett (2003), Kremer (1996), and Geoffard and Philipson (1997)).

Brito, Sheshinski, and Intriligator (1991) did not incorporate an epidemiologically important factor known as herd immunity: the entire population does not have to be immunized in order to eradicate the infectious disease. That is, the presence of a sufficient number of immune individuals in a population leads to the eradication of the disease. Anderson and May (1991) explains that if some individuals in the population were already immune due to vaccination, then the spread of the infection is interrupted when the infected individuals contact an immune rather than a susceptible individual. Therefore, a number of susceptibles will be indirectly protected from infection simply because some others are immune. In fact, Anderson and May (1991) state that the fact that for smallpox "a vaccination coverage around 70-80 percent is sufficient for the eventual eradication ... may help explain the success of global eradication program." Actually, as shown in Table 1 according to Anderson and May (1991), the estimated overall levels of vaccination coverage required for eradication are strictly less than 100 percent.

We incorporate the possibility of eradication through not full but partial vaccination coverage into the model of Brito, Sheshinski, and Intriligator (1991). Then, we extend the compulsory vaccination program to deal with the eradication through partial vaccination coverage, which compels some of individuals to be vaccinated up to the eradication of the infectious disease. Then, we obtain a result that there is a threshold level of the vaccination cost: if the infectious disease can be eradicated by vaccinating up to the individual whose vaccination cost is lower than the threshold, then the partial compulsory vaccination program is strictly preferable to the free choice vaccination program in terms of social welfare, whereas if the infectious disease cannot be eradicated by vaccinating up to the threshold, then the free choice vaccination program is strictly preferable to the partial compulsory vaccination program.

The contribution of this paper is as follows. First, it provides the public health agencies a criterion for judging whether vaccination against an infectious disease should be compulsory or laissez-faire: if epidemiological researches indicate that the infectious disease can be eradicated with vaccination coverage less than the threshold, then the agencies should undertake compulsory vaccination; otherwise, the vaccination decision should be devolved to each individual. The result of this paper means that it is not always the case that the free choice vaccination program is preferable to the compulsory vaccination program without any reservation, as Brito, Sheshinski, and Intriligator (1991).

Second, the result of this paper is a generalization of Brito, Sheshinski, and Intriligator (1991). That is, in the extreme case that the infectious disease can be eradicated only if all

[1] They also show that both vaccination programs are not Pareto optimal.

individuals are vaccinated, our setting is reduced to theirs. Hence, their result can be interpreted as a corollary of ours.

This paper is organized as follows. Section 2 develops the epidemiological model that describes the manner in which eradication is achieved. Section 3 describes the economic model and introduces two vaccination programs. Section 4 presents our proposition. Section 5 concludes the paper.

2 Partial Vaccination

The basic reproductive rate R_0 is the central concept of mathematical epidemiology. Following Anderson and May (1991), R_0 is defined as the average number of secondary infections produced when one infected individual is introduced into a population comprising totally susceptible individuals. Since all individuals are susceptible, the infectious disease in this population would be transmitted to R_0 susceptibles per each infected individual. For the disease to spread, it is required that $R_0 > 1$.

However, in the presence of vaccination programs, the populations are not "totally susceptible"; some individuals are immunized by vaccination. We assume that the vaccination is completely effective. Let $\pi \in [0, 1]$ be the ratio of the vaccinated individuals to the population. The infected individual does not transmit the infectious disease to immunized individuals Therefore, the "actual" average number of secondary infections produced when one infected individual is introduced into the population, referred to as the effective reproductive rate R, is given by discounting the basic reproductive rate R_0 by the ratio of the unvaccinated individuals to the population $1 - \pi$:

$$R = (1 - \pi) \cdot R_0. \tag{1}$$

R indicates that a single infected individual actually transmits the disease to only R individuals and not to R_0 individuals. Equation (1) provides us an important implication: we are able to eradicate the infectious disease if we achieve a sufficiently high π that reduces the effective reproductive rate R below unity. π_e is the critical ratio of vaccinated individuals required to achieve eradication; briefly, it is *the eradication ratio*,

$$(1 - \pi_e) \cdot R_0 = 1, \text{ or } \pi_e = 1 - \frac{1}{R_0}. \tag{2}$$

If π is higher than π_e, then we have $R < 1$ from equation (1), which implies that there will be less than one secondary infection per infected individual, as a result of which the infectious disease will be eradicated. This implies that it is not necessary to vaccinate every individual in order to eradicate the disease, that is, $\pi_e < 1$.

For example, if the basic reproductive rate of an infectious disease is five, then we have

$$\pi_e = 1 - \frac{1}{R_0} = 1 - \frac{1}{5} = 0.8. \tag{3}$$

Hence, only 80 percent of the population needs to be immunized in order to eradicate the infectious disease. Since the value of R_0 varies with the infectious disease, π_e also varies.

p is the probability of a given unvaccinated individual becoming infected; briefly, it is *the probability of being infected.* As assumed by Anderson and May (1991), we have

$$p = \begin{cases} \mu \cdot R_0 \cdot (\pi_e - \pi) & \text{for all } \pi < \pi_e, \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where μ is a parameter.[2] As long as $\pi < \pi_e$, the value of p is strictly decreasing in π. Once the vaccination coverage reaches the eradication ratio π_e, the probability of being infected is eliminated.

3 The Model

Our model is based on that of Brito, Sheshinski, and Intriligator (1991). The primary difference is that the utility of each individual does not depend on his/her income in this paper because we do not treat income distribution aspects. We use their notation to the maximum extent.

We consider an economy with a continuum of individuals whose total number is normalized to unity. u is the utility of a healthy individual, and $\underline{u}$ is that of an infected individual. We assume that $u > \underline{u}$. Each individual is identical except for his/her vaccination cost $\theta \in [0, \bar{\theta}]$, where $\bar{\theta} > 0$. Brito, Sheshinski, and Intriligator (1991) interpret the vaccination cost θ as an individual characteristic that is relevant only if the individual is vaccinated: the different individuals face different prices of being vaccinated or different probabilities of suffering side effects, which is measured in terms of time or pain. The vaccination cost θ is distributed from the cumulative distribution function $F(\theta)$, where $F(0) = 0$ and $F(\bar{\theta}) = 1$. The cumulative distribution function F has the corresponding density function $f(\theta) > 0$. Note that $\int_0^\theta f(z)dz = F(\theta)$.

In Section 2, we assumed that vaccination completely immunizes the recipient individual against the infectious disease. Hence, when an individual with vaccination cost θ is vaccinated, his/her net utility is $u - \theta$. On the other hand, if the individual is not vaccinated, then he/she will be infected with the infectious disease with a probability of being infected $p(\pi)$, which is obtained in Section 2. Hence, his/her expected utility of being unvaccinated is given by

$$v(\pi) \equiv [1 - p(\pi)]u + p(\pi)\underline{u}. \tag{5}$$

The expected utility of being unvaccinated, $v(\pi)$, exhibits

$$\frac{\partial v}{\partial \pi} = (\underline{u} - u)\frac{\partial p}{\partial \pi} \geq 0, \tag{6}$$

since $u > \underline{u}$ and $\frac{\partial p}{\partial \pi} \leq 0$. Note that for any value of π higher than eradication ratio π_e, the expected utility of being unvaccinated is equal to the utility of the healthy individual, i.e., $v(\pi) = u$, since $p(\pi) = 0$ for any $\pi > \pi_e$.

3.1 Vaccination Programs

This study focuses on the following two vaccination programs.

The first vaccination program is referred to as *the free choice vaccination program*, wherein all individuals are free to choose whether or not to be vaccinated. We assume that π is exogenously given for all individuals. An individual with vaccination cost $\theta \in [0, \bar{\theta}]$ decides to be vaccinated if and only if the utility of being vaccinated exceeds the expected utility of being unvaccinated:

$$u - \theta > v(\pi). \tag{7}$$

[2] In Anderson and May (1991), μ represents the natural mortality rate. However, it can be eliminated since it plays no role in this paper.

Brito, Sheshinski, and Intriligator (1991) found that the free choice vaccination program possesses a unique interior equilibrium: there exists a unique individual whose vaccination cost is $\theta^* \in (0, \bar{\theta})$ such that any individual with a vaccination cost less than θ^* chooses to be vaccinated, whereas any individual with a vaccination cost more than θ^* chooses to be unvaccinated, and that the ratio of the vaccinated individuals to the population π is consistent with each individual's vaccination decision. We refer to θ^* as *the free choice solution.* The free choice solution θ^* is characterized by the equation

$$u - \theta^* = v(\pi(\theta^*)). \tag{8}$$

where $\pi(\theta) = F(\theta)$ — the ratio of the individuals whose vaccination cost is equal to or strictly less than θ. We assume that the infectious disease is not eradicated under the free choice vaccination program, or equivalently, $\pi(\theta^*) < \pi_e$. This contribution of Brito, Sheshinski, and Intriligator (1991) is summarized as follows.

Result 1. (Brito, Sheshinski, and Intriligator (1991)). *There exists a unique interior solution $\theta^* \in (0, \bar{\theta})$ where the ratio of the vaccinated individuals to the population, $\pi(\theta)$, is consistent with each individual's vaccination decision, that is,*

$$u - \theta^* = v(\pi(\theta^*)). \tag{9}$$

Proof of Result 1 is in Appendix.

The second vaccination program is referred to as *the partial compulsory vaccination program*, which compels π_e ratio of individuals to be vaccinated in the order of lowest vaccination cost. The implementation of this vaccination program leads to the eradication of the disease. Under the partial compulsory vaccination program, any individual whose vaccination cost is less than θ_e is compelled to be vaccinated, while the rest are not. Here, θ_e satisfies

$$\pi(\theta_e) = \int_0^{\theta_e} f(z)dz = \pi_e. \tag{10}$$

We refer to θ_e as *the partial compulsory solution.* Since $\pi_e < 1$, we have $\theta_e < \bar{\theta}$. Moreover, θ_e is an exogenous parameter since the eradication ratio π_e is exogenously given. Note that this program provides us the lowest total vaccination costs for vaccinating π_e ratio of individuals.

4 Comparison of the Vaccination Programs

Brito, Sheshinski, and Intriligator (1991) showed that the free choice vaccination program does not attain the maximum of social welfare. Hence, it is inevitable to ascertain which of the two vaccination programs is more desirable. We define social welfare in terms of utilitarianism. Let $W(\theta^*)$ be the social welfare under the free choice vaccination program with θ^*. Under this program, any individual whose vaccination cost is lower than θ^* is vaccinated, while the rest are unvaccinated. Moreover, the infectious disease is not eradicated. Hence, $W(\theta^*)$ is given by

$$W(\theta^*) \equiv \int_0^{\theta^*} [u - z]dF(z) + v(\pi(\theta^*)) \cdot (1 - F(\theta^*)). \tag{11}$$

On the other hand, under the partial compulsory vaccination program with $\theta_e \in (0, \bar{\theta})$, any individual whose vaccination cost is lower than θ_e is vaccinated, while the rest are unvaccinated. In addition, the infectious disease is eradicated. Let $\bar{W}(\theta_e)$ be the social welfare under the partial compulsory vaccination program:

$$\bar{W}(\theta_e) \equiv \int_0^{\theta_e} [u - z] dF(z) + u \cdot (1 - F(\theta_e)). \tag{12}$$

Thus, we have the following proposition.

Proposition 1. *There is a unique interior threshold level of the vaccination cost $\hat{\theta} \in (\theta^*, \bar{\theta})$ such that (i) if the partial compulsory solution θ_e is above the threshold $\hat{\theta}$, then the free choice solution is strictly preferable to the partial compulsory solution, and (ii) if the partial compulsory solution θ_e is below the threshold $\hat{\theta}$, then the partial compulsory solution is preferable to the free choice solution. That is, there exists $\hat{\theta} \in (\theta^*, \bar{\theta})$ such that*
(i) *for any $\theta_e > \hat{\theta}$, we have $W(\theta^*) > \bar{W}(\theta_e)$,*
(ii) *for any $\theta_e < \hat{\theta}$, we have $W(\theta^*) < \bar{W}(\theta_e)$.*

Proof of Proposition 1. Consider the level of social welfare at the partial compulsory solution with $\theta_e = \bar{\theta}$:

$$\bar{W}(\bar{\theta}) = \int_0^{\bar{\theta}} [u - z] dF(z), \tag{13}$$

which is strictly lower than the social welfare at the free choice solution since

$$\begin{aligned} W(\theta^*) - \bar{W}(\bar{\theta}) &= \int_0^{\theta^*} [u - z] dF(z) + \int_{\theta^*}^{\bar{\theta}} v(\pi(\theta^*)) dF(z) - \int_0^{\bar{\theta}} [u - z] dF(z) \\ &= \int_{\theta^*}^{\bar{\theta}} [v(\pi(\theta^*)) - u + z] dF(z) > 0. \end{aligned} \tag{14}$$

The last inequality follows from $v(\pi(\theta^*)) - u + \theta > 0$ for all $\theta > \theta^*$ and $\theta^* < \bar{\theta}$.

Further, consider the social welfare at the partial compulsory solution with $\theta_e = \theta^*$:

$$\bar{W}(\theta^*) = \int_0^{\theta^*} [u - z] dF(z) + \int_{\theta^*}^{\bar{\theta}} u dF(z), \tag{15}$$

which is strictly higher than the social welfare at the free choice solution since

$$\begin{aligned} W(\theta^*) - \bar{W}(\theta^*) &= \int_0^{\theta^*} [u - z] dF(z) + \int_{\theta^*}^{\bar{\theta}} v(\pi(\theta^*)) dF(z) \\ &\quad - \left(\int_0^{\theta^*} [u - z] dF(z) + \int_{\theta^*}^{\bar{\theta}} u dF(z) \right) \\ &= \int_{\theta^*}^{\bar{\theta}} [v(\pi(\theta^*)) - u] dF(z) < 0. \end{aligned} \tag{16}$$

The last inequality follows from the fact that $v(\pi(\theta^*))$ is a convex combination of u and $\underline{u}$, and $u > \underline{u}$. Moreover, it easily follows that

$$\frac{\partial \bar{W}}{\partial \theta_e} = -u \theta_e f(\theta_e) < 0, \tag{17}$$

which ensures that there exists a unique interior $\hat{\theta} \in (\theta^*, \bar{\theta})$ satisfying

$$\bar{W}(\hat{\theta}) = W(\theta^*). \tag{18}$$

□

This proposition indicates that there is a threshold level of the vaccination cost $\hat{\theta} \in (\theta^*, \bar{\theta})$ such that if the infectious disease cannot be eradicated by vaccinating up to the individual whose vaccination cost is the threshold level $\hat{\theta}$, then the free choice vaccination program is preferable to the partial compulsory vaccination program; whereas if the infectious disease can be eradicated by vaccinating up to the individual whose vaccination cost is less than the threshold, the partial compulsory vaccination program is preferable to the free choice vaccination program. Note that the threshold $\hat{\theta}$ is strictly more than θ^* and strictly less than $\bar{\theta}$.

Figure 1 illustrates the determination of the threshold level $\hat{\theta}$. The vaccination costs are measured from the left edge of the graph. The net utility of being vaccinated is given by the downward sloping curve labeled $u - \theta$. The curve representing the expected utility of being unvaccinated is upward sloping until it reaches u at $\theta = \theta_e$. Subsequently, it is constant at u since there is no risk of infection for any $\theta \geq \theta_e$. The free choice solution θ^* is given at the point at which both the curves intersect. The social welfare under the free choice vaccination program consists of both the sum of the utility of the vaccinated individuals, given by the area under the graph $u - \theta$ from 0 to θ^*, and that of the unvaccinated individuals, given by the area $(ab\bar{\theta}\theta^*)$. On the other hand, the social welfare under the partial compulsory vaccination program with θ_e consists of both the sum of utility of the vaccinated individuals, given by the area under the graph $u - \theta$ from 0 to θ_e, and that of the unvaccinated individuals, given by the area $(cd\bar{\theta}\theta_e)$. A comparison of the social welfares from both the vaccination programs is reduced to a comparison of the lightly shaded triangular area and the darker rectangular area. If the former is larger than the latter, then the free choice vaccination program is preferable to the partial compulsory vaccination program; otherwise, the partial compulsory vaccination program is preferable to the free choice vaccination program. The threshold level of the vaccination cost $\hat{\theta}$ is obtained as the vaccination cost which equates the lightly shaded triangular area to the darker rectangular area.

The main feature of this Proposition is that when the Pareto optimal vaccination level is infeasible, it provides a criterion that helps the public health agencies to determine which vaccination program should be implemented — the free choice vaccination program or the partial compulsory vaccination program. If the infectious disease can be eradicated by vaccinating up to the individuals whose vaccination cost is lower than the threshold $\hat{\theta}$, then the agencies should select the partial compulsory vaccination program; otherwise, they should select the free choice vaccination program. This criterion is useful especially in the case that the Pareto optimal vaccination level is infeasible.

Another feature is that we generalize the result of Brito, Sheshinski, and Intriligator (1991). In order to observe this, we consider the case where they make the following assumption regarding the infectious disease: the infectious disease is eradicated only if "all" the individuals in the population are vaccinated, that is, $\pi_e = 1$, or equivalently, $\theta_e = \bar{\theta}$. In this case, the partial compulsory vaccination program is reduced to the "full" compulsory vaccination program since not partial but full segment of individuals has to be vaccinated to eradicate the disease. Since the threshold level $\hat{\theta}$ is strictly less than the maximum vaccination cost $\bar{\theta}$ from Proposition 1, the free choice solution dominates the "full" compulsory solution in terms of social welfare. In other words, particularly in the case of $\theta_e = \bar{\theta}$, the public health agencies should select the free choice vaccination program rather than the "full" compulsory vaccination program. This result is same to that obtained by Brito, Sheshinski, and Intriligator (1991). Therefore, their result can be interpreted as a corollary of ours.

Corollary. (Brito, Sheshinski, and Intriligator (1991)). *If all individuals have to be vaccinated to eradicate the infectious disease, i.e.,* $\theta_e = \bar{\theta}$*, then the free choice vaccination program strictly dominates the full compulsory vaccination program, i.e.,* $W(\theta^*) > \bar{W}(\bar{\theta})$.

5 Conclusion

We have shown that if an infectious disease can be eradicated by vaccinating the ratio of the population whose vaccination cost is less than a threshold, then the public health agencies should vaccinate compulsorily up to the eradication of the infectious disease; otherwise, they should provide each individual with the option to be vaccinated or not. This result hinges on the incorporation of the epidemiological fact that the infectious disease can be eradicated by covering only a segment of the population under the vaccination program. Therefore, studies that ignore eradication by partial vaccination when estimating the impact of eradication by compulsory vaccination tend to underestimate the social welfare resulting from compulsory vaccination.

This is different to the result of Brito, Sheshinski, and Intriligator (1991), suggesting that the advantage of the compulsory vaccination program is independent of the characteristics of the infectious diseases. This independent property of the compulsory vaccination programs is attractive for public health agencies that desire to implement vaccination programs because it minimizes the information requirement. In fact, this independent property only holds under a set of ideal conditions. When the independent property does not hold, further information is required for implementing the vaccination program.

Finally, we mention an assumption on the partial vaccination program. The partial compulsory vaccination program used in this paper compels individuals to be vaccinated in the order of lowest vaccination cost. This implies that we implicitly assume that the public health agencies possess information on the vaccination cost of each individual. However, if they do not possess this information, then the individuals to be vaccinated are decided randomly, which decreases the desirability of eradication by the partial compulsory vaccination.

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Appendix

Proof of Result 1. (Brito, Sheshinski, and Intriligator (1991)). For any given $\theta \in [0, \bar{\theta}]$, let $\Psi(\theta)$ be the difference between the utility of being vaccinated and the expected utility of being unvaccinated:

$$\Psi(\theta) \equiv u - \theta - v(\pi(\theta)). \tag{19}$$

Since $\frac{\partial v}{\partial \pi} \geq 0$, it follows that

$$\frac{\partial \Psi}{\partial \theta} = -1 - \frac{\partial v}{\partial \pi} f < 0. \tag{20}$$

A further assumption that when all the individuals are vaccinated, it is not worthwhile to be vaccinated for an individual whose vaccination cost is extremely high: at the other extreme, when no one is vaccinated, it is worthwhile to be vaccinated for an individual whose vaccination cost is zero,

$$u - \bar{\theta} < v(1) \text{ and } u > v(0), \tag{21}$$

leads to the existence of a unique interior solution $\theta^* \in (0, \bar{\theta})$, which satisfies

$$\Psi(\theta^*) = u - \theta^* - v(\pi(\theta^*)) = 0. \tag{22}$$

□

Table 1. Approximate estimates of the vaccination coverage (the degree of herd immunity) required to eradicate a variety of viral, bacterial, and protozoan infections in developed and developing countries.

Infectious disease	Critical proportions of the population to be immunized for eradication
Malaria (*P. falciparum* in a hyperendemic region)	99%
Measles	90-95%
Whooping cough (pertussis)	90-95%
Fifths disease (human parvovirus infection)	90-95%
Chicken pox	85-90%
Mumps	85-90%
Rubella	82-87%
Poliomyelitis	82-87%
Diphtheria	82-87%
Scarlet fever	82-87%
Smallpox	70-80%

Source: Anderson and May (1991), Table 5.1.

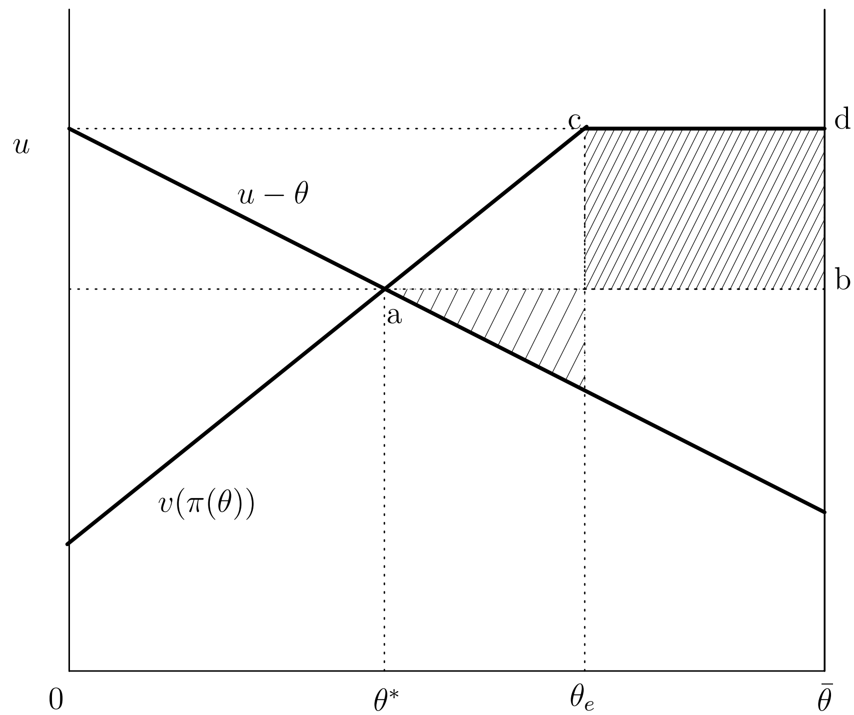

Figure 1.