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Persistence of income shocks and consumption inequality: A case in Japan

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Abstract

The present study investigate the relationship between the persistence of uninsurable income shocks and consumption inequality over a household's life cycle in Japan. Using a life cycle model with incomplete asset markets and calibrated parameters for the Japanese economy, we quantitatively show that moderate persistence of shocks generate a nonlinear consumption inequality profile over the life cycle. The moderate persistence of shocks well replicates the pattern of consumption inequality in Japan.

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1. Introduction

The relationship between uninsurable idiosyncratic income risks and consumption inequality is the key to understanding the consumption-saving behavior of households over the life cycle. As Deaton and Paxson (1994) and Ohtake and Saito (1998) showed, the income and consumption inequality profile takes different shapes in several countries. Figure 1 plots the income and consumption inequality, which is measured by the variance of logarithms, in Japan. We mention two noteworthy features of income and consumption inequality in Japan. First, both the income and consumption inequality increases by age, and they rise sharply after the late 40s. Based on similar observations in the U.S., Storesletten *et al.* (2004a,b) and Blundell and Preston (1998) argue that the persistence of income shock must be very high. The assumption behind their argument is as follows: When the idiosyncratic income risk is insurable in asset markets, consumption inequality does not rise by age. Because the consumption inequality increases with age, the income risks are not perfectly shared through precautionary saving. If the persistence of shocks is strong, it is more difficult to share the risk through savings. Second, the consumption inequality of under-40 households seems to be flat or decreasing. This point is a characteristic feature of the Japanese economy. For example, consumption inequality monotonically rises over the life cycle in the U.S. A natural question we address in this paper is the following: Is it possible to explain the *decreasing* consumption inequality based on a standard life cycle model with *increasing* income inequality? The answer is yes if we allow moderate persistence of labor income shock, which contradicts the view of very persistent labor income shocks in the literature.

There is a conflicting view for the persistence of labor income shocks. One view is that households are subject to very large and persistent labor income shock (MaCurdy 1982, Blundell and Preston 1998, Blundell, *et al.* 2008). If the labor income shock is very persistent, it is difficult to share the income risk with incomplete asset markets through precautionary saving. As a result, consumption inequality rises as the labor income risks revealed.[1] On the contrary, Lillard and Weiss (1979) emphasize that a household have heterogeneous income profile with a moderate persistence of income shock. Recent work by Guvenen (2009) uncovered that consumption inequality in the U.S. is well explained by a heterogeneous income profile with a relatively low income shock persistence and Bayesian learning of households's income risks. We show that consumption inequality in Japan can be explained by moderate income shock persistence.

In this paper, we demonstrate that the persistence of income shock characterizes the shape of consumption inequality using a standard life cycle model similar to that used by Kaplan and Violante (2009). In particular, we suggest that it is

[1] Based on this view, Storesletten et al. (2004b) showed that when the labor income process follows a random walk (a very persistent shock) with iid shocks, the corresponding cross-sectional variance of the logarithm of consumption by age becomes concave in shape. Their model well replicates consumption inequality in the U.S. after adjusting for social security benefits.

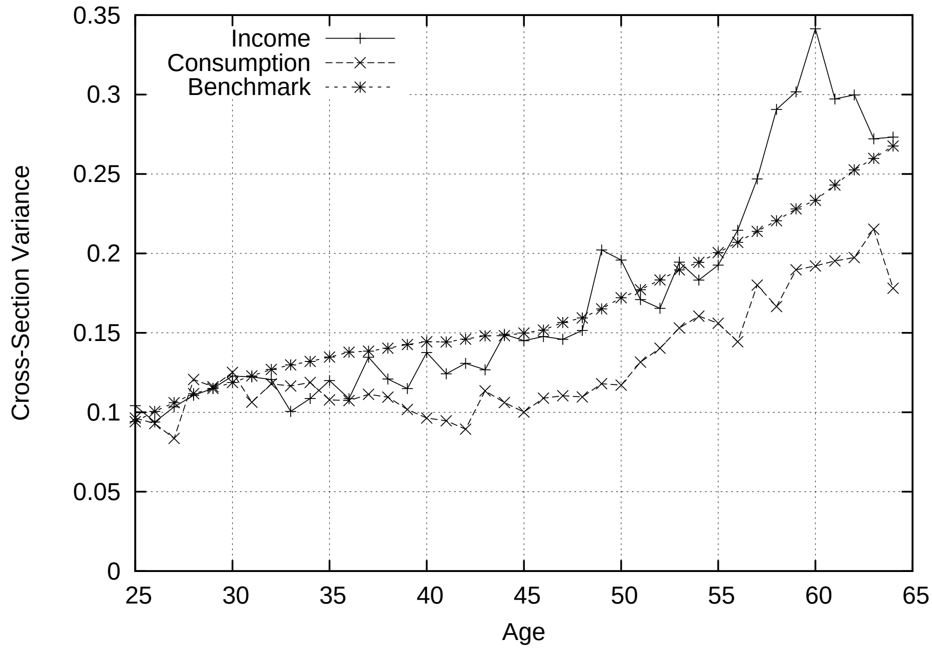

Figure 1: Income and Consumption Inequality over the Life Cycle in Japan

possible to explain the differences in the shape of consumption inequality between the U.S. and Japan simply by the persistence of income shocks. Using a quantitative model with calibrated parameters for the Japanese economy, we show that the consumption inequality profile changes significantly if the persistence parameter is changed incrementally. When the income shock is very persistent, the corresponding consumption inequality profile is weakly concave the over life cycle as shown in Storesletten *et al.* (2004b).When we decrease the persistency parameter, however, the shape of the consumption inequality profile becomes nonlinear and decrease when households are young. We find that the characteristics of the consumption inequality profile in Japan can be replicated by using a relatively low persistence parameter.

The paper is organized as follows; In Section 2, we set up a simple life cycle model. In Section 3, we calibrate the parameters of the model to match the Japanese economy. In Section 4, we discuss quantitative results, and in Section 5 we conclude.

2. A Life Cycle Model

We consider a partial equilibrium life cycle model. A household lives at most T periods, and faces mortality risks. Households of age t earn labor income that is stochastic when they are workers, and retire before age T_r. After retirement, households receive a public pension.

At the beginning of age t, a household i has some asset a_t^i and faces a liquidity constraint, i.e., $a_t^i \geq 0$. We assume that households enter the economy with zero asset, i.e., $a_0 = 0$. The asset yields interest r that is fixed throughout its life cycle. The budget constraint is

$$c_t^i + \zeta_t a_{t+1}^i = (1+r)a_t^i + y_t^i, \tag{1}$$

where y_t is labor income, c_t is consumption and ζ_t is an actuarially fair price of a private annuity. We assume that there is a private annuity market for sharing the mortality risk. When private financial intermediaries offer insurance for the survival risk, the actuarially fair price of the insurance is the survival probability of households ζ_t at age t.[2]

All households face uninsurable idiosyncratic income risks. We assume that asset markets are incomplete and households share the risks via precautionary savings. The logarithm of the labor income y_t^i of household i is the product of a deterministic component κ_t and stochastic components $(\alpha^i, z_t^i, \varepsilon_t^i)$ as follows:

$$\ln y_t^i = \kappa_t + \alpha^i + z_t^i + \varepsilon_t^i, \text{ if } t < T_r. \tag{2}$$

All households have the same deterministic age-efficiency profile $\{\kappa_t\}$. Thus, the average labor income profile becomes an inverse U-shape, although the labor income of each household fluctuates over time. The income risk at age t can be decomposed into three factors: (1) a fixed effect α^i, (2) a persistent component z_t^i, and (3) a transitory shock ε_t^i. We assume that the transitory shock is independent over time and across households, and the fixed effect is independent across households, both of which follows the log-normal distribution: $\varepsilon \sim \mathcal{N}(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2)$ and $\alpha \sim \mathcal{N}(-\sigma_\alpha^2/2, \sigma_\alpha^2)$. The persistent component is determined from the previous period's persistent component z_{t-1}^i, a persistent shock η_t^i, and a persistence parameter ρ, as follows:

$$z_t^i = \rho z_{t-1}^i + \eta_t^i, \ \eta_t \sim \mathcal{N}(-\sigma_{\eta t}^2/2, \sigma_{\eta t}^2), \tag{3}$$

where z_0 is assumed to be one. Note that the variance of the persistent shock is age-dependent.

After retirement, households receive a public pension. For simplicity, we assume that the benefit is the same for all households, which is a constant fraction of the average labor income $\bar{y}$ such that

$$y_t^i = \varphi \bar{y}, \text{ if } t \geq T_r, \tag{4}$$

where φ is the replacement rate.

Therefore, the Bellman equation for a household is as follows:

$$V_t(a,z) = \max \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} + \zeta_t \beta V_t(a', z') \right\}$$
$$\text{subject to (1), (2), (3), and (4).}$$

[2]See, for example, Hansen and İmrohoroğlu (2008)

where $\beta > 0$ is a discount factor, ζ_t is a survival probability from age t to $t+1$, and $V_t(\cdot,\cdot)$ is the value function of age t. We assume that the instantaneous utility function is the CRRA type with γ being the risk aversion parameter.

3. Calibration

We calibrate parameters of the model to match the Japanese economy. A household enters the economy at age 20, retires before 65, and lives at most to age 100. For our purposes, the crucial parameters for the calibration are the persistence ρ and the variances of idiosyncratic income risks.

We choose the income risk parameters to match the variance of the logarithm of the income profile in Japan. As Ohtake and Saito (1998) showed, the cross-sectional variance in the logarithms of income and consumption by age in Japan is convex. We plot the variance of log-income profiles as *Income* in Figure 1. Abe and Yamada (2009) estimate the labor income process from the variances of log-income and log-consumption profiles. They show that the income shock is very persistent, which is consistent with previous research in the U.S. such as Storesletten *et al.* (2004a,b), and in fact it is difficult to reject the possibility of $\rho > 1$.[3] However, it is hard to interpret the case of $\rho > 1$. To replicate the income profiles in our quantitative model, following Abe and Yamada (2009), we assume heteroskedastic innovation, i.e., that the standard deviation of the persistent shock η_t are age-dependent and increases over age: $\sigma_{\eta_{t+1}} = (1+\lambda)\sigma_{\eta_t}$. They find that the standard deviation σ_{η_t} grows after age 47 with a growth rate of 20%, i.e., $\lambda = 0.2$. We approximate equation (3) by Tauchen's (1986) method.[4] The transitory shocks are taken from Abe and Yamada (2009); $\sigma_\varepsilon = 0.218$. We omit the fixed effect because it moves the variances of income and consumption parallel and does not change our results qualitatively.

The autocorrelation coefficient ρ is the choice variable for our purposes. Storesletten *et al.* (2004a,b) estimated the persistence parameter and found that ρ ranges from 0.938 to 0.963, and do not rejected the possibility of $\rho = 1$. We set the persistence parameter to $\rho = 0.95$ and $\sigma_{\eta_0} = 0.1$, which replicates the actual data in Japan as *Benchmark* in Figure 1. Because the persistence parameter changes the distribution of z_t, high persistence needs a wide grid of z_t to approximate precisely. We change the standard deviation σ_{η_0} with ρ. The relationship between ρ and σ_{η_0} is summarized in the lower row of Table I. The age-efficiency profile $\{\kappa_t\}$ is also taken from the parameters used in Abe and Yamada (2009).

Figure 2 plots the simulated variance of log-income profile for each ρ. The income inequality profile in the model well replicate the actual inequality in Japan,

[3]Abe and Yamada (2009), in fact, show that a standard life cycle model can explain the consumption inequality with plausible preference parameters. However, they fail to explain decreasing consumption inequality of young households, which is this paper's main purpose.

[4]We discretized z_t by 30 grids, ε_t by 13 grids respectively. For very persistent stochastic process, Flodén (2007) points out that it should be more than 15 grids to approximate the AR(1) process with a finite Markov chain precisely.

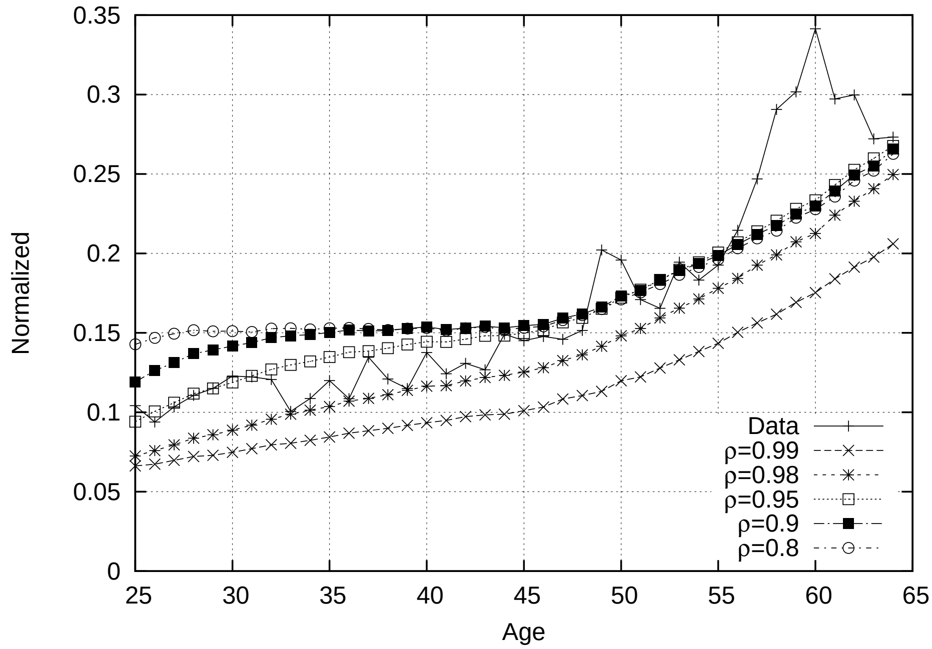

Figure 2: Cross Section Variance of Income with Several ρ

especially when $\rho = 0.95$.[5] As we calibrate the sharp rise of persistent shock after 47, the simulated income profile traces the shape of the data.

We summarize the fundamental parameters in the upper row of Table I. We set the rate of return per year to be 4% and the discount factor is $\beta = 0.96$. The relative risk aversion γ is 2, which implies the intertemporal elasticity of substitution to be 0.5. These parameters are standard in both the Japanese and U.S. literature. The replacement rate φ is targeted at 50% of average income for workers. The survival probability is taken from the life table estimated by the *National Institute of Population and Social Security Research* in 2006.

β	γ	r	φ	σ_ε
0.96	2	0.04	0.5	0.218
ρ	0.99	0.95	0.90	0.80
σ_{η_0}	0.0452	0.1	0.1396	0.1922

Table I: Calibration Parameters

[5]The level of income inequality is low when the shock is very persistent such as $\rho = 0.99$ and 0.98. Because we approximate the AR(1) process with a finite Markov chain, it is infrequent to reach a maximum and minimum state with a finite working period in the very persistent case. This problem will be overcome when we consider z_t as a continuum state variable, although it implies an additional computational burden. Because we focus on the *shape* of consumption inequality, this point does not matter for our purposes.

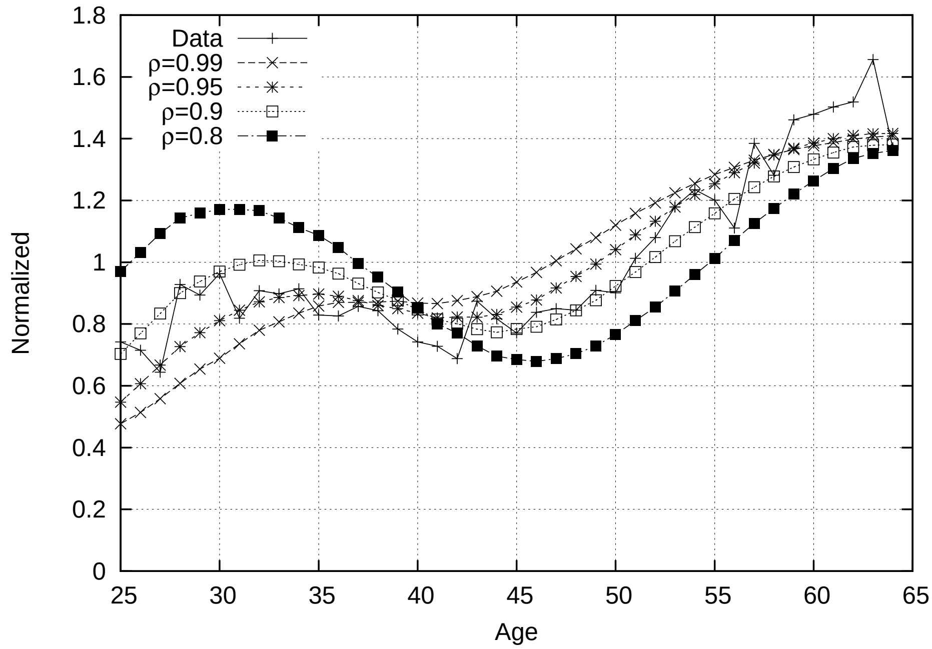

Figure 3: Cross Section Variance of Consumption (Adjusted σ_0 for Each ρ)

4. Are Labor Income Shocks Very Persistent? A Case in Japan

We describe the cross-sectional variances of the logarithms of consumption for each age in Figure 3. Because the average of the variances of the consumption inequality profile differs for each ρ, we normalize the average of the variances to be one in order to focus on the shape of the profile. Storesletten *et al.* (2004b) show that the consumption inequality monotonically increases over age if the income shock is very persistent. In our model, the shape does not seem to be monotonic, even in a persistent case, because the standard deviation of the persistent shock σ_{η_t} increases after middle age.[6] Therefore, the consumption inequality profile becomes a stepwise pattern.

When the persistence of income shock is close to unity, e.g., $\rho = 0.99$, the corresponding consumption variance profile becomes weakly concave before the 40s. Storesletten *et al.* (2004) used $\rho = 1$ as a benchmark case, and found that the consumption variance profile becomes concave. In our model, consumption inequality rises when households are in their 20s, and slow down to increase in the late 30s. Because households enter the economy with zero assets, it is difficult to share the income risk when they are young, and consumption inequality rises as the persistent shocks realize. However, as they accumulate some wealth for precautionary saving, the increase of consumption inequality slows down and the

[6]When the standard deviation of the shock is constant even after 47, our sumulation result is similar to that of Storesletten et al. (2004b).

consumption inequality profile becomes concave. This shape matches the actual consumption variance profile in Japan.

A main finding of our paper is as follows: When the persistence parameter is smaller than $\rho = 0.95$, the consumption variance profile does not become monotonic.[7] When the persistence of income shocks is moderate, $\rho = 0.95$ or 0.9, it is relatively easy to smooth consumption by precautionary saving. Consumption inequality does not increase so much when households are in their 30s because the households diversify their income risk via savings. After the sharp rise of income risks after 47, consumption inequality rises again. Such a shape of the cross-sectional variance of consumption is close to the shape of the Japanese economy, as shown in Figure 3. When the shock is very low, less than 0.8, the consumption inequality varies up and down extremely at young and middle ages. When the shock is not persistent, it is easy to share the risk by saving. Thus, although young households first accumulate assets and consumption inequality increases, the consumption inequality quickly decreases after accumulating sufficient savings. Using micro-data in Japan, Abe and Yamada (2009) showed that the variance of the log-consumption profile is flat or weakly decreasing when a household head is young. It increases rapidly when the household is over 40. In contrast to the U.S., this shape does not match the simulated consumption variance profile when the persistence is close to one. Figure 3 shows that the shape of consumption inequality in Japan is consistent with a moderate persistent shock ρ.

We have adjusted σ_0 for each ρ to match the level of the income inequality profile. We need to show that our finding is not a specific result of our choice of calibration parameters. We plot the consumption inequality profile simply by replacing ρ with σ_0 being fixed at 0.1 in Figure 4. Note that when households face very persistent income shocks, the corresponding income inequality becomes large than actual data in Japan. However, as shown in Figure 4, changing ρ without adjusting σ_0 does not change our result significantly. Similarly, the consumption inequality profile becomes concave when the shock is very persistent. When the persistence is moderate, consumption inequality decreases in the 20s and 30s, and this humped up pattern increases, as shown in Figure 4.

Lastly, we should mention the sensitivity of our analysis regarding two points. First, when the labor supply is endogenous as in Kaplan (2007), our results do not change; i.e., as the persistence parameter decreases, the consumption inequality becomes more non-linear. The mechanism is the same although labor supply is also available to share labor income risk. Second, the replacement rate is key to understanding the sharp rise in consumption inequality in middle and old age. We confirm that when the replacement rate is almost zero, consumption inequality is more nonlinear and closer to the actual data.

[7] Storesletten et al. (2004) also pointed out that the consumption inequality profile becomes bimodal when the persistence is low.

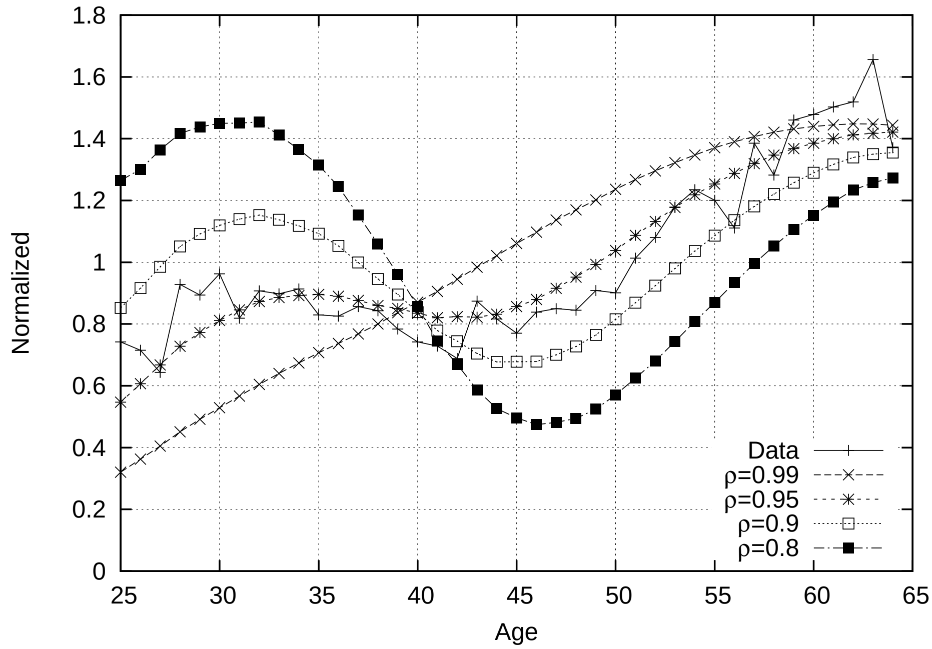

Figure 4: Cross Section Variance of Consumption (Unadjusted σ_0 for Each ρ)

5. Conclusions

We showed that the autocorrelation coefficient ρ is a significant parameter in explaining the shape of consumption inequality over the life cycle in a model with incomplete asset markets. When the persistence parameter is close to unity as in Storesletten *et al.* (2004b), the corresponding consumption inequality profile becomes concave. In contrast, when the persistence parameter is below 0.95, the consumption variance does not become monotonically increasing due to precautionary saving behavior. We find that when the persistence of shocks is moderate, it fits the Japanese economy very well. We need further investigation of the shape of labor income and consumption inequality. In particular, why the income inequality sharply rises after middle age is an important question for considering the labor income risk in Japan. Moreover, we need more empirical investigation to conclude that low persistence is the most significant parameter for characterizing decreasing consumption inequality. For example, Guvenen (2007) concludes that the combination of moderate persistence and heterogeneous income profile with Bayesian learning are important for understanding consumption inequality in the U.S. We do not pursue this point simply due to limited availability of micro-data in Japan. These are future research topics.

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