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Efficient Nash equilibria, individual rights and Pareto principle: an impossibility result

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Abstract

This paper shows an impossibility result similar to the liberal paradox concerning two games, each of one with an efficient Nash equilibrium. We show that our result holds also in dictatorial games.

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1. Introduction

Ever since the publication of the "impossibility of a liberal paretian" (Sen 1970a, 1970b) which showed the mutual inconsistency between individual rights and the Pareto principle, a huge amount of literature followed during the years. This literature may be subdivided in two main streams. The first one was developed in a social choice theoretic framework and was followed by Sen himself (Sen 1970a, 1970b, 1976, 1983, 1992) and other authors[1]. The second one was developed later and interpreted individual rights in terms of game forms[2]: the basic idea is that individual rights can be seen as the possibility given to individuals of choosing a certain strategy among a set of permissible strategies.

The formulation of rights in terms of game form is not intended to deny the contrast between individual rights and the Pareto principle. Indeed, with the game formulation of rights, the liberal paradox becomes a sort of prisoner's dilemma: the Nash equilibrium of the game is not Pareto optimal[3]. However, as Sen himself noted about twenty-five years ago, (Sen 1983, p. 22), the liberal paradox is a 'wider' result than the prisoner dilemma: "[...] Even when each individual can choose his personal 'feature' or 'issue' independently of the choice of others, the impossibility of Paretian liberal can hold without the game's being a variant of the Prisoner's Dilemma." The aim of this paper is to add further evidence and motivation to this claim. In this paper we will consider two efficient games, i.e. two games whose Nash equilibria are Pareto efficient[4], and we will show that that there is no social choice mechanism defined over the outcomes of the games, satisfying individual rights and the Pareto principle.

A similar, but distinct result, from ours can be found in Suzumura (1996, 1999). In these papers, among other things, the author shows that in the attribution of individual rights - which, roughly speaking, concerns the choice of the game to be played - there can emerge a contrast between efficiency and rights themselves. A game is chosen through the "extended social welfare function"[5] (e.g. game *A*) since the Nash equilibrium outcome of this game is preferred (by the "extended social welfare function") to the Nash equilibrium of the other game (e.g. game *B*). Game *A*, however is inefficient[6]. Therefore, as noted by Suzumura (1996, p. 35): "Sen's Pareto libertarian paradox recurs not only in the context of realizing game forms rights, but also in the context of initial conferment of game form rights". Our result is different from Suzumura's (1996, 1999) because all the games we consider are efficient. Furthermore, we follow the classic Arrow-Sen social choice approach.

The rest of the paper is organized as follows: in section two, we will present some mathematical notation together with the basic definitions. In section three we will show our impossibility result: by using both a social choice and a game form articulation of rights we

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[1] See, among others, Gibbard (1974) who found another important impossibility result, according to which there is no social decision mechanism which can satisfy a somewhat more articulated notion of individual rights, even without invoking the Pareto principle. The main results of this kind of literature are surveyed in Sen (1976), Suzumura (1983), Wriglesworth (1985) and Suzumura (1996).

[2] See, among others, Gaertner *et al*. (1992), Deb *et al*. (1997).

[3] One of the first authors who highlighted this similarity was Fine (1975); see also Aldrich (1977).

[4] Therefore the prisoner's dilemma situation is ruled out by definition.

[5] The "extended social welfare function" is a new tool introduced by Pattanaik and Suzumura (1994, 1996). The "extended social welfare function" maps the "extended individual preference orderings" into an "extended social welfare ordering". The "extended individual preference orderings" are individual preferences defined not only on the outcomes of the games but also on the games by which outcomes are generated. It is assumed that individuals are able to say if they prefer to obtain an outcome x under a game G_A or an outcome y under a game G_B. The pair (x, G_A) and (y, G_B) are called "extended social states". (Suzumura 1996, pp. 31-32).

[6] We are aware that we are oversimplifying this rather complex framework and we apologize with the readers.

will prove that in situations in which there are more than one game, there is no social choice mechanism defined over the outcomes of the games respecting both rights (interpreted in Sen-Gibbard way) and paretianism. It should be noted that our result holds also if each game exhibits an efficient Nash equilibrium. Section four shows that our result also holds in presence of 'dictatorial' game forms. Section five, at the end, concludes the paper.

2. Two different ways of formalizing individual rights

Consider a set of individuals $N=\{1, \ldots, n\}$, $\#N = n$, and a set of social states $X = \{x, y, \ldots\}$. We indicate by 2^X the set containing all possible non void subsets[7] of X. Every individual i has an ordering R_{iX} over X (i.e. a complete, reflexive and transitive binary relation). The symmetric part of this relation is denoted by I_{iX}, while the asymmetric part is denoted by P_{iX}. The set of individual orderings over X is denoted by $\mathcal{R}_X$. As we noted in the introduction, in economic literature there are basically two ways of defining individual rights: in this paper we will use both the representations[8].

The second one - from a 'historical' perspective – presents rights in terms of game forms: a *normal game form* G_A is a $(n+3)$-tuple $<N, S_{1A}, \ldots, S_{nA}, f>$ where N is the set of players; for all $i \in N$, S_{iA} is the set of strategies of i; $f : S_{1A} \times \ldots \times S_{nA} \rightarrow A \subset X$ is the outcome function, where A is the set of feasible alternatives (social states or outcomes).

We initially assume that f is a bijection and in particular that f is an identity function. Thanks to this assumption it results that a social state can be presented as the combination of the strategies the agents play. This means that we initially interpret rights by attributing to the individual(s) the entitlement of fixing the characteristic or the feature of the social state which concerns himself[9] and which belongs to the set of his permissible strategies.

We drop this assumption on f when dictatorial game forms will be introduced (section 4). In this case f is assumed to be a surjective function. Given a normal game form G_A and a preference profile $(R_{1A}, \ldots, R_{nA}) \in \mathcal{R}_A^n$, $\Gamma_A = (G_A, R_{1A}, \ldots, R_{nA})$ defines a non cooperative game. $\mathbf{NE}_A$ denotes the set of Nash equilibria outcomes of the game Γ_A and $\mathbf{ENE}_A$ denotes the set of Efficient Nash equilibria outcomes of the game.

We will consider two normal game forms $G_A = < N, S_{1A \ldots} S_{nA}, f >$ and $G_B = <N, S_{1B} \ldots S_{nB}, f>$ (as said above, we initially assume that the outcome function is a identity function both in G_A and G_B). The set of outcomes of the games are denoted by A for G_A and B for G_B, indeed f: $S_{1A} \times \ldots \times S_{nA} \rightarrow A$ and f: $S_{1B} \times \ldots \times S_{nB} \rightarrow B$. The preference profiles are denotes by $(R_{1A}, \ldots, R_{nA}) \in \mathcal{R}_A^n$ and $(R_{1B}, \ldots, R_{nB}) \in \mathcal{R}_B^n$. We assume that $X = A \cup B$ and therefore every individual i not only is able to compare social states in the same game form, but he is also able to compare outcomes belonging to different game forms. The individual ordering over X, as said above, is noted by R_{iX} and includes the orderings over A and B: $R_{iX} = R_{iA} \cup R_{iB} \in \mathcal{R}_A \cup \mathcal{R}_B = \mathcal{R}_X$.

The other formulation of individual rights capitalizes on the notion of some social choice mechanism and was introduced by Sen (1970a, 1970b) and subsequently extended by Gibbard (1974). A (General) Social Choice Rule (**GSCR**) is a function C: $K \times 2^X \rightarrow 2^X$ where

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[7] For simplicity we abused the notation: the correct notation would be $2^X \setminus \{\emptyset\}$.

[8] The following notations and definitions are based on Pattanaik (1996a).

[9] One of the first authors to express this view was Robert Nozick who, commenting on the "liberal paradox", says: "individual rights are co-possible; each person may exercise his rights as he chooses. The exercise of these rights fixes some features of the world. […] Rights do not determine a social ordering but instead set the constraints within which a social choice is to be made, by excluding certain alternatives, fixing others, and so on…". Nozick (1974, p. 166). There are also other definitions of rights when game forms are involved. A relevant one is based on the concept of effectivity function (see, among others, Peleg 1998).

K is a non empty subset of $\mathcal{R}_X{}^n$, such that for all $H \in 2^X$ and for all $R_{1X}, \ldots, R_{nX} \in K$, $C(H; R_{1X}, \ldots, R_{nX}) \subseteq H$. $C(H; R_{1X}, \ldots, R_{nX})$ denotes the choice set; H denotes the set of feasible social states[10]. We assume that $H = X$ where, as said above, $X = A \cup B$. If $K = \mathcal{R}^n$ the (General) Social Choice Rule incorporates in its domain all the logically possible individual orderings and this condition is often called a 'Universalism Condition' (**U**). Otherwise, if $K \subset \mathcal{R}^n$ there is some restriction on individual preferences.
Sen's original definition of rights (condition **L**)[11] assumes that for each individual[12] $j \in N$ there are two distinct alternatives, $x, y \in A$ (or B) such that j is decisive over them. For convenience we denote by D_j the set containing the couple of social states over which j is decisive. We say that the **GSCR** satisfies individual rights iff for all j in N, there exists a couple of social states over which each j in N is decisive. The decisiveness of j over x and y means that for all $(H; R_1, \ldots, R_n) \in 2^X \times \mathcal{R}^n$, [if $\{x,y\} \in D_j$ and xP_jy, then $y \notin C(H; R_1, \ldots, R_n)$] and [if $\{x,y\} \in D_j$ and yP_jx, then $x \notin C(H; R_1, \ldots, R_n)$].
A subsequent definition of individual rights was developed by Gibbard (1974). According to Gibbard's framework, it is assumed that each social state can be decomposed in its elementary aspects or features, each one of these for every individual. In this paper, the set of feasible outcomes A and B of game forms G_A and G_B are product sets[13]: $A = A_1 \times A_2 \times \ldots \times A_n$ and $B = B_1 \times B_2 \times \ldots \times B_n$, where A_i and B_i ($i \in N$) denote the set of characteristics which concern individual i in respectively A and B. We can write, therefore, for each social state $x \in A$, $x = (x_1, \ldots, x_{n-1}, x_n) \in A_1 \times A_2 \times \ldots \times A_n$. For every social state x and y in A and for all j in N, we say that x and y are j-variant iff $x_j \neq y_j$ and for all i in $N \backslash \{j\}$, $x_i = y_i$. Since we have assumed that the outcome function f is an identity function it results: $S_{1A} \times \ldots \times S_{nA} = A = A_1 \times \ldots \times A_n$ and $S_{1B} \times \ldots \times S_{nB} = B = B_1 \times \ldots \times B_n$. Accordingly two social states (outcomes) are j-variant if they only differ in the strategy that agent j plays.
With reference to individual rights, we say that the **GSCR** satisfies condition **L** iff for all j in N, j is decisive over the couple(s) of j-variants. Also in this case we denote by $D_j = D_{jA} \cup D_{jB}$ the set containing the couple(s) of social states over which j is decisive[14]. The decisiveness of j over x and y means that for all $(H; R_1, \ldots, R_n) \in 2^X \times \mathcal{R}^n$, [if $\{x,y\} \in D_{jA}$ (or D_{jB}) and xP_jy, then $y \notin C(H; R_1, \ldots, R_n)$] and [if $\{x,y\} \in D_{jA}$ (or D_{jB}) and yP_jx, then $x \notin C(H; R_1, \ldots, R_n)$]. The idea of condition **L** is quite simple: being x and y different only for a feature which concerns a personal aspect of individual j (since the other features are not variant), it is thought to be convenient to attribute to individual j the right to discard the social state he does not prefer. As pointed out by Pattanaik (1996a, p.103) "[…] the power of i takes the following form: if i prefers x (y respectively) to y (x respectively), then given that x (y respectively) is feasible, y (x respectively) must be thrown out of the set of socially chosen social alternatives."
We say that a **GSCR** satisfies the Pareto condition (**P**) iff for all $(H; R_1, \ldots, R_n) \in 2^X \times \mathcal{R}^n$, [if $x, y \in H$, and xP_iy $\forall i \in N$, then $y \notin C(H; R_1, \ldots, R_n)$] and [if $x, y \in H$, and yP_ix $\forall i \in N$, then $x \notin C(H; R_1, \ldots, R_n)$].

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[10] For notational convenience, we will use hereafter for each individual i, R_i instead of R_{iX} and we will use $\mathcal{R}$ instead of $\mathcal{R}_X$

[11] In the original paper, Sen called this condition "liberty condition".

[12] Condition **L** can be weakened to two individuals.

[13] Gibbard's framework includes also a set A_0 for the 'public features', which are common to all individuals. We do not consider this aspect.

[14] D_{jA} denotes the couples of social states belonging to A over which j is decisive. Symmetrically D_{jB} denotes the couples of outcomes in B which belong to j's protected sphere.

We will assume a restriction of individual preferences (condition **U***). In particular we restrict individual preferences so that each game has a non void set of efficient (pure strategy) Nash equilibria outcomes.

By assuming that each game has a pure strategy Nash equilibrium we are discarding those individual preferences which give rise to the so-called "Gibbard paradox" (Gibbard 1974) which consists in a conflict between conditions **L** and **U**. By assuming that each game has an efficient Nash equilibrium we are not allowing those preferences which give rise to a prisoner's dilemma situation, since our aim is to show that the liberal paradox is a 'wider' result than the prisoner's dilemma. Therefore it follows that in each game conditions **L**, **P** and **U*** are perfectly compatible, since an efficient Nash equilibrium (outcome) belongs to the set of Pareto optima outcomes of the games.

3. An impossibility result

Proposition 1. Let $N = \{i, j\}$. Suppose that both Γ_A and Γ_B are Nash efficient games and that for all $h = i,j$ $\#S_{hA}$, $\#S_{hB} \geq 2$ and $S_{hA} \cap S_{hB} = \varnothing$. Then there exists no **GSCR** defined over $X = A \cup B$ satisfying **L**, **P** and **U***.

Proof.

First of all, we note that by the assumption that f is an identity function, and that the cardinality of individual strategies in both games is at least two, it results that the sphere of rights for both individuals in both games contains at least two different couples of outcomes.

Let $S_{iA} = \{s_{iA}^1, s_{iA}^2\}$, $S_{jA} = \{s_{jA}^1, s_{jA}^2\}$, $S_{iB} = \{s_{iB}^1, s_{iB}^2\}$, $S_{jB} = \{s_{jB}^1, s_{jB}^2\}$, where S_{iA} is the set of strategies of the first individual in game form G_A, S_{iB} is the set of strategies of the first individual in game form G_B, S_{jA} is the set of strategies of the second individual in game form G_A, and S_{jB} is the set of strategies of the second individual in game form G_B.

The matrix representation of G_A is the following:

	s_{jA}^1	s_{jA}^2
s_{iA}^1	(s_{iA}^1, s_{jA}^1)	(s_{iA}^1, s_{jA}^2)
s_{iA}^2	(s_{iA}^2, s_{jA}^1)	(s_{iA}^2, s_{jA}^2)

By condition **U***, assume that individual orderings are so defined over A:

$(s_{iA}^1, s_{jA}^2)\ P_i\ (s_{iA}^2, s_{jA}^2)\ P_i\ (s_{iA}^1, s_{jA}^1)\ P_i\ (s_{iA}^2, s_{jA}^1)$ for i, and

$(s_{iA}^1, s_{jA}^1)\ I_j\ (s_{iA}^2, s_{jA}^1)\ P_j\ (s_{iA}^1, s_{jA}^2)\ I_j\ (s_{iA}^2, s_{jA}^2)$ for j.

The matrix representation of G_B is the following:

	s_{jB}^1	s_{jB}^2
s_{iB}^1	(s_{iB}^1, s_{jB}^1)	(s_{iB}^1, s_{jB}^2)
s_{iB}^2	(s_{iB}^2, s_{jB}^1)	(s_{iB}^2, s_{jB}^2)

By condition **U*,** assume that individual orderings over B are so defined:

$(s_{iB}^1, s_{jB}^1)\ I_i\ (s_{iB}^1, s_{jB}^2)\ P_i\ (s_{iB}^2, s_{jB}^1)\ I_i\ (s_{iB}^2, s_{jB}^2)$ for i, and

$(s_{iB}^2, s_{jB}^1)\ P_j\ (s_{iB}^2, s_{jB}^2)\ P_j\ (s_{iB}^1, s_{jB}^1)\ P_j\ (s_{iB}^1, s_{jB}^2)$ for j.

It is easy to see that Γ_A has $\mathbf{ENE}_A = \{(s_{iA}^1, s_{jA}^1)\}$ and that Γ_B has $\mathbf{ENE}_B = \{(s_{iB}^1, s_{jB}^1)\}$.

Always by **U***, assume that individual's orderings over $A \cup B = X$ are so defined:

i	j
(s_{iA}^{1}, s_{jA}^{2})	(s_{iB}^{2}, s_{jB}^{1})
(s_{iA}^{2}, s_{jA}^{2})	(s_{iB}^{2}, s_{jB}^{2})
(s_{iB}^{1}, s_{jB}^{1}) (s_{iB}^{1}, s_{jB}^{2})	(s_{iA}^{1}, s_{jA}^{1}) (s_{iA}^{2}, s_{jA}^{1})
(s_{iB}^{2}, s_{jB}^{1}) (s_{iB}^{2}, s_{jB}^{2})	(s_{iA}^{1}, s_{jA}^{2}) (s_{iA}^{2}, s_{jA}^{2})
(s_{iA}^{1}, s_{jA}^{1})	(s_{iB}^{1}, s_{jB}^{1})
(s_{iA}^{2}, s_{jA}^{1})	(s_{iB}^{1}, s_{jB}^{2})

Note that when a social state is written over another it means that, according to individual's ordering, the former is strictly preferred to the latter and when two social states are in the same row it means that there is a relation of indifference between them.
According to our definition of **L** we have:
$D_{iA} = \{\{(s_{iA}^{1}, s_{jA}^{2}), (s_{iA}^{2}, s_{jA}^{2})\}, \{(s_{iA}^{1}, s_{jA}^{1}), (s_{iA}^{2}, s_{jA}^{1})\}\}$, $D_{iB} = \{\{(s_{iB}^{1}, s_{jB}^{1}), (s_{iB}^{2}, s_{jB}^{1})\}, \{(s_{iB}^{1}, s_{jB}^{2}), (s_{iB}^{2}, s_{jB}^{2})\}\}$, $D_{jA} = \{\{(s_{iA}^{1}, s_{jA}^{1}), (s_{iA}^{1}, s_{jA}^{2})\}, \{(s_{iA}^{2}, s_{jA}^{1}), (s_{iA}^{2}, s_{jA}^{2})\}\}$, $D_{jB} = \{\{(s_{iB}^{2}, s_{jB}^{1}), (s_{iB}^{2}, s_{jB}^{2})\}, \{(s_{iB}^{1}, s_{jB}^{1}), (s_{iB}^{1}, s_{jB}^{2})\}\}$.
It is straightforward to see, even if boring, that by condition **L** we have: (s_{iA}^{1}, s_{jA}^{2}), (s_{iA}^{2}, s_{jA}^{2}), (s_{iA}^{2}, s_{jA}^{1}), (s_{iB}^{1}, s_{jB}^{2}), (s_{iB}^{2}, s_{jB}^{1}), $(s_{iB}^{2}, s_{jB}^{2}) \notin C(A \cup B; R_i, R_j)$ so that condition **L** alone does not yield to an empty choice set.
Finally, in order to prove the impossibility result, by condition **P** we have:
$[(s_{iA}^{2}, s_{jA}^{2})\ P_{i,j}\ (s_{iB}^{1}, s_{jB}^{1})] \rightarrow (s_{iB}^{1}, s_{jB}^{1}) \notin C(A \cup B; R_i, R_j)$ and
$[(s_{iB}^{2}, s_{jB}^{2})\ P_{i,j}\ (s_{iA}^{1}, s_{jA}^{1})] \rightarrow (s_{iA}^{1}, s_{jA}^{1}) \notin C(A \cup B; R_i, R_j)$.
Thus we obtain $C(A \cup B; R_i, R_j) = \varnothing$ ■

It is well-known in literature that the definition of rights in term of game forms is not sufficient to avoid the conflict between paretianism and individual rights[15]. Indeed **Proposition 1** shows another kind of inconsistency: even if in the single games there is not any tension, in the 'enlarged situation' which includes all the outcomes of the two games, the liberal paradox still returns. This result, therefore, gives support to Sen's claim (Sen 1983, p. 22) that the liberal paradox is a wider result than the prisoner's dilemma.

Example 1
There are two individuals which are a prude and a lewd and two books, Lady Chatterley's Lover and The Adventures of Pinocchio. Regarding Lady Chatterley's Lover, we assume, following Sen's original example, that the individual preferences can be so described:
"1) Lewd essentially wants prude to read the book [Lady Chatterley's Lover] and wants to read the book himself. 2) Prude essentially wants lewd not to read the book [Lady Chatterley's Lover] and does not want to read it himself.[16]"
With reference to The Adventures of Pinocchio, we assume that individual preferences are reversed, i.e.: 1) Prude essentially wants lewd to read The Adventures of Pinocchio and wants to read The Adventures of Pinocchio himself. 2) Lewd essentially wants prude not to read The Adventures of Pinocchio and does not want to read it himself.
Given these preferences, we consider the following game form *A*:

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[15] Such conflict "persists under virtually every plausible concept of individual rights that we can think of"(Gaertner *et al.* 1992, p. 161). For a different view, see Pattanaik (1996b).
[16] Fine (1975, p. 1279).

	r_{jA}	n_{jA}
r_{iA}	(r_{iA}, r_{jA})	(r_{iA}, n_{jA})
n_{iA}	(n_{iA}, r_{jA})	(n_{iA}, n_{jA})

where i stands for Lewd and j for the Prude. In G_A we assume that j has the availability of Lady Chatterley's Lover and may decide to read it (r_{jA}) or not to read it (n_{jA}). At the same time i has the The Adventures of Pinocchio and may decide to read it (r_{iA}) or not to do it (n_{iA}). Obviously, given the individual preferences, n_{iA} is the dominant strategy for i, n_{jA} is the dominant strategy for j and therefore the pure strategy Nash equilibrium of the game is (n_{iA}, n_{jA}).

	r_{jB}	n_{jB}
r_{iB}	(r_{iB}, r_{jB})	(r_{iB}, n_{jB})
n_{iB}	(n_{iB}, r_{jB})	(n_{iB}, n_{jB})

In game form B we assume that i has the availability of Lady Chatterley's Lover and j has the availability of The Adventures of Pinocchio. Obviously, given the individual preferences, r_{iB} is the dominant strategy for i and r_{jB} is the dominant strategy for j and therefore game B has (r_{iB}, r_{jB}) a pure strategy Nash equilibrium.
We also assume that individual's orderings over $X=A \cup B$ are defined as follows:

i	j
(r_{iB}, n_{jB}) (n_{iA}, r_{jA})	(n_{iB}, r_{jB}) (r_{iA}, n_{jA})
(r_{iA}, r_{jA})	(n_{iB}, n_{jB})
(r_{iB}, r_{jB})	(n_{iA}, n_{jA})
(n_{iB}, n_{jB})	(r_{iA}, r_{jA})
(n_{iA}, n_{jA})	(r_{iB}, r_{jB})
(n_{iB}, r_{jB}) (r_{iA}, n_{jA})	(r_{iB}, n_{jB}) (n_{iA}, r_{jA})

Therefore it can be easily seen that each game is not a prisoner's dilemma since the Nash equilibrium (outcome) is efficient. However each Nash equilibrium (outcome) is Pareto dominated by an outcome belonging to the other game. Indeed it results: (n_{iB}, n_{jB}) P_{ij} (n_{iA}, n_{jA}) and (r_{iA}, r_{jA}) P_{ij} (r_{iB}, r_{jB}).

As it is well known the liberal paradox gives strong reasons to question the ethical validity of the Pareto principle. A relevant one is the following: a way to bypass the paradox is to assume that individuals could stipulate a Pareto improving contract in order to reach the most efficient outcome (i.e. (n_{iB}, n_{jB}) or (r_{iA}, r_{jA})). However, since these outcomes are not Nash equilibria outcomes, there would be a natural tendency to break this agreement. Therefore, these contracts in order to be 'effective' should be publicly enforceable. However, "[t]he role of an enforcer checking […] whether the prude has broken his agreement to read Lady Chatterley's Lover every morning […] is morally problematic, aside from being deeply chilling[17]". Indeed it is questionable that a society in which a policeman checks that prude or lewd are reading (or that they are forbidden from reading) the books is a liberal society.

[17] Sen (1983, p. 25).

Therefore, if it is problematic to accept the moral validity of such contracts in a "liberal's dilemma"[18] situation, it is, probably, even more troublesome to accept this in a game whose Nash equilibrium outcome is also efficient[19].

A distinct but related question concerns the choice of game. In other terms, which is better? The Nash equilibrium outcome of game *A* or the Nash equilibrium outcome of game *B*? In order to address this question we have to tackle the issue concerning the "initial conferment of individual rights" (Suzumura and Yoshihara 2008)[20]. Suzumura and Yoshihara use a two stage procedure. The game form is socially chosen in the first stage, using an extended social welfare function, while in the second stage agents play the game previously chosen. In this paper we do not try to tackle this issue, since our aim is to show that the liberal paradox is a wider result than the prisoner's dilemma; in doing so we use the classic Arrow-Sen social choice framework.

4. Dictatorial game forms

In this section we consider the situation in which games are dictatorial and we ask whether our impossibility result is present or not. We begin by defining a dictator.
Let $N = \{i, j\}$, let S_{iC} and S_{jC} denote the set of strategies of the agents, let C ($\#C \geq 2$) denotes the set of outcomes of game form G_C and let $f: S_{iC} \times S_{jC} \rightarrow C$ denotes the outcome function.
We say that a game form G_C is *dictatorial* if there exists one and only individual $i \in N$ such that for all $s_{iC} \in S_{iC}$, for all $s_{jC}, s_{jC}' \in S_{jC}$ $(s_{jC} \neq s_{jC}')$: $f(s_{iC}, s_{jC}) = f(s_{iC}, s_{jC}')$.
This means that individual *i* is able to 'force' the game towards his desired outcome, irrespective of the strategy chosen by the other agent *l*. Indeed the strategies the other agent plays do not affect the outcome. Agent *i* is thus called a *dictator*.
Before stating the following result, we note that in a dictatorial game form, the outcome function f is a surjective function[21].
Given $N = \{i, j\}$, we also say that two game forms G_A and G_B are *reversed dictatorial* if one individual is a dictator in one of them and the remaining individual is a dictator in the other game form. For reversed dictatorial game forms it is possible to show the following result:

Proposition 2.
Let $N = \{i, j\}$. Suppose that both Γ_A and Γ_B are *reversed dictatorial* games with $\#A, \#B \geq 2$ and $A \cap B = \emptyset$. Then there exists no **GSCR** defined over $X = A \cup B$ satisfying **P**, **U** and **L**.

Proof.
Let $S_{iA} = \{s_{iA}^1, s_{iA}^2\}$, $S_{jA} = \{s_{jA}^1, s_{jA}^2\}$, $S_{iB} = \{s_{iB}^1, s_{iB}^2\}$, $S_{jB} = \{s_{jB}^1, s_{jB}^2\}$, where S_{iA} is the set of strategies of the first individual in game form G_A, S_{iB} is the set of strategies of the first individual in game form G_B, S_{jA} is the set of strategies of the second individual in game form G_A, and S_{jB} is the set of strategies of the second individual in game form G_B.
The matrix representation of G_A is the following:

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[18] The expression "liberal's dilemma" is due to Fine (1975).
[19] In this latter situation a Pareto improving contract should: 1) redistribute individual rights (by 'shifting' from a game to another); 2) oblige individuals to a certain behaviour in the new game.
[20] See, also, the line of research inaugurated by Pattanaik and Suzumura (1994, 1996), Suzumura (1996) and Suzumura (1999).
[21] In **Proposition 1**, on the contrary we assumed that the outcome function *f* was a bijection.

	s_{jA}^1	s_{jA}^2
s_{iA}^1	x	x
s_{iA}^2	y	y

It can be easily checked that G_A is a dictatorial game form and that i is the dictator. The matrix representation of G_B is the following:

	s_{jB}^1	s_{jB}^2
s_{iB}^1	w	z
s_{iB}^2	w	z

Also G_B is a dictatorial game form and j is the dictator.
By condition **U**, assume that individual orderings over $A \cup B$ are so defined:

i	j
w	y
x	z
y	w
z	x

It is easy to see that Γ_A has $\mathbf{ENE_A} = \{x\}$ and that Γ_B has $\mathbf{ENE_B} = \{z\}$.
According to the social choice framework, the individual rights can be described in the following way: $D_{iA} = \{\{x, y\}\}$, $D_{iB} = \{\varnothing\}$, $D_{jB} = \{\{z, w\}\}$ and $D_{jA} = \{\varnothing\}$. Therefore by **L** we have: $y, w \notin C(A \cup B; R_i, R_j)$. Finally, by condition **P** we have:
$[w\ P_{i,j}\ x] \rightarrow x \notin C(A \cup B; R_i, R_j)$ and $[y\ P_{i,j}\ z] \rightarrow z \notin C(A \cup B; R_i, R_j)$.
Thus we obtain $C(A \cup B; R_i, R_j) = \varnothing$. ■

The following example presents Sen's original example in terms of dictatorial game forms.

Example 2.
Let's consider again the "Lady Chatterley's Lover" example, by assuming, as in the original Sen's example, that there is a single copy of this book.
In G_A agent i (lewd) has the availability of the book and may decide to read (r_{iA}) it or not (n_{iA}). In the same game form the other player (prude) may just approve (a_{jA}) or disapprove (d_{jA}) i's choice without affecting it (and without affecting the payoffs of the game). Agent i is thus a dictator for G_A. In G_B j has the availability of the book, and thus G_A and G_B are *reversed dictatorial* game forms.
In G_A we have:

	a_{jA}	d_{jA}
r_{iA}	x	x
n_{iA}	y	y

In G_B we have:

	r_{jB}	n_{jB}
a_{iB}	z	w
d_{iB}	z	w

where the outcomes are so defined:
x = (lewd reads the book, prude does not read it);
y = (lewd does not read the book, prude does not read it);
w =(lewd does not read the book, prude does not read it) [22];
z = (lewd does not read the book, prude reads it).
Now, let's assume that individual preferences are the following:

i	j
z	y w
x	z
y w	x

According to individual preferences, x is the Nash equilibrium outcome of game A and w is the Nash equilibrium outcome of game B. Both games are obviously efficient. By condition **P**, x is ruled out since it results $zP_{i,j}x$. By condition **L**, y and z are ruled out. Therefore the chosen state is w.[23]

We note that the definition of individual rights in this example is similar to Sen's original example (each individual has only one couple of social states over which she/he is decisive). Therefore there can be an analogy between the individual decisiveness over a couple of social states and the fact that the individual is a dictator over a game with two outcomes. However, while in Sen's original example there is just one social state for the situation in which nobody reads the book (i.e. y = (lewd does not read the book, prude does not read it)) in our example it seems more proper to have two different social states (i.e. y and w) for the same situation. This explains why in our example the impossibility result does not occur[24]. Although Example 2 sheds a new light on the original Sen's example, it is not intended, obviously, to be a "solution" to the liberal paradox. Indeed, it is always possible to identify suitable individual orderings such that the impossibility result is present.

5. Concluding remarks

In this paper we show an impossibility result similar to the liberal paradox by using a game form definition of rights. Since, as it is well known, the definition of rights in term of game forms is not sufficient to avoid the contrast with the Pareto principle, we find a particular kind of inconsistency. By considering two different games, our results show that even if in each game there is no tension between paretianism and rights (indeed each game exhibits one

[22] We note that, on principle, outcome y in game A is different from outcome w in game B. The fact that I do not read the book, although I have the right to do so is different from the fact that the book is not available to me.

[23] If we decide to give up the Pareto principle because of the reasons we mentioned in section 3, the chosen states become x and w. This brings us back again to the question of "initial conferment of individual rights" (Suzumura and Yoshihara 2008).

[24] If we had only one social state for this situation (i.e. y), then by **L**, individual i would be able to discard this social state, since $D_{iA} = \{\{x, y\}\}$ and xP_iy. However, this would involve that i is able to rule out an outcome (y) which belongs also to the other game form (G_B) over which j is dictator. This is obviously arbitrary and does not make much sense.

efficient Nash equilibrium which is, by definition, a Pareto optimal result), in the 'enlarged situation' defined over all the outcomes of the two games the paradox returns again.
Finally, we introduce the *reversed dictatorial game forms* category and show that also in this case the impossibility result can be found.

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