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Estimating the distribution of inflation expectations

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Abstract

In this paper, we use survey data to estimate the shape of the distribution of inflation expectations. However, unlike previous studies, we do not assume a distribution a priori. We employ an applied approximation method using normal distribution: Cornish-Fisher expansion. Skewness and kurtosis may provide necessary information for understanding the shape of the distribution of inflation expectations. The estimated inflation expectations contain slight biasedness and are not fully efficient, but some superiority can be verified.

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1. Introduction

This paper presents a technique for estimating the distribution of inflation expectations using survey data. Although inflation expectations are frequently used as a reference for economic decision-making, unlike in the case of interest rates or commodity prices, the exact values of inflation expectations are not published in Japan. Therefore, we can consider two types of methods to infer inflation expectations as numerical values. Assuming an economic model, we can calculate inflation expectations according to a model that uses financial data or price levels. Alternatively, we can administer a questionnaire survey and then quantify the responses.

We specifically address the latter method: administering a questionnaire survey and quantifying the responses. Survey research of inflation expectations is conducted in many countries, and such surveys classify possible responses to questions for the convenience of the respondents. In a survey seeking information pertaining only to the tendency of future prices, the results are published as qualitative data. The Carlson and Parkin method (1975) is a recognized conversion method of such qualitative data that provides a numerical value that is comparable to the actual inflation rate.

Surveys often present respondents with subdivided possible responses, which might be classified into numerical ranges. The Japanese Cabinet Office's "Monthly Consumer Confidence Survey" has used such inquiries since April 2004. Each respondent selects from among the following possible responses related to personal inflation expectations.

> Price decrease: "greater than or equal to –5%," "less than –5% to greater than or equal to –2%," and "less than –2%"
>
> Price increase: "less than 2%," "greater than or equal to 2% to less than 5%," and "greater than or equal to 5%"
>
> Remain the same: around 0%

Then, the rates of the respective responses are published. The results of such surveys are easy to evaluate because the threshold value is defined.

For this study, using Cabinet Office data, we estimate the distribution of inflation expectations. The shape of the distribution of the respondents is explained using the approximation method by normal distribution. For the mean and standard deviation, skewness and kurtosis are examined as the parameters that characterize this distribution. Further, we check the unbiasedness and efficiency of the estimated results.

2. Method

Previous studies on the estimation of inflation expectations often assume a certain distribution that appears reasonable. The distribution is then fitted to the data and evaluated[1]. In such studies, the possible distributions that are not selected as candidates might be discarded[2].

This paper does not assume a distribution *a priori*; rather, it employs an approximation method—Cornish-Fisher expansion—using normal distribution.

Cornish-Fisher expansion is a method by which a certain distribution has a *j*-th cumulant; it can calculate the inverse value of the cumulative distribution using the value obtained from normal distribution. If the value of the inverse of the cumulative standard normal distribution is denoted by z_α when α is the input, and that of standardized distribution is estimated as x_α, then

$$x_\alpha = z_\alpha + \frac{1}{6}\lambda_3(z_\alpha^2 - 1) + \frac{1}{24}\lambda_4(z_\alpha^2 - 3z_\alpha) - \frac{1}{36}\lambda_3^2(2z_\alpha^3 - 5z_\alpha), \qquad (1)$$

where $\lambda_3 \equiv K_3/\sigma^3$, $\lambda_4 \equiv K_4/\sigma^4$, K_j is the *j*-th cumulant, and σ is the standard deviation[3],[4].

For standardization, we must introduce the mean and standard deviation into the left-hand side. We can obtain z_α from the survey data. Our data are available for rates of "up to –5%," "up to –2%," "up to 2%," and "up to 5%." Therefore, the standardized value based on the distribution to estimate is, for example, denoted as $x_{-5\%} = \frac{-0.05 - \mu}{\sigma}$, where μ is the mean. We can obtain the following.

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[1] The Carlson-Parkin method assumes that the distribution of inflation expectations is normal. This assumption is often justified by the central limit theorem when the respondents are sufficiently numerous. See, for example, Carlson (1975) and Batchelor (1982).

[2] Batchelor and Dua (1987) show that skewness and kurtosis are significantly different from the results of normal distribution. Batchelor (1982) and Foster and Gregory (1977) argue that symmetric distribution should not be expected when upward trends are evident in prices. Carlson (1975) also reports that the distribution is skewed rightward.

[3] Note that this expansion is up to second order. Although Balcombe (1996) also uses second-order Cornish-Fisher expansion, the last term of the right hand side of eq. (1) is omitted; omitting such a term is questionable.

[4] See, for example, Minotani (2003) for this derivation.

$$
\begin{aligned}
\frac{0.05-\mu}{\sigma} &= z_{5\%} + \frac{1}{6}\lambda_3(z_{5\%}^2 - 1) + \frac{1}{24}\lambda_4(z_{5\%}^2 - 3z_{5\%}) - \frac{1}{36}\lambda_3^2(2z_{5\%}^3 - 5z_{5\%}) \\
\frac{0.02-\mu}{\sigma} &= z_{2\%} + \frac{1}{6}\lambda_3(z_{2\%}^2 - 1) + \frac{1}{24}\lambda_4(z_{2\%}^2 - 3z_{2\%}) - \frac{1}{36}\lambda_3^2(2z_{2\%}^3 - 5z_{2\%}) \\
\frac{-0.02-\mu}{\sigma} &= z_{-2\%} + \frac{1}{6}\lambda_3(z_{-2\%}^2 - 1) + \frac{1}{24}\lambda_4(z_{-2\%}^2 - 3z_{-2\%}) - \frac{1}{36}\lambda_3^2(2z_{-2\%}^3 - 5z_{-2\%}) \\
\frac{-0.05-\mu}{\sigma} &= z_{-5\%} + \frac{1}{6}\lambda_3(z_{-5\%}^2 - 1) + \frac{1}{24}\lambda_4(z_{-5\%}^2 - 3z_{-5\%}) - \frac{1}{36}\lambda_3^2(2z_{-5\%}^3 - 5z_{-5\%})
\end{aligned}
\tag{2}
$$

Therein, $z_{r\%}$ denotes the inverse of the cumulative standard normal distribution of the ratio for the reply of "up to $r\%$." We can calculate four unknown parameters from the four equations: μ, σ, λ_3, and λ_4.

From the calculation presented above, we can estimate the first to fourth moments. Using eq. (1) and the moments, we can draw a figure of the estimated distribution.

3. Results

We apply the previously described method to the Japanese Cabinet Office's "Monthly Consumer Confidence Survey." Figure 1 shows the estimated inflation expectations using Cornish-Fisher expansion, the estimated inflation expectations obtained from normal distribution, and the actual inflation rate. The estimated inflation expectations obtained from normal distribution correspond to the Carlson-Parkin method; however, we need not estimate the threshold values. The actual inflation rate is calculated from the Japanese Ministry of Internal Affairs and Communications' "Consumer Price Index (general)." Two series are obtained from Cornish-Fisher expansion: that based on the mean of the distribution and that based on the mode.

Figure 1: Expected and actual inflation: 2004:4 – 2008:10

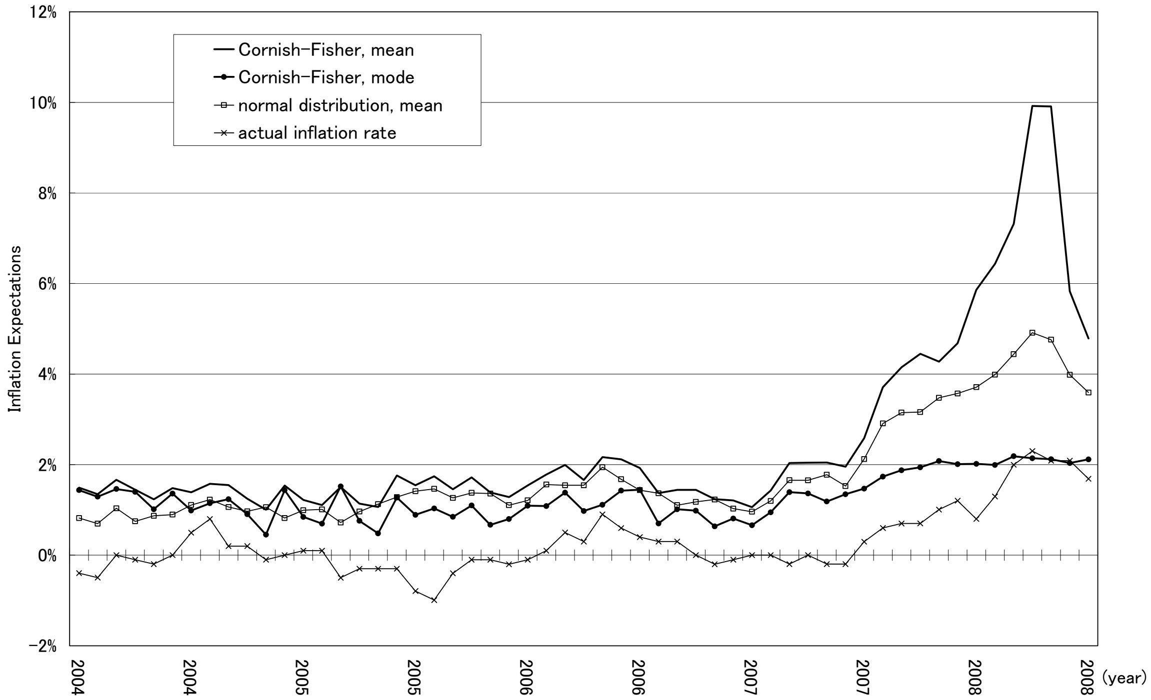
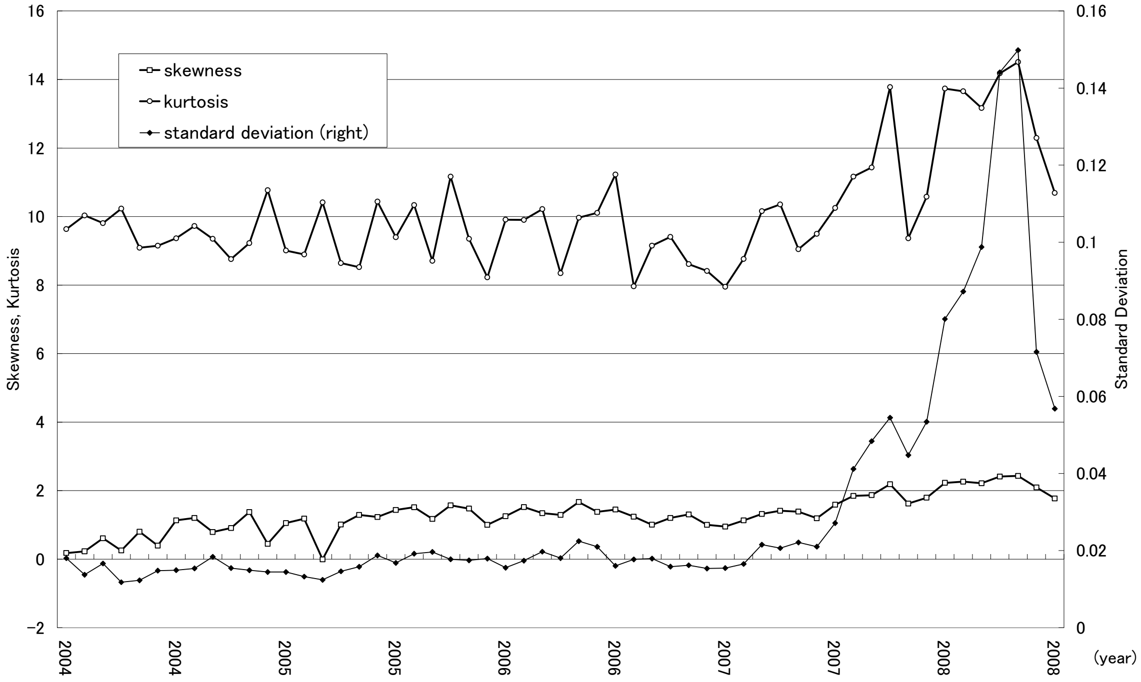


Figure 2 portrays the estimated skewness and kurtosis, which are based on the standardized estimated distribution. This figure also presents the standard deviation estimated by Cornish-Fisher expansion.

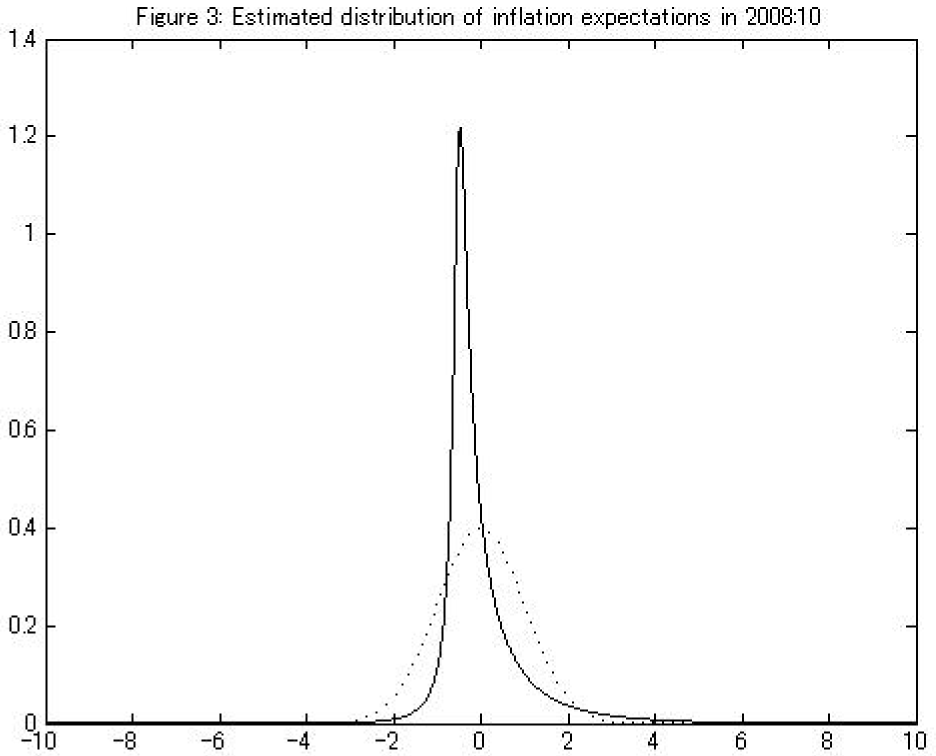
Figure 2: Skewness, kurtosis, and standard deviation: 2004:4 – 2008:10



For almost all periods, inflation expectations based on the mean of Cornish-Fisher expansion are higher than those based on the mode. A positively skewed property is confirmed[5]. Furthermore, for almost all periods, inflation expectations based on the mean of Cornish-Fisher expansion are higher than those based on normal distribution.

Figure 3 shows the estimated standardized distribution based on the data of October 2008. For comparison, we also present the standard normal distribution. This figure confirms that the estimated distribution has a right-hand side fat-tail.

[5] We can verify these results using the Kolmogorov-Smirnov test to determine whether the data follow normal distribution. In every period, the null hypothesis that data follow normal distribution is rejected. We also use the Pearson-Hartley test to verify whether the data follow the hypothesis that skewness equals 0 or that kurtosis equals 3. This hypothesis is rejected for almost all periods when we apply the conventional significance level (5%), except for the skewness of June 2006.



Note: The solid line is the estimated distribution and the dotted line is the normal distribution.

As the analysis described above suggests, introducing skewness and kurtosis changes yield great estimations of inflation expectations.

4. Testing rationality

In this section, we test whether the estimated inflation expectations predict actual inflation efficiently. In the case of inflation expectations, it is worth mentioning whether the series is rational. Therefore, we check whether the estimated inflation expectations satisfy rationality. First, we check the sufficient condition by testing the stationarity. Second, we check the weak form efficiency by testing unbiasedness and dynamic properties. Finally, we check the semi-strong form efficiency by testing macroeconomic efficiency.

4.1. Testing stationarity

In this subsection, we apply the unit root test to actual inflation, and to the estimated inflation expectation series. For rationality, if the actual series are stationary, this is a sufficient condition for the expectation series also being stationary, and vice versa. We use the expectation series based on the Cornish-Fisher expansion and normal distribution. We applied the augmented Dickey-Fuller (ADF) test, and applied three types of models: with intercept, with intercept and trend, and with none.

Table 1: ADF test

	ADF	p-value	model
Actual Inflation	-0.29316	0.917	intercept
	-1.97214	0.5983	intercept, trend
	0.045223	0.6914	none
Cornish-Fisher mean	-2.57625	0.1062	intercept
	-2.98029	0.1502	intercept, trend
	-0.01613	0.6714	none
Cornish-Fisher median	-3.26361	0.0235 **	intercept
	-3.304	0.0802 *	intercept, trend
	-0.33468	0.5583	none
Cornish-Fisher mode	-4.1017	0.0026 ***	intercept
	-3.98554	0.0173 **	intercept, trend
	-0.72241	0.3974	none
Normal distribution	-2.32759	0.1686	intercept
	-2.95799	0.1564	intercept, trend
	0.213771	0.7432	none

Note: ** denotes significance at the 0.01 level. * denotes significance at the 0.05 level.

Table 1 shows the results. For all models, the actual inflation series does not reject the null hypothesis at the conventional significance level. For the inflation expectation series, we use the series based on the Cornish-Fisher expansion and normal distribution. The Cornish-Fisher mean series rejects the null hypothesis for all models. The Cornish-Fisher median and mode series contains test statistics that do not reject the null. The normal distribution series does not reject the null.

However, the sample size of our dataset is only 42; that is, a small sample size. We should be careful about the power of the ADF test. Indeed, other tests that consider a small sample size are developed; however, the Elliott-Rothenberg-Stock ADF-GLS test requires at least 50 observations. Therefore we cannot apply this test.

These results show that we cannot immediately reach conclusions about rationality from the unit root test.

4.2. Testing weak form efficiency

In this subsection, we check the weak form efficiency. This is checked by the root mean square error (RMSE), unbiasedness, and the orthogonality condition.

First, we check for accuracy by calculating RMSE. It is calculated for the deviation of the actual and expected inflation, that is, expectation errors. For comparison, we calculate the naïve expectation series in addition to the previous subsection series[6].

The results are shown in the left column of Table 2. Each result is about 1%, and the predictions are not poor. From Figure 1, we can also confirm that the inflation expectations generally did not deviate from the actual inflation.

Table 2: Test for accuracy and orthogonality condition

	RMSE	Box-Pierce
Cornish-Fisher, mean	0.014183	2.3731263
Cornish-Fisher, mode	0.014183	0.9301343
Cornish-Fisher, median	0.011811	1.4667085
Normal distribution	0.011357	2.6595715
naive expectations	0.010096	0.4137786

Next, we test for unbiasedness by estimating the following equation.

$$\pi_t = \alpha + \beta \pi_t^e + \varepsilon_t \qquad (3)$$

Here π_t denotes the actual inflation at period *t*, and π_t^e denotes the inflation expectation for one year ahead, as formed at period *t – 12*. If inflation expectations are unbiased, then the null hypothesis, H0: $(\alpha, \beta) = (0,1)$, will not be rejected.

6 Naïve expectations is actual inflation 12 months old.

Table 3: Test for unbiasedness

	α	β	R^2	Wald test	F
Cornish-Fisher, mean					
coefficient	–0.00575	0.588106	0.057687	40.47099	20.2355
std. error	0.010401	0.735742			
p-value	0.5835	0.5787		0	0
Cornish-Fisher, mode					
coefficient	0.003414	–0.01456	0.000031	11.51671	5.758353
std. error	0.004602	0.517702			
p-value	0.741947	0.057		0.0032	0.0063
Cornish-Fisher, median					
coefficient	–0.00064	0.320282	0.011913	19.77834	9.88917
std. error	0.008276	0.7588			
p-value	0.9385	0.3757		0.0001	0.0003
Normal distribution					
coefficient	–0.00766	0.900136	0.127749	31.78771	15.89385
std. error	0.007627	0.717487			
p-value	0.321	0.89		0	0
naive expectations					
coefficient	0.003106	–0.66688	0.107245	30.91785	30.91785
std. error	0.002048	0.299778			
p-value	0.1373	0		0	0

Note: Wald test and F are the test statistics of the joint hypothesis H0: $(\alpha, \beta) = (0, 1)$. The equations are estimated by ordinary least squares (OLS), and standard errors are calculated by the Newey and West (1987) method.

Table 3 shows the results. In the case of all estimations, $\alpha = 0$ is not rejected for the conventional significance level. Moreover, $\beta = 1$ is not rejected for Cornish-Fisher or for normal distribution. On the other hand, naïve expectations reject $\beta = 1$. This may express the usefulness of the Cabinet Office data. However, joint hypothesis H0 is rejected for both Cornish-Fisher and the normal distribution estimation. A slight improvement is found in the Cornish-Fisher mode case, although for the conventional significance level we reject H0. Therefore, we cannot state that inflation expectations are fully unbiased in the Japanese data.

Finally, we check the orthogonality condition. This is checked by the Box-Pierce test, using the residuals of eq. (3). Test statistics are modified according to the proposals of Batchelor (1986) and Batchelor and Orr (1988), and are calculated for the 13–24 order, because the expectation is formed about 1 year ahead.

The results are shown in the right column of Table 2. Each value is not rejected for the conventional significance level.

4.3. Testing macroeconomic efficiency

In this subsection, we check for macroeconomic efficiency: that is, whether the expectation error is explained by demand variables, monetary variables, or other price variables. For these variables, we choose the Indices of Industrial Production (IIP), the unemployment rate, monetary base, M2, call rate, the long-term interest rate, Yen/Dollar exchange rate, and the Corporate Goods Price Index (CGPI). These are checked by estimating the following equation.

$$\pi_t - \pi_t^e = \delta + \gamma X_{t-12} + \varepsilon_t, \qquad (4)$$

where X_{t-12} represents the above variables.

Table 4 shows the results. In all cases, call rate, Yen/Dollar rate, and CGPI are significant. This is particularly noteworthy because these variables are under the effect of monetary policy. Monetary policy may not be taken into consideration for inflation expectations. Therefore, it is possible that the bias shown in Table 1 is based on monetary policy.

Table 4: Test for efficiency

Variable	Cornish-Fisher, mean coefficient	std error	p-value	Cornish-Fisher, mode coefficient	std error	p-value	Cornish-Fisher, median coefficient	std error	p-value
C	–0.01262	0.005404	0.0257 *	–0.00525	0.005996	0.3873	–0.00895	0.005574	0.1177
IIP	0.014872	0.041704	0.7237	–0.00769	0.049618	0.8778	0.000905	0.043346	0.9835
Unemployment	–0.00509	0.003016	0.1009	–0.00301	0.00329	0.3667	–0.00352	0.002962	0.2438
Monetary base	0.025897	0.013636	0.0663	0.023374	0.01518	0.1331	0.021541	0.013871	0.13
M2	0.084658	0.27861	0.7631	–0.00761	0.314272	0.9808	0.097845	0.288928	0.737
Call rate	0.016529	0.00329	0 **	0.016639	0.003662	0.0001 **	0.016521	0.003154	0 **
Long-term interest rate	–0.002	0.001992	0.3226	–0.00067	0.002732	0.8083	–0.00107	0.002301	0.6451
Yen/Dollar	–0.03692	0.010125	0.0009 **	–0.02681	0.010109	0.0122 *	–0.03058	0.008927	0.0017 **
CGPI inflation rate	–0.51349	0.071829	0 **	–0.55816	0.070248	0 **	–0.53804	0.063447	0 **
R^2	0.84879			0.841851			0.860542		

Variable	Normal distribution coefficient	std error	p-value	Naïve coefficient	std error	p-value
C	–0.01442	0.004823	0.0053 **	–0.0038	0.005442	0.4898
IIP	0.06798	0.028615	0.0235 *	0.11456	0.033486	0.0017 **
Unemployment	–0.00548	0.002914	0.069	–0.00411	0.003102	0.1943
Monetary base	0.022415	0.013795	0.1137	0.034169	0.01572	0.037 *
M2	0.296632	0.267685	0.2758	0.471573	0.301914	0.1278
Call rate	0.015237	0.002818	0 **	0.013357	0.003702	0.001 **
Long-term interest rate	–0.00251	0.001687	0.1457	–0.00012	0.001975	0.9502
Yen/Dollar	–0.04125	0.0093	0.0001 **	–0.0254	0.01025	0.0185 *
CGPI inflation rate	–0.50918	0.068813	0 **	–0.80966	0.080184	0 **
R^2	0.86001			0.89815		

Note: ** denotes significance at the 0.01 level. * denotes significance at the 0.05 level.

IIP, Monetary base, M2, and Yen/Dollar are on a year-over-year basis. Unemployment, Call rate, Long-term interest rate, and CGPI inflation rate are differences from the previous year.

Equations are estimated by OLS, and standard errors are calculated by the Newey and West (1987) method.

In the Cornish-Fisher mean case and the normal distribution case, the constant terms are also significant. This shows that systematic bias cannot be removed by the explanation of various macroeconomic variables. On the other hand, in the Cornish-Fisher median and mode cases, the constant term is not significant. In other words, although they are not fully efficient, these variables provide desirable characteristics of inflation expectations, as compared to the Cornish-Fisher mean and normal distribution cases.

Moreover, in the normal distribution case, IIP is also significant, whereas in all Cornish-Fisher cases it is not significant. Therefore, in this case, the inflation expectations estimated by Cornish-Fisher are superior.

To evaluate these points, we can consider the influences of the outlier. In similar cases of survey research, the problem of many replies of the multiple of 5 has been pointed out (Kamada, 2008). Even when inflation is steady, replies such as 5% or 10% are provided, and these are detected as outliers. When we take account of the sensitivity of the mean from the outlier, the estimation of inflation expectations by mean distribution may be problematic.

5. Concluding remarks

This paper presented the estimation of inflation expectations using Japanese Cabinet Office survey data. In contrast to previous studies, we did not assume a distribution as *a priori*. Our analysis suggests that skewness and kurtosis may provide necessary information for understanding the shape of the distribution of inflation expectations.

In many financial datasets, price variation is well known to have fat-tail properties. The results described in this paper reinforce this.

The estimated inflation expectations contain slight biasedness. Although these variables may not reflect all of the information efficiently, some superiorities can be verified.

Understanding the distribution of inflation expectations is becoming more important. Mankiw, Reis, and Wolfers (2003) argue this point as “disagreement over inflation expectations,” and state that this may be a key to macroeconomic dynamics. Future studies should be undertaken to elucidate this point.

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