The Choice of Tax Bases under Fiscal Federalism and the Unitary System

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Abstract
This paper analyzes the government’s choice of three proportional tax bases (a consumption tax, a wage tax and a capital income tax) and its influence on the steady-state levels of capital accumulation and social welfare under fiscal federalism and the unitary system. We report two main findings: first, the system that uses a consumption and a capital income tax bases yields a higher steady state level of capital accumulation than that which uses a wage tax base. When each system uses the same tax base, then the steady state levels of capital accumulation under the two systems are equivalent. Second, the social welfare levels in fiscal federalism and the unitary system under a consumption tax base are equivalent if individuals’ rate of time preference just equal to the interest rate, while the social welfare levels in the two systems under the wage and capital income tax bases are equivalent if and only if the rate of time preference is equal to the population growth rate.

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1. Introduction
The proper choice of tax bases has important implications for the course of saving and economic growth, welfare distribution among generations, and the level of economic efficiency (Auerbach and Kotlikoff, 1987). This paper presents an analysis on the choice of three proportional tax bases (a consumption tax, a wage tax and a capital income tax) and its influence toward the steady-state capital accumulation and social welfare under fiscal federalism and the unitary system in an overlapping generations model. We introduce three tax regimes: the consumption tax, the wage tax, and the capital income tax regimes which are independently applied to both systems, fiscal federalism and the unitary system. Under each regime, we analyze two possible cases which correspond to each system. In this sense, under the consumption tax regime, we introduce case A, which is when a consumption tax is imposed by the government on young and old generations under fiscal federalism, and case D when it is imposed on both generations under the unitary system. On the same fashion under the wage tax regime, we introduce case B when a wage tax is imposed by the government only on young generation under fiscal federalism and case E when it is imposed only on young generation under the unitary system. Lastly, under the capital income tax regime, when a capital income tax is imposed by the government only on old generation under fiscal federalism, we call it case C, while when it is imposed only on old generation under the unitary system, we name it case F. We then analyze and make comparisons among these cases in terms of their steady-state levels of capital accumulation and social welfare.

The basic theoretical principle of fiscal federalism is perhaps due to Tiebout (1956) who hypothesize that competition among communities might result in an efficiency level of the public good provision at the local level, if fully mobile households could choose a jurisdiction or a locality that provides the best fiscal packages, which met their preferences. This conjecture is further elaborated by Oates (1972, 1993, 1999), and supported by, among others, Bird (1993), Gramlich (1993) and Brueckner (1999, 2006). However, Bewley (1981) and Gordon (1983), among others, present the opposite views, suggesting that the Tiebout-Oates conjecture in favor of fiscal federalism may no longer hold.

Despite few attempts at theoretical analyses, there has been substantial research in the empirical arena focusing on the relationship between fiscal decentralization and economic growth. Empirical evidences on this relationship are mixed, however. Weingast (1995), Lin and Liu (2000), Akai and Sakata (2002), Thiesen (2003), Stansel (2005), Iimi (2005), and Jin, Qian, and Weingast (2005), among others, have found a positive relationship between fiscal decentralization and economic growth after conducting a variety of country case studies, while some authors find a negative relation (e.g., Zhang and Zou, 1998; Davoodi and Zou, 1998; Xie, Zou and Davoodi, 1999), or no relation (e.g., Woller and Phillips, 1998; Thornton, 2007).

The objective of this paper is to fill the gap in the ongoing theoretical literatures of fiscal federalism that focuses on the dynamic aspects of the choice of tax bases toward the steady-state levels of capital accumulation and social welfare. This analysis, to our knowledge, is not well established in academic literatures. Our basic model mainly relies on the work of Brueckner (1999) and some parts of our formulation exhibit a similar pattern to that of Brueckner (2006). We differ from these studies in two respects. First, we formulate the behavior of the government under the two systems in maximizing social welfare by introducing six possible cases of the government’s taxing policy toward individuals (in order to finance the public goods provision), where previous models described the behavior of both individuals and government in a simultaneous-move Nash game. In this sense, we consider the government’s choice on the proportional tax bases, where previous models use a head tax
instrument.\footnote{In fact, we have also analyzed the government taxing policy by using a head tax instrument, following the similar pattern of analysis we use in this paper. The result is consistent with the findings which this paper has drawn.} Second, we clarify the social welfare comparison between the two systems, which is informally argued in the Brueckner (1999) while we depart from Brueckner (2006) by abstracting our analysis from human capital and economic growth.

We report two main findings: first, the system that uses a consumption and a capital income tax bases will have a higher steady state level of capital accumulation than the system that uses a wage tax base. Our first finding provides another interpretation on the understanding of steady-state level of capital accumulation in both systems, as previously argued by Brueckner (1999). In fact, he claimed that the steady-state level of capital accumulation in fiscal federalism is higher (lower) than that of under the unitary system if the young generation has a lower (higher) demand for public goods (which will influence the savings level). In this formulation, our finding suggests that, as long as each system uses the same tax base, then the steady state levels of capital accumulation under the two systems are equivalent.

Second, the social welfare levels in fiscal federalism and the unitary system under a consumption tax base are equivalent if individuals’ rate of time preference is equal to the interest rate at which they choose their level of consumption stream. In addition, the social welfare levels in the two systems under the wage and capital income tax bases are equivalent if and only if the rate of time preference is equal to the population growth rate.

Our second finding might be in contrast with the result suggested by Brueckner (1999), which showed that, as long as the golden rule welfare condition—the condition in which the marginal product of capital in the steady state is defined to be equal to the population growth rate—is satisfied, the social welfare level under fiscal federalism is greater than that under the unitary system. In our formulation, this golden rule welfare condition is a sufficient condition which makes the level of social welfare level under fiscal federalism is equal to that of under the unitary system.

The rest of the paper is organized as follows. Section 2 presents the model, while section 3 provides equilibrium characteristics and the solutions of the model. Section 4 presents the comparison between the two systems. Section 5 concludes the paper.

2. The Model

The basic framework relies on the model of Brueckner (1999), modified to include the six possible cases on how government’s choice of the tax bases policy could be formulated. In our model, each region is populated by two generations, the young and old, who are assumed to live for two periods. When young, individual works and divides the resulting labor income between consumption, saving and a tax payment. Then, during the old period, the individual consumes the savings and any interest he or she earns, pays a tax and dies. In all cases, we assume that the population grows at a constant rate \( n \), where \( n > 0 \). In this case, we assume that the population of the young generation is as large as \((1+n)\) of the old population. The consumption of both generations is divided into consumption of private goods, \( c_i \), and of public goods, \( g_i \), where subscript \( i \) denotes cases A, B, C, D, E and F. Following Brueckner (1999), public good is provided by the government and could be consumed by both generations. It is assumed that this public good is a publicly produced private good. In addition, the difference between fiscal federalism and the unitary system is that, under fiscal federalism, each generation is living in a segregated homogenous community; while in the unitary system, both generations are living together in the same
community. In this federalist system, any kinds of public goods such as police protection and recreational place could then be provided specifically by following a specific demand of the young and old. Although the assumption of a segregate community under the federalist system is lack of realism due to the facts that, as in the spirit of Brueckner (1999), most communities are usually inhabited by both young and old generations, we might hope that this formulation might present a reference for academic exercises and a practical relevance for any related policies on this ground. Needless to say, for all public goods provisions, we abstract from the constraint of capacity and congestion.

The public goods provision is financed by a tax, \( \tau \), imposed on young and/or old generations. In addition to the subscript \( i \) mentioned earlier, we also use the time subscripts \( t \) and \( t + 1 \), throughout this paper, which denote the periods, and the superscripts \( y \) and \( o \), which denote the young and old, respectively. The per-capita consumption of private goods are \( c^y_i \) (for young individuals born at \( t \) in case \( i \)) and \( c^o_{i+1} \) (for old individuals born at \( t \) in case \( i \)), while analogous definitions also apply to consumption taxes, \( \tau^y_{i,t} \) and \( \tau^o_{i,t+1} \); a wage tax, \( \tau_{i,wt} \), and a capital income tax, \( \tau_{i,ct} \).

2.1. Individual behavior under Fiscal Federalism (case A, B and C)

The respective budget constraints for the young and old in case A are

\[
\begin{align*}
\text{(1)} & \quad w_{t,y} = c^y_{t,y} (1 + \tau^y_{t,y}) + s_{t,y}, \\
\text{(2)} & \quad (1 + r_{t+1}) s_{t,y} = c^y_{t+1} (1 + \tau^y_{t+1}),
\end{align*}
\]

where \( s_{t,y} \) and \( w_{t,y} \) respectively, are the level of saving and wage of the young individual at \( t \), while \( r_{t+1} \) is the level of interest rate. From (1) and (2), the lifetime budget constraint is

\[
\begin{align*}
\text{(3)} & \quad w_{t,y} = c^y_{t,y} (1 + \tau^y_{t,y}) + \frac{c^o_{t+1} (1 + \tau^o_{t+1})}{1 + r_{t+1}}.
\end{align*}
\]

The formulation of individuals’ utility function adopts the Brueckner’s (2006) type. The utility function is separable for both generations, in which, utility of the old is discounted by a rate of time preference, \( \rho \). For simplicity, we define the level of \( \rho \) as well as the level of \( \alpha \) are all identical for all cases. Thus, the utility function of generation \( t \) individual is assumed to be a log-linear utility function and can be given as

\[
U_t = \alpha \log c^y_t + (1 - \alpha) \log g^y_t + \frac{1}{1+\rho} [\alpha \log c^o_{t+1} + (1 - \alpha) \log g^o_{t+1}], \quad 0 < \alpha < 1. \tag{4}
\]

where in this function, \( g^y_t \) and \( g^o_{t+1} \) respectively, denote the consumption of public goods by the young and old born at \( t \). Under this function, individuals maximize their utility subject to budget constraint as described in equation (3). By defining \( \lambda \) as a Lagrange-multiplier, we can perform an optimization procedure to obtain

\[
\begin{align*}
\text{(5)} & \quad c^y_{t,y} = \frac{(1 + r_{t+1})(1 + \tau^y_{t,y})}{(1 + \rho)(1 + \tau^y_{t,y})},
\end{align*}
\]

in which, by using (5) and (3), we can derive the \( c^y_t \) and \( c^o_{t+1} \) respectively as

\[
\begin{align*}
\text{(6)} & \quad c^y_t = \frac{1 + \rho}{2 + \rho} \frac{w_{t,y}}{1 + \tau^y_{t,y}}.
\end{align*}
\]
From equations (6) or (7) and (3), an individual’s saving function can be stated as

\[ s_{At} = \frac{1}{2 + \rho} w_{At}. \]  

(8)

In case B, the budget constraints for the young and old respectively, are

\[ w_{Bt} (1 - \tau_{Bw}) = c_{Bt}^y + s_{Bt}, \]  

(9)

\[ (1 + r_{Bt+1}) s_{Bt} = c_{Bt+1}^o. \]  

(10)

By following an optimization procedure as explained above, we can derive \( c_{Bt}^y, c_{Bt+1}^o \) and \( s_{Bt} \) as follows:

\[ c_{Bt}^y = \frac{1 + \rho}{2 + \rho} w_{Bt} (1 - \tau_{Bw}), \]  

(11)

\[ c_{Bt+1}^o = \frac{1 + r_{Bt+1}}{2 + \rho} w_{Bt} (1 - \tau_{Bw}), \]  

(12)

\[ s_{Bt} = \frac{1}{2 + \rho} w_{Bt} (1 - \tau_{Bw}). \]  

(13)

Finally, in case C, we formulate the budget constraints as follows:

\[ w_{Ct} = c_{Ct}^y + s_{Ct}, \]  

(14)

\[ (1 + r_{Ct+1}) s_{Ct} = c_{Ct+1}^o. \]  

(15)

By following a similar standard optimization procedure we can obtain \( c_{Ct}^y, c_{Ct+1}^o \) and \( s_{Ct} \) as follows:

\[ c_{Ct}^y = \frac{1 + \rho}{2 + \rho} w_{Ct}, \]  

(16)

\[ c_{Ct+1}^o = \frac{1 + r_{Ct+1} (1 - \tau_{Ct+1})}{2 + \rho} w_{Ct}, \]  

(17)

\[ s_{Ct} = \frac{1}{2 + \rho} w_{Ct}. \]  

(18)

2.2. Individual behavior under the Unitary System (case D, E and F)

In the cases D, E and F, we follow the same formulation as previously conducted in the cases A, B and C respectively. We might then get the levels of private consumption and saving under these cases which are similar to those under fiscal federalism by adjusting the relevant subscripts for each case.

2.3. Firm’s production function

In each system, firm produces goods, pays wages for the labor input, \( L_t \), and makes rental payments for the capital input, \( K_t \). Technology is represented by a production function:
\[ Y_i = K_i^\beta L_i^{1-\beta}, \] which exhibits constant returns to scale \((0 < \beta < 1)\). The per-capita term of the production function is
\[ y_i = k_i^\beta, \] (19)
where the output-labor ratio and capital-labor ratio, respectively, are:
\[ y_i = \frac{Y_i}{L_i}, k_i = \frac{K_i}{L_i}. \]
The profit maximizing condition of a representative firm yields
\[ r_i = \beta k_i^{\beta-1}, \] (20)
\[ w_i = (1 - \beta)k_i^\beta, \] (21)
where \(r_i\) and \(w_i\) both describe the factor prices of production inputs.

2.4. Equilibrium
Capital market clearing condition is defined such that a total saving of the young generation is equal to a capital stock in the next period. This condition could be stated as
\[ s_i = (1 + n)k_{i+1}. \] (22)
By substituting the level of saving of each case to this equation, we can get the levels of the next period capital stock for the six cases as follows:
\[ k_{A_{i+1}} = \frac{1}{(2 + \rho)}(1 - \beta)k_{A_i}^\beta, \] (23)
\[ k_{B_{i+1}} = \frac{1}{(2 + \rho)}(1 - \beta)k_{B_i}^\beta (1 - \tau_{B_{i+1}}), \] (24)
\[ k_{C_{i+1}} = \frac{1}{(2 + \rho)}(1 - \beta)k_{C_i}^\beta (1 - \tau_{C_{i+1}}), \] (25)
\[ k_{D_{i+1}} = \frac{1}{(2 + \rho)}(1 - \beta)k_{D_i}^\beta, \] (26)
\[ k_{E_{i+1}} = \frac{1}{(2 + \rho)}(1 - \beta)k_{E_i}^\beta (1 - \tau_{E_{i+1}}), \] (27)
\[ k_{F_{i+1}} = \frac{1}{(2 + \rho)}(1 - \beta)k_{F_i}^\beta. \] (28)

3. Government’s behavior
To see the effect of government’s behavior, suppose that the economy is on the steady state. In this sense, the capital stock per worker is constant from one period to the next, which implies the marginal products of capital and labor are then constant over time, yielding a constant wage \(w\) and interest rate \(r\). To further analyze the capital stock accumulation in a steady state, let \(k_{i+1}^*, k_{i}^*, \tau_i^y, \tau_i^c, \tau_i^\tau, \tau_i^u, \tau_i^{\tau+1} = \tau_i^y\) where \(k_i^*, \tau_i^c, \tau_i^\tau, \tau_i^u\) represent the steady-state values of, respectively, the capital stock, a consumption tax for each young and old, a wage tax and a capital income tax in case \(i\). We then consider this steady state condition while conducting analyses on the government’s behavior.

3.1. Under fiscal federalism
In this system, the regional government chooses the optimal values of public goods by considering the behavior of individuals’ born at certain generation. Since we are considering the behavior of the government in the steady state, maximizing the social welfare of certain generation is similar to the maximizing the social welfare of generations living at certain period. We formulate the budget constraint of government as
\[ ag_A = \tau_A^y c_A^y, \] (29)
\[ a g_A^\circ = \tau_A^\circ c_A^\circ, \]  
for the case A, and
\[ a\left( g_B^\gamma + \frac{g_B^\circ}{1+n} \right) = \tau_B^w w_B, \]  
for the case B, and finally,
\[ a\left((1+n)g_C^\gamma + g_C^\circ \right) = \tau_C^w r_C s_C, \]  
for the case C.

3.2. Under the Unitary System
In this system, the government can only provide a common level of public goods for both generations. The budget constraints of the government in cases D, E and F respectively could be given as
\[ ag_D = \frac{1}{(2+n)} \tau_{D_D} \left((1+n)c_D^\gamma + c_D^\circ \right), \]  
\[ ag_E = \frac{1}{(2+n)} \left((1+n)\tau_{E_E} w_E \right), \]  
\[ ag_F = \frac{1}{(2+n)} \left(\tau_{F_E} r_F s_F \right), \]  
In the equations (29)-(35), \( a \) is a linear technology parameter in the production of public goods and assumed to be equivalent in all six cases.

3.3. The levels of public goods and proportional taxes
We first rearrange the equations (20) and (21) to become
\[ r_i = \beta k_i^{\beta-1}, \]  
\[ w_i = (1 - \beta) k_i^\beta. \]  
In fiscal federalism, the regional government could determine each level of consumption tax for both generations in order to provide a specific level of public goods for them. In order to maximize the generation \( t \)'s utility level in each period of its life, the regional government defines its objective function as
\[ W_i = \alpha \log c_i^\gamma + (1-\alpha) \log g_i^\gamma + \frac{1}{1+\rho} (\alpha \log c_i^\circ + (1-\alpha) \log g_i^\circ). \]  
By using (6), (7), (29) and (30), we can solve (36) for the case A and get
\[ \tau_{A_k} = \tau_A^\circ = \frac{1-\alpha}{\alpha}. \]  
and accordingly,
\[ g_A^\gamma = \frac{(1-\alpha)(1+\rho)}{a(2+n)} (1-\beta) k_A^\beta, \]  
\[ g_A^\circ = \frac{(1-\alpha)(1+\beta k_A^{\beta-1})}{a(2+n)} (1-\beta) k_A^\beta. \]  
By following a similar fashion, we can get
\[ g_B = \frac{(1 - \alpha)(1 + \rho)}{a(2 + \rho)} (1 - \beta)k_B^\beta, \]  
\[ g_C = \frac{(1 - \alpha)(1 + n)}{a(2 + \rho)} (1 - \beta)k_C^\beta, \]  
\[ \tau_B = (1 - \alpha), \]  
\[ \tau_C = (1 - \alpha), \]  
for the case B, and
\[ g_C = \frac{(1 - \alpha)(1 + \rho) + 1 + \beta k_{B}^{\beta - 1}}{a(2 + \rho)(1 - \alpha)} (1 - \beta)k_C^\beta, \]  
\[ g_C = \frac{(1 - \alpha)}{a(2 + \rho)(1 - \alpha)} (1 - \beta)k_C^\beta, \]  
\[ \tau_C = \frac{(2 + \rho)(1 - \alpha)}{[\alpha + (2 + \rho)(1 - \alpha)] (\beta k_C^{\beta - 1})}, \]  
for the case C (Please see Appendices A, B and C for more details).

In the unitary system, taxes impose on both generations follow the similar formulation to that of under fiscal federalism (Please see Appendices D, E and F for more details). For the case D, the consumption tax is equivalent between the two generations, which is equal to \((1 - \alpha)/\alpha\), and accordingly, the public good level for this case is
\[ g_D = \frac{(1 - \alpha)(1 - \beta)k_D^\beta}{a(2 + n)(2 + \rho)} (1 + n)(1 + \rho) + (1 + \beta k_D^{\beta - 1}), \]  
As for the case E, we again follow the similar aforementioned procedure to get
\[ g_E = \frac{(1 - \alpha)(1 + n)}{a(2 + n)} (1 - \beta)k_E^\beta, \]  
\[ \tau_E = (1 - \alpha), \]  
and finally in the case F, we can obtain
\[ g_F = \frac{(1 - \alpha)(1 + \rho) + 1 + \beta k_{F}^{\beta - 1}}{a(2 + n)(1 - \alpha)} (1 - \beta)k_F^\beta, \]  
\[ \tau_F = \frac{(2 + \rho)(1 - \alpha)}{[\alpha + (2 + \rho)(1 - \alpha)] (\beta k_F^{\beta - 1})}. \]

3.4. The Steady State of Capital Accumulation

After inserting the relevant wage tax values into the equations (24) and (27), we can obtain the values of steady state of capital accumulation as previously stated in equations (23)-(28) as follow
\[ k_A^* = \left(\frac{1 - \beta}{(1 + n)(2 + \rho)}\right)^{\frac{1}{1-\beta}}, \]  
\[ k_B^* = \left(\frac{\alpha(1 - \beta)}{(1 + n)(2 + \rho)}\right)^{\frac{1}{1-\beta}}, \]  
\[ k_C^* = \left(\frac{(1 - \beta)}{(1 + n)(2 + \rho)}\right)^{\frac{1}{1-\beta}}, \]  
\[ k_D^* = \left(\frac{(1 - \beta)}{(1 + n)(2 + \rho)}\right)^{\frac{1}{1-\beta}}, \]  
\[ k_E^* = \left(\frac{\alpha(1 - \beta)}{(1 + n)(2 + \rho)}\right)^{\frac{1}{1-\beta}}, \]  
\[ k_F^* = \left(\frac{(1 - \beta)}{(1 + n)(2 + \rho)}\right)^{\frac{1}{1-\beta}}. \]
4. The comparisons between the two systems

4.1. The comparison of steady-state levels of capital accumulation

It is easy to see that since $1 > \alpha$, $k^*_A = k^*_c > k^*_b$ under fiscal federalism and $k^*_D = k^*_F > k^*_E$ under the unitary system. In addition, when we consider the same tax base under the two systems, we might conclude that: $k^*_A = k^*_D; k^*_B = k^*_E; k^*_C = k^*_F$. We summarize our finding in the following proposition:

**Proposition 1.** In both systems—fiscal federalism and the unitary system,—the system that uses a consumption and a capital income tax bases will have a higher steady state level of capital accumulation than the system that uses a wage tax base. In addition, as long as each system uses the same tax base, then the steady state levels of capital accumulation under the two systems are equivalent.

4.2. The comparison of social welfare levels

Let $W_i$ be the social welfare levels under case A-F respectively. In comparing the two systems, we measure the welfare levels for the same tax base, for instance, the social welfare under fiscal federalism and the unitary system if they use a consumption tax base, a wage tax and a capital income tax respectively. We consider the government objective function stated in equation (36) as the social welfare function. Then, after inserting the relevant values of per-capita consumption of private goods and public goods for each case, we could make a simple logarithmic comparison between the systems. Since from the proposition 1 we know that steady state levels of capital accumulation under the two system which use the same tax base are equivalent, we could proceed the comparisons more easily as follows:

$$W_A - W_D = \alpha \log \frac{(2 + n)(1 + \rho)}{(1 + n)(1 + \rho) + (1 + \beta k_A^{\beta - 1})} + \frac{1}{1 + \rho} (1 - \alpha) \log \frac{(2 + n)(1 + \beta k_D^{\beta - 1})}{(1 + n)(1 + \rho) + (1 + \beta k_D^{\beta - 1})},$$

(57)

$$W_B - W_E = (1 - \alpha) \log \frac{(2 + n)(1 + \rho)}{(1 + n)(2 + \rho)} + \frac{1}{1 + \rho} (1 - \alpha) \log \frac{(2 + n)}{(2 + \rho)},$$

(58)

$$W_C - W_F = (1 - \alpha) \log \frac{(2 + n)(1 + \rho)}{(1 + n)(2 + \rho)} + \frac{1}{1 + \rho} (1 - \alpha) \log \frac{(2 + n)}{(2 + \rho)}.$$  

(59)

From (57), we can observe $W_A - W_D$ will depend on the magnitude of $n, \beta k_A^{\beta - 1}$ and $\rho$. To get a clear result of it, it is necessary to assume certain conditions. By recalling (20)’ in which, in the steady state, we might assume that $\beta k_A^{\beta - 1} = r_A$, which implies $\beta k_D^{\beta - 1} = r_D$. Thus, $W_A = W_D$ if $r_A = r_D = \rho$. As for the (58) and (59), we can clearly see that the value of $W_B - W_E$ and $W_C - W_F$ will depend on the magnitude of $\rho$ and $n$. Thus, $W_B = W_E$ and $W_C = W_F$ if and only if $\rho = n$. Unless this condition is satisfied, the comparison stated in (58) and (59) will yield ambiguous values. We summarize these findings in the following proposition.

**Proposition 2.** Suppose that the economy is on the steady state. The social welfare levels under fiscal federalism and the unitary system under a consumption tax base are equivalent if
individuals’ rate of time preference is equal to the interest rate. The social welfare levels under fiscal federalism and the unitary system under the wage and capital income tax bases are equivalent if and only if the rate of time preference is equal to the population growth rate.

The intuition behind this proposition could be stated as follows. First, the condition of $r_A = \rho$ means that individuals’ rate of time preference is equal to the interest rate at which they choose their level of consumption stream. In this case, there is a stable level of consumption, as in the spirit of Olson and Bailey (1981). In addition, although the condition of $r_A > \rho$ is more consistent to the common condition in the real world since in almost cases, capital has a positive net marginal product, the condition that individuals choose a level of consumption stream if the interest rate is equal to the rate of time preference clearly holds for multiperiod as well as two-period cases (Samuelson, 1937). Finally, by following the condition of $r_A = \rho$, $\rho = n$ implies that $r_A = n$, which is known as the golden rule welfare condition. If this condition is satisfied, then we might conclude that the social welfare level under fiscal federalism is equivalent to that of under the unitary system. This finding might suggest that fiscal federalism is not necessarily superior to the unitary system.

On the other hand, we also compare the levels of social welfare in the same system as a result of the choice of a different tax base. However, the results show that these comparisons yield ambiguous values, except for the comparison between a consumption tax base and a wage tax base.

5 Conclusion

The analyses in this paper suggest that the greater steady-state levels of capital accumulation and social welfare under fiscal federalism and the unitary system may constitute an additional benefit of the proper choice of tax bases. These results, deriving from the six possible cases of the government’s tax bases policy toward individuals, suggest that the level of steady-state capital accumulation and social welfare under fiscal federalism is equal to that of under the unitary system as long as certain conditions are satisfied. While the present results emerge from a model based on a very simple formulation, the additional theoretical works are clearly needed. Exploring the richer model by incorporating, for instance, the conditions of capital and household mobility and the taxation mix policy might be fruitful. These possible expansions will be for further research.
Appendix A
Since we are considering a steady state condition, we must rearrange the relevant values of the levels of private consumption for all cases in the steady state. As for the case A, we utilize the equations (6), (7), (29), and (30), and then insert them into (36) by incorporating (20)’ and (21)’ to get
\[
\begin{align*}
W_A &= \alpha \log \left( \frac{1 + \rho (1 - \beta)k_A^{\beta}}{2 + \rho} \right) + (1 - \alpha) \log \left( \frac{\rho (1 - \beta)k_A^{\beta}}{2 + \rho} \right) \\
&= \frac{1}{1 + \rho} \left( \alpha \log \left( \frac{(1 + \beta)k_A^{\beta-1}}{2 + \rho} \right) + (1 - \alpha) \log \left( \frac{(1 + \beta)k_A^{\beta-1}}{2 + \rho} \right) \right) \left(1 + \tau_A^{y} \right) \\
&\quad + \frac{1}{1 + \rho} \left( \alpha \log \left( \frac{1 + \beta k_A^{\beta-1}}{2 + \rho} \right) - a \left( g_B^y + g_B^o \right) \right) + (1 - \alpha) \log g_B^y \\
&= \frac{1}{1 + \rho} \left( \alpha \log \left( \frac{1 + \beta k_A^{\beta-1}}{2 + \rho} \right) - a \left( g_B^y + g_B^o \right) \right) + (1 - \alpha) \log g_B^y \\
&= \frac{1}{1 + \rho} \left( \alpha \log \left( \frac{1 + \beta k_A^{\beta-1}}{2 + \rho} \right) - a \left( g_B^y + g_B^o \right) \right) + (1 - \alpha) \log g_B^y.
\end{align*}
\]
By performing an optimization problem in respect to \(\tau_A^{y}\) and \(\tau_A^{o}\), we can get the relevant first order conditions as follows:
\[
\begin{align*}
(1 - \alpha) \tau_A^{y} &= \frac{1}{(1 + \tau_A^{y})}, \\
(1 - \alpha) \tau_A^{o} &= \frac{1}{(1 + \tau_A^{o})},
\end{align*}
\]in which, we can obtain
\[
\tau_A^{y} = \tau_A^{o} = \frac{1}{\alpha}.
\]
Thus, the levels of public good for young and old, respectively, are as stated in equation (38) and (39).

Appendix B
We follow the similar process as in Appendix A by using equations (11), (12), and (31) for the case B to get
\[
W_B = \alpha \log \left( \frac{1 + \rho (1 - \beta)k_B^{\beta}}{2 + \rho} \right) + (1 - \alpha) \log \left( \frac{\rho (1 - \beta)k_B^{\beta}}{2 + \rho} \right) \\
= \frac{1}{1 + \rho} \left( \alpha \log \left( \frac{(1 + \beta)k_B^{\beta-1}}{2 + \rho} \right) + (1 - \alpha) \log \left( \frac{(1 + \beta)k_B^{\beta-1}}{2 + \rho} \right) \right) \left(1 + \tau_A^{y} \right) \\
= \frac{1}{1 + \rho} \left( \alpha \log \left( \frac{1 + \beta k_B^{\beta-1}}{2 + \rho} \right) - a \left( g_B^y + g_B^o \right) \right) + (1 - \alpha) \log g_B^y.
\]
Note that, in this case, we reformulate the regional government to choose the level of public goods and, by using these values, we can determine the level of a wage tax. Performing a standard optimization procedure with respect to \(g_B^y\) and \(g_B^o\), we can obtain the levels of public goods as stated in equations (40) and (41). The level of wage tax is accordingly given by equation (42).

Appendix C
As for the case C, by using equations (16), (17), and (32), we can formulate
\[ W_c = \alpha \log \frac{1 + \rho}{2 + \rho} (1 - \beta)k_c^{* \beta} + (1 - \alpha) \log g_c^{*} \]
\[ + \frac{1}{1 + \rho} \left( \alpha \log \frac{(1 - \beta)k_c^{* \beta}}{2 + \rho} \left( 1 + \beta k_c^{* \beta - 1} \left( 1 - \frac{(2 + \rho)a((1 + n)g_c^{*} + g_c^{*})}{(\beta k_c^{* \beta - 1})(1 - \beta)k_c^{* \beta}} \right) \right) + (1 - \alpha) \log g_c^{*} \right). \]

(C1)

Solving (C1) for \( g_c^{*} \) and \( g_c^{*} \), we can get the levels of public goods and eventually a capital income tax as previously stated in equations (43)-(45).

**Appendix D**

Since the levels of private consumption under case D are similar to that of case A, we recall (6) and (7), replace its subscript A’s to become subscript D’s and consider its value in the steady state. We then get

\[
\begin{align*}
\bar{c}_D^{c} &= \frac{1 + \rho}{2 + \rho} w_D, \quad (D1) \\
\bar{c}_D^{\omega} &= \frac{1 + r_D}{2 + \rho} w_D. \quad (D2)
\end{align*}
\]

From the government’s budget constraint in the case D as stated in equation (33), we can rearrange (D1) and (D2) to become

\[
\begin{align*}
\bar{c}_D^{c} &= \frac{(1 + \rho)(1 - \beta)k_D^{* \beta}((1 + n)(1 + \rho) + (1 + \beta k_D^{* \beta - 1})) - a g_D(1 + \rho)(2 + n)(2 + \rho)}{(2 + \rho)((1 + n)(1 + \rho) + (1 + \beta k_D^{* \beta - 1}))}; \quad (D3) \\
\bar{c}_D^{\omega} &= \frac{(1 + \beta k_D^{* \beta - 1})(1 - \beta)k_D^{* \beta}((1 + n)(1 + \rho) + (1 + \beta k_D^{* \beta - 1})) - a g_D(1 + \rho)(2 + n)(2 + \rho)}{(2 + \rho)((1 + n)(1 + \rho) + (1 + \beta k_D^{* \beta - 1}))}. \quad (D4)
\end{align*}
\]

Plugging (D3) and (D4) into (36) and solving it for \( g_D \), we can obtain (46) and accordingly, the level of \( \tau_{c_D} \).

**Appendix E**

We follow similar formulation to that of case D by using (11) and (12) adjusted to have an E’s subscripts, such that, in the steady state we can obtain

\[
\begin{align*}
\bar{c}_E^{c} &= \frac{1 + \rho}{2 + \rho} w_E(1 - \tau_{E_c}), \quad (E1) \\
\bar{c}_E^{\omega} &= \frac{1 + r_E}{2 + \rho} w_E(1 - \tau_{E_w}). \quad (E2)
\end{align*}
\]

Then, we use the government’s budget constraint as stated in (34) to get

\[
\begin{align*}
\bar{c}_E^{c} &= \frac{(1 + \rho)}{(2 + \rho)} \left( 1 - \beta \right) k_E^{* \beta} - a g_E \frac{2 + n}{1 + n}; \quad (E3) \\
\bar{c}_E^{\omega} &= \frac{(1 + \beta k_E^{* \beta - 1})}{(2 + \rho)} \left( 1 - \beta \right) k_E^{* \beta} - a g_E \frac{2 + n}{1 + n}. \quad (E4)
\end{align*}
\]

Plugging (E3) and (E4) into (36) and solving it for \( g_E \), we can get the levels of public good and wage tax in case E as previously stated in equations (47) and (48).
Appendix F

We follow similar formulation to that of case D by using (16) and (17) adjusted to have an F’s subscripts, such that, in the steady state we can obtain

\[ c^*_F = \frac{1+\rho}{2+\rho} w_F, \quad (F1) \]
\[ c^\rho_F = \frac{1 + \rho F}{2 + \rho + F} \]

Then, we use the government’s budget constraint as stated in (35) to get

\[ c^\beta_F = \frac{(1+\rho)}{(2+\rho)} (1-\beta) k_F^\beta; \quad (F3) \]
\[ c^\alpha_F = \frac{(1-\beta) k_F^\beta}{(2+\rho)} \frac{\alpha(1+\beta k_F^\beta - 1)}{(2 - \alpha + \rho - \alpha \rho)}. \quad (F4) \]

Plugging (F3) and (F4) into (36) and solving it for \( g_F \), we can get the levels of public good and wage tax in case F as previously stated in equations (49) and (50).
References


