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# Incomplete third-degree price discrimination, and market partition problem

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## Abstract

We introduce in this paper the "incomplete" third-degree price discrimination, which is the situation where a monopolist must charge at most k different prices while the total market is composed of n markets, with n>k. We thus study the optimal partition problem of the n markets in k groups. As a byproduct, when k=2, we are able to reconsider the so-called (Robinsonian) weak-strong partition.

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# 1 Introduction

The common feature of the models in the third-degree price discrimination literature in monopoly is that there are $n$ local markets, independent or not, each characterized by a local demand function, linear or not. The typical problem (e.g., Schmalensee 1981, Malueg 1993, Varian 1988, Cowan 2008) is to compare, from a welfare point of view, the case in which the monopolist is not allowed to price discriminate, i.e., she must charge a uniform price, with the case in which she may (third-degree) price discriminates, i.e., she may charge $n$ different prices. The monopolist charge thus either one price, or $n$ (different) prices.

Suppose now that for some reason (e.g., regulatory, marketing...), the monopolist must charge at most $k$ different prices, with $2 \leq k < n$ so that some markets must be grouped together. The problem of the monopolist is now to find the best partition of the $n$ markets in $k$ groups, and the price associated to each group that maximizes the aggregate profit. To solve this optimization problem, the monopolist must thus consider each possible partition $\mathcal{A} = (A_1, ...A_k)$ of the markets, and then find the optimal vector of prices $\mathbf{P}^*_{\mathcal{A}} = (P^*_{A_1}, ...P^*_{A_1})$. The aim of this paper is to provide a simple *necessary condition* that must be satisfied by the solution of this combinatorial-like optimization problem. Do to so, we consider a firm composed of a headquarters, whose job is to propose a "menu" $\mathbf{P}$ containing $k$ prices, and a set of $n$ "experts", whose job is to choose a price in $\mathbf{P}$. Each expert, implanted on a local market, perfectly knows her (own) demand function (hence the name of expert) and chooses the price in $\mathbf{P}$ that maximizes her (own) profit function. The menu $\mathbf{P}$ induces thus a *partition* denoted $\mathcal{A}(\mathbf{P})$ of the $n$ experts in at most $k$ distinct groups. Let $\mathbf{P}^*_{\mathcal{A}}$ the vector of prices that maximizes the aggregate profit of the *fixed* partition $\mathcal{A}$. When the headquarters proposes $\mathbf{P}^*_{\mathcal{A}}$ to the experts, for the partition $\mathcal{A}$ to be a possible candidate of the optimization problem, $\mathbf{P}^*_{\mathcal{A}}$ must induce the partition $\mathcal{A}$ and not another one. When this is the case, we say that the partition $\mathcal{A}$ is *admissible*.

Using a decentralized decision process eliminates thus partitions which are not admissible, but also partitions for which no price induces them. In the particular case in which $k = 2$, we are able to re-examine the so-called Robinsonian weak-strong markets partition (e.g., Robinson 1933, Schmalensee 1981, Cowan 2008), which is implicitly supposed to be the optimal partition of the markets in two groups.

This paper is organized as follows. We first present the assumptions of the model and the required terminology; we then define what an *admissible partition* is, and we show that such a partition is in general not unique. We prove that an optimal partition exists and eventually provide a simple way to get a unique admissible partition.

# 2 The model

## 2.1 Assumptions

We consider a firm which is in a monopoly situation on $n$ separated markets but is allowed to charge at most $k$ different prices, with $2 \leq k < n$. We call this situation an "incomplete" third-degree price discrimination. One may assume that the firm has $n$ subsidiaries, and that each subsidiary $j \in J \equiv \{1, 2...n\}$ is implanted in different region of the world. Let

$$q_j(P_j) = (a_j - P_j)^\alpha \qquad \alpha > 0 \tag{1}$$

with $a_j > 0\ \forall j \in J$, be the demand function of market $j$, which is only function of the price $P_j$, so that trade among markets is (implicitly) not allowed. Note that when $\alpha$ is lower, equal, or higher than one, the demand function is respectively concave, linear, or convex. In each subsidiary $j$, there is a local head $j \in J$ that we call an "expert" because she is assumed to perfectly know her profit function given by

$$\pi_j(P_j) = (P_j - C)(a_j - P_j)^\alpha \tag{2}$$

where $C$ is the constant marginal cost. It is easy to show that the profit function given by equation (2) is *single-peaked* and that

$$P_j^* = \frac{a_j + \alpha C}{1 + \alpha} \tag{3}$$

is the monopoly price of the (local) market $j$, i.e., the price that maximizes the profit function $\pi_j(P_j)$. We shall assume that $a_{j+1} > a_j\ \forall j \in J$ so that $P_1^* < P_2^* < ...P_n^*$. For economic interest, we shall also assume that $C < a_1$ so that $\pi_j(P_j)$ is a concave function on $]0, a_j[$ $\forall j \in J$. In our model, the price decision process of the firm is *decentralized* in the following sense: the headquarters proposes a menu $\mathbf{P} = (P_{A_1}, P_{A_2}...P_{A_k})$, with $0 \leq P_{A_1} \leq P_{A_2} \leq ... \leq P_{A_k}$ and each expert $j$ is *free* to choose one price in $\mathbf{P}$. Since each expert must choose one price, the vector $\mathbf{P}$ induces thus a *partition* $\mathcal{A}(\mathbf{P}) \equiv \mathcal{A} = (A_1, A_2, ...A_k)$ of the experts in $k$ groups, where $A_i \subseteq J$ is the subset of experts of $J$ who choose the price $P_{A_i}$. By definition of a partition, $\bigcup_{i=1}^{k} A_i = J$ with $A_i \bigcap A_{i'} = \emptyset$ for all $i \neq i'$. Note that for some $i$ but not all, $A_i$ may be empty.

## 2.2 Set of prices and admissible (consecutive) partition of experts

Let $V(\mathbb{R}^{k+})$ be the set of prices defined as follows.

$$V(\mathbb{R}^{k+}) = \{\mathbf{P} \in \mathbb{R}^{k+} : 0 \leq P_{A_1} \leq P_{A_2} \leq ... \leq P_{A_k} \leq \bar{P}\} \tag{4}$$

where $P_n^* \leq \bar{P}$, with $\bar{P}$ finite. Note that $V(\mathbb{R}^{k+})$ is compact and convex[1], and let $\Pi(\mathcal{A}(\mathbf{P}))$ be the aggregate profit generated by the induced partition $\mathcal{A}(\mathbf{P})$.

$$\Pi(\mathcal{A}(\mathbf{P})) := \Pi(\bigcup_{i=1}^{k} A_i) = \sum_{i=1}^{k} \Pi_{A_i}(P_{A_i}) \tag{5}$$

$$\Pi_{A_i}(P_{A_i}) = \sum_{j \in A_i} \pi_j(P_{A_i}) \tag{6}$$

$$j \in A_i \iff \pi_j(P_{A_i}) \geq \pi_j(P_{A_{i'}}),\ \forall i \neq i' \tag{7}$$

[1]Compacity follows since $V(\mathbb{R}^{k+})$ is a closed and bounded subset of $\mathbb{R}^{k+}$. Let $\mathbf{P}, \mathbf{P}' \in V(\mathbb{R}^{k+})$ with $\mathbf{P}' \neq \mathbf{P}$. Convexity follows since for all $\lambda \in [0, 1]$, $\lambda\mathbf{P} + (1 - \lambda)\mathbf{P}' \in V(\mathbb{R}^{k+})$.

Equations (5) and (6) uses the fact that the aggregate profit is an additive function. Equation (7) is an individual rationality condition since each agent $j$ chooses the component $P_{A_i}$ of $\mathbf{P}$ that maximizes her (own) profit $\pi_j(P)$. Let us now examine more closely the kind of partition $\mathcal{A}$ one may have.

**Definition 1**. The partition $\mathcal{A} = (A_1, ...A_k)$ is said to be *consecutive* whenever $j'$ and $j''$, with $j' < j''$, belong to the set $A_i$ (i.e., they choose the price $P_{A_i}$), experts $j \in J$ such that $j' < j < j''$ also belong to $A_i$.

**Lemma 1** . Assume that $\pi_j(P_j) = (P_j - C)(a_j - P_j)^\alpha$, with $a_{j+1} > a_j > 0$ $\forall j \in J$ and $\alpha > 0$. Then, the partition $\mathcal{A} = (A_1, ...A_k)$ is consecutive.

**Proof**. See the appendix.

Let

$$V(\mathcal{A}(\mathbf{P})) = \{\mathbf{P} \in V(\mathbb{R}^{k+}) : \mathbf{P} \text{ induces the (consecutive) partition } \mathcal{A}\} \tag{8}$$

When $k = 2$, $V(\mathcal{A}(\mathbf{P}))$ is never empty since for any (consecutive) partition $\mathcal{A} = (A_1 = \{1, 2...h\}; A_2 = \{h+1, ...n\})$, one can always choose $\mathbf{P} = (P^*_h, P^*_{h+1})$. However, when $k \geq 3$, $V(\mathcal{A}(\mathbf{P}))$ may be empty. To see this, consider the case in which $J = \{1, 2...6\}$, $C = 0$, $\bar{P} = P^*_n$, and $\alpha = 1$ so that $P^*_j = \frac{a_j}{2}$ for all $j \in J$. Assume moreover that three (different) prices are allowed, i.e., $\mathbf{P} = (P_{A_1}, P_{A_2}, P_{A_3}) \in V(\mathbb{R}^{3+})$. Consider now the parameters $a_1, ...a_6$ such that $P^*_1 = 1, P^*_2 = 1.9, P^*_3 = 2, P^*_4 = 20, P^*_5 = 20.1, P^*_6 = 21$, and the partition $\mathcal{A} = (A_1 = \{1, 2\}; A_2 = \{3, 4\}; A_3 = \{5, 6\})$. If $P_{A_2} \in ]4, 21[$, then, at least expert 1,2,3 choose the price $P_{A_1}$. If $P_{A_2} \in ]0, 4]$, then, at least experts 4,5 and 6 choose the price $P_{A_3}$. As a consequence, there is no way to choose $P_{A_2} \in ]0, 21[$ to induce the partition $\mathcal{A}$ so that $V(\mathcal{A}(\mathbf{P}))$ is empty. When $V(\mathcal{A}(\mathbf{P})) \neq \emptyset$, it is the subset of prices in $V(\mathbb{R}^{k+})$ that leaves *invariant* the partition $\mathcal{A}$. Let $\mathcal{P}(\mathcal{A})$ be the set of (consecutive) partitions for which $V(\mathcal{A}(\mathbf{P})) \neq \emptyset$.

**Definition 2**. We say that $\mathcal{A} \in \mathcal{P}(\mathcal{A})$ is a *feasible* partition for the price $\mathbf{P}$ if $\mathbf{P} \in V(\mathcal{A}(\mathbf{P}))$

Let $\Pi_{\mathcal{A}}(\mathbf{P})$ be the aggregate profit function assuming that the partition $\mathcal{A}$ is *fixed*.

**Definition 3**. We say that $\mathcal{A} \in \mathcal{P}(\mathcal{A})$ is an *admissible* partition if there exists a vector of prices $\mathbf{P}^*_{\mathcal{A}} = (P^*_{A_1}, ...P^*_{A_k})$ such that

$$\nabla \Pi_{\mathcal{A}}(\mathbf{P}^*_{\mathcal{A}}) = \mathbf{0} \Longleftrightarrow \Pi'_{A_i}(P^*_{A_i}) = \sum_{j \in A_i} \pi'_j(P^*_{A_i}) = 0 \ \ \forall i = 1, 2...k \tag{9}$$

$$\mathbf{P}^*_{\mathcal{A}} \in V(\mathcal{A}(\mathbf{P})) \tag{10}$$

We show in lemma A1 in appendix that if $P^*_{A_i}$ maximizes (perhaps locally) the profit function $\Pi_{A_i}(P_{A_i})$ then, $P^*_{A_i} \neq a_j$ $\forall j \in A_i$, $\forall i = 1, 2...k$. As a consequence, $\mathbf{P}^*_{\mathcal{A}}$ has to satisfy the optimality condition given by equation (9). We show in lemma A2 that if $\mathbf{P}^*_{\mathcal{A}}$ satisfy equation (9), then, the second order condition for a (local) maximum is also satisfied. Since the price decision process is decentralized, for the partition $\mathcal{A}$ to be admissible, $\mathbf{P}^*_{\mathcal{A}}$ must induce the partition $\mathcal{A}$ and not another one, i.e., $\mathcal{A}$ must be feasible for $\mathbf{P}^*_{\mathcal{A}}$.

Let $\mathcal{A}^*$ be an admissible partition and let $\mathcal{P}(\mathcal{A}^*)$ be the set of all admissible partitions. Clearly, $\mathcal{P}(\mathcal{A}^*) \subseteq \mathcal{P}(\mathcal{A})$. We shall denote $\mathbf{P}^*_{\mathcal{A}^*}$ the optimal vector of prices associated to the admissible partition $\mathcal{A}^*$. There may indeed exist (finitely) many admissible partitions[2]. To see this, consider a "simple" partition $\mathcal{A} = (A_i = \{i\},\ \forall\ i = 1, 2...k-1; A_k = \{k, k+1, ...n\})$ and let $\mathbf{P}^*_{\mathcal{A}} \in V(\mathbb{R}^{k+})$ be such that $\mathbf{P}^*_{\mathcal{A}} = (P^*_{A_1} = P^*_1; ...P^*_{A_k-1} = P^*_{k-1}; P^*_{A_k} \in [P^*_k, P^*_n]$. Since $P^*_{A_i} = P^*_i$, each expert $i$ chooses the price $P^*_i$ for $i = 1, 2...k-1$. If moreover expert $k$ is such that $\pi_k(P^*_{A_k}) > \pi_k(P^*_{A_{k-1}})$, then the partition $\mathcal{A}$ is admissible. Of course, there are finitely many such simple partitions and some of them are generally admissible.

**Definition 4**. We say that $\mathcal{A}^{**} \in \mathcal{P}(\mathcal{A}^*)$ is the *optimal partition* of $J$ if

$$\Pi(\mathbf{P}^*_{\mathcal{A}^{**}}) > \Pi(\mathbf{P}^*_{\mathcal{A}^*}) \quad \forall \mathcal{A}^* \in \mathcal{P}(\mathcal{A}^*) \quad \mathcal{A}^{**} \neq \mathcal{A}^* \tag{11}$$

**Lemma 2**. If $\mathcal{A}^* \in \mathcal{P}(\mathcal{A}^*)$ contains at least one empty group (i.e., there exists at least one $i \in \{1, 2...k\}$ such that $A_i = \emptyset$), then, $\mathcal{A}^*$ can not be the optimal partition.

**Proof** . Assume that $\mathbf{P}^* = (P^*_{A_1}, ....P^*_{A_k})$ induces the optimal partition $\mathcal{A}^* \in \mathcal{P}(\mathcal{A}^*)$ and such that at least $A_i$ is empty, i.e., nobody choose the price $P^*_{A_i}$. Since $n > k$, one can at least find one expert $j \in J$ such that the chosen price in $\mathbf{P}^*$ is different than her optimal price $P^*_j$. Fix now $P^*_{A_i} = P^*_j$. Clearly, this expert $j$ will choose this price $P^*_{A_i}$, and her profit will increase. But then, the partition change since now $A_i$ is not empty. Hence, $\mathcal{A}^*$ can not be the optimal partition.

**Proposition 1**. There exists a price $\mathbf{P}^{**} \in V(\mathbb{R}^{k+})$ that induces the optimal partition $\mathcal{A}^{**} \in \mathcal{P}(\mathcal{A}^*)$

**Proof**. The aggregate profit function $\Pi(\mathcal{A}(\mathbf{P}))$ is a continuous function since it is a finite sum of continuous function. Since $V(\mathbb{R}^{k+})$ is a compact (and convex) subset of $\mathbb{R}^{k+}$, $\Pi(\mathcal{A}(\mathbf{P}))$ reaches its minimum, and its maximum at $\mathbf{P}^{**}$, and $\mathcal{A}^{**}$ is the optimal partition induced by $\mathbf{P}^{**}$ so that $\mathbf{P}^{**} \in V(\mathcal{A}^{**}(\mathbf{P}))$. From lemma A1 and A2, we know that $\mathbf{P}^{**}$ satisfies equation (9), which is also a sufficient condition for a maximum. Let $\mathcal{A}^{**} = (A^{**}_1, ...A^{**}_k)$, with $\bigcup_{i=1}^k A^{**}_i = J$. From lemma 2, $\mathcal{A}^{**}$ contains $k$ non-empty groups, i.e., $A^{**}_i \neq \emptyset\ \forall i = 1....k$
□

[2]An analogy may be done with the coalition structures literature that assumes *free mobility* (i.e., each agent is free to choose one coalition) in which there are multiple inefficient free mobility equilibria. See Demange (2005).

## 2.3 Application when $k = 2$: a reexamination of the weak-strong markets partition

### 2.3.1 The weak-strong markets partition

Let $\Pi(P) = \sum_{j \in J} \pi_j(P)$ be the profit function when discrimination is not allowed, and let $P^*$ be the uniform monopoly price. The *weak-strong markets partition* (see e.g., Cowan 2008, Robinson 1933, Schmalensee 1981, Malueg 1993, Shih et al 1988), denoted $W$-$S$ is the partition in which the two sets $W$ and $S$ are defined as follows:

$$W = \{j \in J : P_j^* < P^*\} \;\; ; \;\; S = \{j \in J : P_j^* > P^*\} \tag{12}$$

Since $\pi_j'(P^*) < 0$ for all $j \in W$ and $\pi_j'(P^*) > 0$ for all $j \in S$, when the monopolist is allowed to move from $k = 1$ to $k = 2$, it is clear that the profit of the group $W$ ($S$) increases when we decrease (increase) the price. Probably for this reason, in the third-degree price discrimination literature, it is implicitly believed that the weak-strong partition is the optimal partition of the $n$ markets in two groups.

### 2.3.2 Linear demand functions

Suppose that each expert is endowed with a linear demand function $q_j(P_j) = a_j - P_j$ (i.e., $\alpha = 1$) and assume that $C = 0$ for simplicity, so that the profit function is simply $\pi_j(P_j) = a_j P_j - P_j^2$, for $P_j \leq a_j$. Let $\mathcal{A}_h$ be the following partition

$$\mathcal{A}_h = (A_1 = \{1, ...h\}; A_2 = \{h+1, ...n\}) \tag{13}$$

where $V(\mathcal{A}_h(\mathbf{P}))$ is the set of prices that induces $\mathcal{A}_h$. Since the partition is consecutive, to induce the partition $\mathcal{A}_h$, it suffices that $P_{A_1}$ and $P_{A_2}$ are such that $\pi_h(P_{A_1}) > \pi_h(P_{A_2})$ and $\pi_{h+1}(P_{A_1}) < \pi_{h+1}(P_{A_2})$. It is easy to show that the two inequalities implies that:

$$V(\mathcal{A}_h(\mathbf{P})) = \{(P_{A_1}, P_{A_2}) \in V(\mathbb{R}^{2+}) : (P_{A_1} + P_{A_2}) \in ]a_h; a_{h+1}[\} \tag{14}$$

so that $V(\mathcal{A}_h(\mathbf{P})$ is a convex set. The partition $\mathcal{A}_h$ will thus be admissible if there exists a vector of prices $\mathbf{P}^*_{\mathcal{A}_h} = (P^*_{A_1}, P^*_{A_2})$ such that $(P^*_{A_1} + P^*_{A_2}) \in ]a_h; a_{h+1}[$. Our model allows us to examine whether or not the weak-strong partition of the markets in two groups is the optimal partition. In fact, as we shall now show, the weak-strong partition may not be admissible while, as in Schmalensee (1981), all the markets are served under the uniform monopoly price. To consider such a case, let $\Psi(n, h)$, with $1 \leq h \leq n-1$, and $n$ is a finite integer, be the set of parameters $\mathbf{a} = (a_1, ...a_n) \in \mathbb{R}^{n+}$, with $a_1 < a_2 < ...a_n$ such that:

1. under the uniform monopoly price $P^*$, all markets are served, i.e., $q_j(P^*) > 0 \; \forall j \in J$
2. the weak-strong partition is $\mathcal{A}_h = (W = \{1, ...h\}; S = \{h+1, ...n\}$
3. $\mathbf{P}^*_{\mathcal{A}_h} = (P^*_W, P^*_S) \notin V(\mathcal{A}_h(\mathbf{P}))$

If one can find a couple $(n, h)$ such that $\Psi(n, h)$ is not empty, this implies that the weak-strong partition is not always the best way to divide the set $J$ in two groups. But then, the meaning of the weak-strong partition is not clear. The label weak-strong partition should indeed be attributed only to the optimal partition.

We shall now provide an element of $\Psi(5, 2)$. The "difficult" part is to find $\mathbf{a}$ such that the weak-strong markets partition is not admissible. To obtain this property, we thus look for a vector $\mathbf{a}$ such that the set $V(\mathcal{A}_2(\mathbf{P}))$ is *thin*. To this end, consider the case in which $a_1 = 10$, $a_2 = 15$, $a_3 = 15.5$, $a_4 = 16$, $a_5 = 20$, and recall that $P_j^* = (1/2)a_j$. It is easy to show that the uniform monopoly price is $P^* = 7.65$ so that all the markets are served. As a consequence, the weak-strong markets partition is $\mathcal{A}_2 = (W = \{1, 2\}; S = \{3, 4, 5\})$, and $\mathbf{P}^*_{\mathcal{A}_2} = (6.25; 8.58)$. Since $(P_W^* + P_S^*) = 14.83$ and $14.83 \notin ]15; 15.5[$, it thus follows that $\mathbf{P}^*_{\mathcal{A}_2} \notin V(\mathcal{A}_2(\mathbf{P}))$. In fact, since expert 3 chooses the price $P_W^*$, $\mathbf{P}^*_{\mathcal{A}_2} \in V(\mathcal{A}_3(\mathbf{P}))$. In this example, $\mathcal{P}(\mathcal{A}^*) = \{\mathcal{A}_1, \mathcal{A}_3, \mathcal{A}_4\}$, and the optimal partition is $\mathcal{A}_1$, which turns out to be a "simple" one.

For the weak-strong markets partition to be the *unique* admissible partition, the experts of $W$ and $S$ must be "homogeneous" enough in terms of their optimal price $P_j^*$. To see this, consider first the case in which there are only two experts, $a$ and $b$, with demand function $q(x, P) = x - P$, for $x \in \{a, b\}$, $a < b$. It is easy to show that when $b < 2a$, the profit function is single-peaked, and the uniform monopoly price is $P^* = \frac{a+b}{4}$. Moreover, the two markets are served. Let $\Delta$ be a "heterogeneity" parameter. To construct the set $W$ and $S$, at each time we add an expert in the set $W$, we also add an expert in $S$ so that the monopoly price $P^* = \frac{a+b}{4}$ remains invariant. There are thus $N$ experts in each group $W$ and $S$, and the total number of experts is $n = 2N$. This construction allows us to control the heterogeneity of the optimal prices $P_j^*$ of each group $W$ and $S$ with the parameter $\Delta$, but also $|P^* - P_j^*|$ for some $j \in J$. When $\Delta$ is small enough, then, the weak-strong markets partition is the unique admissible one. The following proposition gives an example when $N = 3$.

**Proposition 2**. Let $a > 0$, $b \in ]a, 2a[$, $\Delta > 0$, $n = 2N$, $N = 3$. Assume that $a_j = a + \epsilon_j$ and $b_j = b - \epsilon_j$ for $j = 1, 2, 3$, with $\epsilon_1 = -\Delta$, $\epsilon_2 = 0$ and $\epsilon_3 = \Delta$. When $\Delta < \frac{b-a}{6}$, the weak-strong markets partition given by $W = \{a_1, a_2, a_3\}; S = \{b_3, b_2, b_1\}$ is the *unique* admissible partition.

**Proof**. See the appendix

The above proposition is obviously not intended to give the most general conditions under which the weak-strong partition is the unique admissible partition. It is rather to show how one get this property by restricting the "heterogeneity" of the optimal price of each group $W$ and $S$.

# 3 Conclusion

In this paper, we have examined, from the profit-maximizing monopolist point of view, the optimal partition problem of the $n$ markets in $k$ groups. In Braouezec (2009), we consider the welfare problem and we show that, for linear demands, incomplete third-degree price discrimination is (generally) optimal from a total welfare point of view.

## Appendix

**Proof of lemma 1**. Let $\mathbf{P} = (P_{A_1}, ....P_{A_k})$ be the menu of prices. For simplicity, assume that $C = 0$, and consider experts 1,2 and 3, with $a_1 = a$, $a_2 = b$, and $a_3 = c$ such that $0 < a < b < c$. Assume now that experts $a$ and $c$ choose the price $P_{A_i}$. Consider the case $P_{A_i} > P_{A_{i-1}} > ... > P_{A_1} > 0$. By assumption, $\pi_a(P_{A_i}) > \pi_a(P_{A_{i-1}})$, and $\pi_c(P_{A_i}) > \pi_c(P_{A_{i-1}})$, which is equivalent to $f(x) := \frac{P_{A_i}}{P_{A_{i-1}}}\left(\frac{x - P_{A_i}}{x - P_{A_{i-1}}}\right)^{\alpha} > 1$ for $x \in \{a, c\}$. Note that we necessarily have $a > P_{A_i}$. Since $f'(x) > 0$ for $x > P_{A_i}$, this implies that $1 < f(a) < f(b) < f(c)$ for any $b$ between $a$ and $c$. Consider now the case in which $0 < P_{A_i} < P_{A_{i+1}}$. Now, $\pi_a(P_{A_i}) > \pi_a(P_{A_{i+1}})$, and $\pi_c(P_{A_i}) > \pi_c(P_{A_{i+1}})$ is equivalent to $f(x) := \frac{P_{A_{i+1}}}{P_{A_i}}\left(\frac{x - P_{A_{i+1}}}{x - P_{A_i}}\right)^{\alpha} < 1$ for $x \in \{a, c\}$. Note that $a > P_{A_i}$, otherwise expert $a$ would have chosen a price lower than $P_{A_i}$. However, it may be the case that $c < P_{A_{i+1}}$, so that $c - P_{A_{i+1}} = 0$. If $a > P_{A_{i+1}}$, since $f'(x) > 0$, then, $0 < f(a) < f(b) < f(c) < 1$. If $c < P_{A_{i+1}}$, then, $f(a) = f(b) = f(c) = 0$. If $c > P_{A_{i+1}}$ and $a < P_{A_{i+1}}$, then, $f(a) = 0 < f(b) < f(c) < 1$ for $b > P_{A_{i+1}}$ and $f(a) = f(b) = 0 < f(c) < 1$ for $b < P_{A_{i+1}}$. In all cases, if $a$ and $c$ choose $P_{A_{i+1}}$, so will do $b$ $\square$

**Lemma A1**. If $P_{A_i} = a_j$, for some $j \in A_i$, $\forall i = 1, 2...k$, then, this price $a_j$ can not maximize the profit function.

**Proof**. Fix a given $h \in A_i$ and let $A_i^h = \{j \in A_i : j \geq h\}$ and $A_i^{h+1} = \{j \in A_i : j \geq h+1\}$. If $\sum_{j \in A_i^h} \pi'_j(P_{A_i}) > 0$ for $P_{A_i} \in ]a_{h-1}, a_h[$, then, $\sum_{j \in A_i^{h+1}} \pi'_j(a_h + \epsilon) > 0$, $\forall \epsilon > 0$. Note first that $\sum_{j \in A_i^h} \pi'_j(P_{A_i}) = \pi'_h(P_{A_i}) + \sum_{j \in A_i^{h+1}} \pi'_j(P_{A_i}) > 0$ for $P_{A_i} \in ]a_{h-1}, a_h[$. Fix $\epsilon > 0$. Since $\pi'_h(a_h - \epsilon) < 0$ while $\pi'_h(P) = 0$ for $P > a_h$ and since $\sum_{j \in A_i^{h+1}} \pi'_j(P_{A_i})$ is continuous for $P_{A_i} \in [a_h - \epsilon; a_h + \epsilon]$, one must have $\sum_{j \in A_i^{h+1}} \pi'_j(P_{A_i}) > 0$. As a consequence, the derivative of the aggregate profit function can not change of sign around a point $a_j$ $\square$

**Lemma A2**. If $\mathbf{P}^*_{\mathcal{A}} = (P^*_{A_1}, ...P^*_{A_k})$ solves equation (9), then, the second order condition is satisfied.

**Proof**. Let $A_i$ be a set of consecutive indices and recall that $\Pi_{A_i}(P_{A_i}) = \sum_{j \in A_i} \pi_j(P_{A_i})$. Since each $\pi_j(P_{A_i})$ is a concave (and twice continuously differentiable) function of $P_{A_i}$ on $]0, a_j[$, $\Pi_{A_i}(P_{A_i})$ is a concave (and twice continuously differentiable) function of the price on $]a_j, a_{j+1}[$, $\forall j \in A_i$, for $i = 1, ...k$. As a consequence, if there exists $P^*_{A_i} \in ]a_j, a_{j+1}[$, such that $\Pi'_{A_i}(P^*_{A_i}) = 0$, then, by concavity, $P^*_{A_i}$ is unique and such that $\Pi''_{A_i}(P^*_{A_i}) < 0$ $\square$.

**Proof of proposition 2**. It is easy (but cumbersome) to construct the profit function and show that the aggregate profit function is single-peaked if $\Delta < (2/11)a$, so that this single-peakedness property is satisfied for $\Delta < \frac{b-a}{6}$ since $b \in ]a, 2a[$. It is also easy to show that $P^*_{a_3} < P^*$ and $P^*_{b_3} > P^*$ are satisfied if $\Delta < \frac{b-a}{2}$. By symmetry of each profit function $\pi_j(P)$, if $\gamma' > \gamma$, then $\pi_j(P^*_j - \gamma') < \pi_j(P^*_j + \gamma)$ or $\pi_j(P^*_j - \gamma) > \pi_j(P^*_j + \gamma')$. Let $P$ and $P'$ two prices. It thus follows that $\pi_j(P') > \pi_j(P)$ if $|P^*_j - P'| < |P^*_j - P|$. Consider $(P^*_W, P^*_S)$, with $P^*_W < P^* < P^*_S$, which is the optimal vector of prices associated to the weak-strong partition. Since the partition is consecutive, if expert $a_3$ chooses the price $P^*_W$, then $a_1$ and $a_2$ make the same choice. In the same way, if expert $b_3$ chooses the price $P^*_S$, then, $b_1$ and $b_2$ make the same choice. When $\Delta < \frac{b-a}{6}$, $P^*_{b_3} - P^* > \Delta$ and $P^*_{a_3} - P^* > \Delta$. Since

$P_W^* \in [P_{a_1}^*, P_{a_3}^*]$, and $P_{a_3}^* - P_{a_1}^* = \Delta$ but also $P_S^* \in [P_{b_3}^*, P_{b_1}^*]$, and $P_{b_3}^* - P_{b_1}^* = \Delta$, it thus follows that the weak-strong partition is admissible. To show that it is unique, let $B \subset S$ (i.e., $B$ is either $\{b_3\}$ or $\{b_3; b_2\}$) and consider the consecutive partition $\mathcal{A} = (W \cup B), (S \setminus B)$. Note that $P_W^* < P_{W \cup B}^* < P^*$ and $P_{S \setminus B}^* > P_S^* > P^*$. The partition $\mathcal{A}$ is not admissible since $|P_{b_3}^* - P_{S \setminus B}^*| < \Delta$ while $|P_{b_j}^* - P_{W \cup B}^*| > \Delta \ \forall j \in S$, i.e., all experts of $B$ choose the price $P_{S \setminus B}^*$ and not the price $P_{W \cup B}^*$. Apply the same reasoning for $\mathcal{A} = (W \setminus B), (S \cup B)$, for $B \subset W$ $\square$

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