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Non-linear unit root properties of stock prices: Evidence from India, Pakistan and Sri Lanka

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Abstract

This study applies a threshold autoregressive (TAR) model to monthly stock prices for three South Asian countries over the period from 1991:01 to 2009:09. Two main conclusions are drawn. Firstly, the results indicate that all the stock prices in this study exhibit non-linear behavior. Secondly, a partial unit root was found to be present in one of the regimes indicating that the stock prices are weak form efficiency, but not all the time.

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1. Introduction

Since the seminal work by Fama (1970), a large body of literature in finance has reported the testing of market efficiency. The weak-form efficiency market hypothesis (EMH), which is the focus of this study, states that the current stock price is determined only by historical prices of that particular stock. An important implication derived is that stock price changes behave like a "random walk" or a process with no memory and the volatility in stock markets will increase without bound (Chaudhuri and Wu, 2003).

There have been numerous approaches to examine the EMH and one approach commonly sited was to test for a unit root. Along these lines, both conventional and panel unit roots tests are applied (Grieb and Reyers, 1999; Narayan and Prasad, 2007). Recent advances in non-linear modeling techniques, however, questioned the reliability of these earlier findings. Different types of non-linearities, for instance, chaos (Atchison and White, 1996), Smooth Transition Autoregressive (STAR) models (Lim and Liew, 2004) and threshold autoregressive model (Narayan, 2005; Qian *et al*., 2008; Munir and Mansur, 2009) have all been modeled based on stock prices, resulting in convincing evidences that non-linearity has a significant impact on stock markets.

This study aims to test the efficiency of three South Asian stock markets by examining the non-linearity and unit root properties of stock prices in India, Pakistan and Sri Lanka by adopting the Caner and Hansen (2001) threshold autoregressive model. Two motivations for this study are identified. Firstly, most of these South Asian stock markets have undergone rapid transformation after the implementations of deregulation and liberalization of the financial markets. As shown in Table 1, over 1990-2007, the stock market capitalization in these countries has increased from four to ten times. The ratio of stock market capitalization to GDP of India has increased to a remarkable 154.6 percent in year 2007 as compared to 12.2 percent in year 1990. For Sri Lanka and Pakistan, the percentage has surged up more than twofold and sevenfold respectively from 1990 to 2007. It is therefore crucial to investigate if the removal of restrictions on foreign investment in South Asia has led to weak-form efficiency of the stock markets in this region. Identifying the nature of informative efficiency is of great importance not only to policy makers but also to investors. For policy makers, the efficiency in the stock market is an important factor to attract foreign portfolio investment which further improves the overall economic development. For investors, an accurate pattern in stock prices is beneficial in risk management and portfolio optimization.

Secondly, although it appears that an impasse has been reached in the empirical evidence, few studies have been sited to carry out the simultaneous presence of non-linearity and non-stationarity in South Asian countries. In an effort to reduce the literature gap, this study attempts to examine the possible presence of non-linear stationary in the South Asian stock markets, using the Threshold autoregressive (TAR) models. The TAR is capable of capturing the possible non-linear and non-stationarity behavior in the stock market.

Table 1. Stock Market Capitalization (in US$ million and percent of GDP) for South Asian Countries, 1990-2008

Year	Stock Market Capitalization (US$ million)			Stock Market Capitalization (% of GDP)		
	India	Sri Lanka	Pakistan	India	Sri Lanka	Pakistan
1990	-	-	-	12.2	11.4	7.1
1995	127199	1998	9286	35.7	15.3	15.3
2000	148064	1074	6581	32.2	6.6	8.9
2005	553074	5720	45937	68.3	23.4	42
2006	818879	7769	45518	89.5	27.5	35.7
2007	1819101	7553	70262	154.6	23.4	49.2
2008	645478	4326	23491	56.9	10.6	16.6

Source: Asian Development Bank (2009), Key Indicators for Asia and the Pacific 2009, http://www.adb.org/Documents/Books/Key_Indicators/2009/Part-III.asp#money, access ed 26 October 2009.

In brief, the empirical results reveal that all the stock prices under study support the non-linearity, however, the evidence of weak-form efficiency is partially supported.

The structure of the study is as follows. Section 2 provides the details of the methodology used in this study. Section 3 describes the data used and presents the empirical results. The conclusion is drawn in Section 4.

2. Methodology

In order to jointly test for non-stationarity and non-linearity, TAR model proposed by Caner and Hansen (2001) was utilized. A two-regime TAR model, with an autoregressive unit root, can be described by the following data generating process:

$$\Delta s_t = \theta_1' x_{t-1} I_{\{Z_{t-1}<\lambda\}} + \theta_2' x_{t-1} I_{(Z_{t-1}\geq\lambda)} + e_t \tag{1}$$

where s_t is the Stock Price index after taking logarithm for t = 1, …, T, $x_{t-1} = (s_{t-1} r_t' \Delta s_{t-1} \ldots \Delta s_{t-k})'$, $I_{\{.\}}$ is the indicator function, e_t is an identical and independently distributed error term, r_t is a vector of deterministic components including an intercept and possibly a linear time trend; and the threshold variable, $Z_{t-1} = s_{t-1} - s_{t-m-1}$ is predetermined and strictly stationary with m the delay order, $k \geq 1$ is the autoregressive order.

In this study, the variable Z_t acts as return at the time horizon of m months. The optimal delay order, m, is chosen so that it minimizes the residual variance for the TAR model of each deviation series. The unknown threshold parameter is given by λ. λ takes values in the interval $\lambda \in \Lambda = [\lambda_1, \lambda_2]$ and is picked so that $P(Z_t \leq \lambda_1) = \pi_1 > 0$ and $P(Z_t \leq \lambda_2) = \pi_2 < 1$. Following convention, π_1 and π_2 are treated symmetrically so that $\pi_2 = 1 - \pi_1$. The particular choice for π_1, though, is somewhat arbitrary it must be guided by the consideration that each "regime" needs to have sufficient observations to adequately identify the regression parameters[1].

[1] See, in particular, Caner and Hansen (2001).

The vectors of coefficients in threshold regime one and threshold regime two, θ_1 and θ_2, can be partitioned as $\theta_1 = (\rho_1\ \beta_1\ \alpha_1)'$, $\theta_2 = (\rho_2\ \beta_2\ \alpha_2)'$ where (ρ_1, ρ_2) are the slope coefficients on lagged levels, s_{t-1}, (β_1, β_2) are the slopes on the deterministic components r_t, and (α_1, α_2) are the slope coefficients on the lagged differences in the two regimes. Test on ρ_1 and ρ_2 is the core of this study as they control the stationarity of s_t.

The TAR model is estimated in two steps using Ordinary Least Squares (OLS). In the first step of estimating equation (1), for each $\lambda \in \Lambda$, the threshold λ is selected by minimizing $\sigma^2(\lambda)$. The OLS estimates of other parameters are found in the second step by inserting the point estimate $\hat{\lambda}$, viz. $\hat{\theta}_1 = \hat{\theta}_1(\hat{\lambda})$ and $\hat{\theta}_2 = \hat{\theta}_2(\hat{\lambda})$.

The estimated model can be rewritten below

$$\Delta s_t = \hat{\theta}_1' x_{t-1} I_{\{Z_{t-1} \le \hat{\lambda}\}} + \hat{\theta}_2' x_{t-1} I_{(Z_{t-1} > \hat{\lambda})} + \hat{e}_t \tag{2}$$

The TAR model in equation (2) is estimated for two central issues: whether there is a threshold effect and whether the process of s_t is stationary.

2.1. Testing the Threshold Effect

To check if there is a threshold effect, the null hypothesis of linearity, that is H_0: $\theta_1 = \theta_2$ is tested against the alternative of a threshold. If the null hypothesis is rejected, the threshold effect will be supported and thus the vectors of coefficients θs will not be identical between two regimes ($\theta_1 \ne \theta_2$). The test of H_0: $\theta_1 = \theta_2$ is the standard Wald statistic W_T:

$$W_T = W_T(\hat{\lambda}) = \sup_{\lambda \in \Lambda} W_T(\lambda) = T\left(\frac{\hat{\sigma}_0^2}{\hat{\sigma}^2(\lambda)} - 1\right) \tag{3}$$

where $\hat{\sigma}_0^2$ and $\hat{\sigma}^2$ are the residual variances from OLS estimation of the null linear and TAR models, respectively. Given that the $\sup_{\lambda \in \Lambda} W_T(\lambda)$ has a non-standard asymptotic null distribution with critical values that cannot be tabulated, Caner and Hansen (2001) proposed two bootstrap approaches – unconstrained and unit-root-constrained – to compute asymptotical critical values and p-values. Caner and Hansen (2001) further suggested making inference on the more conservative (the larger) p-value from these two approaches.

2.2. The Threshold and Partial Unit Root Tests

The null of $H_0: \rho_1 = \rho_2 = 0$ is built to test if the process of s_t is stationary. If it holds, then the stock index has a unit root and can be described as a favor of efficient market hypothesis. Given that the stationarity of the series can be characterized as a threshold unit root (a unit root in both regimes) or a partial unit root (unit root in only one of the regimes), there would be two alternatives. The alternative in the threshold unit root case is: $H_{1A}: \rho_1 < 0$ and $\rho_2 < 0$. Under H_{1A}, the stock price is non-linear stationary and can be inferred as a rejection of EMH.

In the case of a partial unit root, the alternative is specified as:

$$H_{2A}: \begin{cases} \rho_1 < 0 \text{ and } \rho_2 = 0 \\ \text{or} \\ \rho_1 = 0 \text{ and } \rho_2 < 0 \end{cases}$$

The standard test for null against the unrestricted alternative $\rho_1 \neq 0$ or $\rho_2 \neq 0$ is the two-side Wald statistic, expressed as $R_{2T} = t_1^2 + t_2^2$ where t_1 and t_2 are the t statistics for $\hat{\rho}_1$ and $\hat{\rho}_2$ from the OLS regression of equation (2).

As indicated by Caner and Hansen (2001), the alternatives H_{1A} and H_{2A} are one-sided and the two-sided Wald statistic (the R_{2T}) may be less powerful than a one-sided version (the R_{1T}). Caner and Hansen (2001) suggested a simple one-sided Wald statistic: $R_{1T} = t_1^2 I_{\{\hat{\rho}1<0\}} + t_2^2 I_{\{\hat{\rho}1<0\}}$ to test the H_0 against the one-sided alternative $\rho_1 < 0$ or $\rho_2 < 0$. While a bootstrap distribution that imposes an identified threshold effect or imposes an unidentified threshold effect can be constructed to determine "significance" of the test statistics, Caner and Hansen (2001) suggest the use of the unidentified threshold model as it appears to be less sensitive to the nuisance parameters.

Although a "significant" test statistic justifies the rejection of the null, it cannot discriminate between the stationary case H_{1A} and the partial unit root case H_{2A}. To distinguish between the alternative of threshold stationarity (given by H_{1A}) and partial stationarity (given by H_{2A}), Caner and Hansen (2001) suggested examining the individual t-statistics, t_1 and t_2. Under H_{2A}, if the bottom case holds, then the stock index behaves like a "non-stationary process" in first regime; but exhibits a "stationary process" in the second regime, vice versa. Therefore, it will be interesting to distinguish between the cases H_0, H_{1A} and H_{2A}.

3. Empirical Results

This study evaluates the non-linear dynamics of stock prices using a two-regime, non-linear threshold random-walk model and monthly data of India, Pakistan and Sri Lanka's stock indices from 1991:01 to 2009:09. The FTSE (India and Pakistan) and the All Share Index (Sri Lanka) are the stock indices used in this study. The data are extracted from DataStream® database and the closing prices of the last trading days of all months are utilized.

The orders of integration of the stock price indices in the context of the linear autoregressive model is first considered by using the Augmented Dickey-Fuller unit root test (Dickey and Fuller, 1981) to establish a basis for comparison. As shown in Panel A of Table 2, these results fail to reject the unit root null hypothesis for the stock price series at conventional significance levels. This implies that the linear representation for each of the stock prices is found to have a unit root.

The results for the threshold tests are summarized in Panel B of Table 2. According to the estimates obtained, for India, the Wald statistic is maximized (W_T =47.50) when m = 11. Hence, for India, m = 11 is the preferred model. The bootstrap p-value is carried out with 5,000 replications. Based on the same criteria, m = 4 and m = 7 are the preferred models for Sri Lanka and Pakistan respectively. Given the bootstrap p-values of less than 10 percent, each of the series was found to be non-linear. These results thus give strong support to a TAR model and to the existence of a threshold effect.

Having established that stock prices follow the non-linear data generating process, this study continues to examine if stock prices contain a unit root. The threshold unit root test statistics for the R_{1T}, R_{2T}, t_1 and t_2 are calculated. The bootstrap p-values calculated under the assumption of unidentified threshold are reported in Panels C and D of Table 2. R_2 test results are not reported since they are almost identical to the R_1 test results. As shown in Panel C, The one-sided Wald R_{1T} test rejects the null of a unit root at the 10 percent level for all stock markets at the optimal value of the delay parameter. These indices are therefore non-linear stationary.

Table 2. Threshold Effects and Unit Root Tests

Stock Market	**India**	**Pakistan**	**Sri Lanka**
A: ADF unit root test	-0.56	-1.34	-1.62
B: Threshold effect			
Wald statistic	47.5	62.9	40.1
Bootstrap p-value	0.05	0.00	0.07
Optimal delay parameter, m	11	7	4
Threshold parameter, $\hat{\lambda}$	0.40	0.16	0.19
Observations in Regime 1 (%)	81.3	85.8	79.5
C: Threshold unit root			
R_{1T} statistic	11.4	11.2	13.1
Bootstrap p-value	0.06	0.09	0.03
D: Partial unit root			
Regime 1, t_1 statistic	0.11	0.85	0.75
Bootstrap p-value	0.81	0.60	0.58
Regime 2, t_1 statistic	3.38	3.24	3.54
Bootstrap p-value	0.02	0.04	0.01

The one-side test statistic of R_{1T}, however, is not able to discriminate between the full unit root and the partial unit root. Following this, the individual t-statistics, t_1 and t_2 are referred (Panel D). For the optimal delay parameter (m), the bootstrap p-values-related t_1 statistics are 0.81, 0.60 and 0.58, while the bootstrap p-values-related t_2 statistics are 0.02, 0.04 and 0.01 for India, Pakistan and Sri Lanka respectively. These bootstrap p-values for the t_1 and t_2 statistics indicate that the unit root null hypothesis cannot be rejected in regime 1 only.

It is worth noting that the TAR model identifies two regimes depending on whether the threshold variable lies above or below the threshold parameter, $\hat{\lambda}$. Take Pakistan for example, with $\hat{\lambda}$ of 0.16 and m = 7, it indicates that the first regime is when $Z_T < 0.16$, and it occurs when the stock price has fallen less than 16 percent within seven months. 85.8 percent of the sample belongs to this category and given the insignificant bootstrap p-value for t_1, the stock price follows a random walk. Another threshold regime (regime 2) is for $Z_T \geq 0.16$. Approximately 14.2 percent of the observations belong to the second regime and are essentially mean reverting, given the significant p-value of t_2 statistic.

Turning to the other two stock prices, India and Sri Lanka, the regime (in particular, regime 2) not containing the partial unit root consists of 18.7 and 20.5 percent of observations, respectively. Thus, for one-fifth of the period studied, the stock markets in India and Sri Lanka were stationary. This finding partly echoes the recent evidence by Lim *et al*. (2008) which concludes that most of the Asian stock markets are weak-form efficient, but not all the time.

4. Conclusions

This study investigates the behavior of stock prices of India, Pakistan and Sri Lanka using the TAR model proposed by Caner and Hansen (2001). An important message emerging from the present analysis is that all three stock prices can be characterized by non-linearities. However, all the stock prices reject the threshold and partial unit root and therefore are subjected to pricing inefficiency. The mean reverting process in these stock markets implies that a few brief periods exist in which stock prices do not adjust rapidly and unbiasedly to the arrival of new information.

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