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Stochastic Dominance, Poverty and the Treatment Effect Curve

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Abstract

The paper proposes a simple framework for the evaluation of anti-poverty programs based on single means differences, FGT poverty measures and stochastic dominance theory. A Treatment Effect Curve (TEC) is derived and its use illustrated with simulated data.

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1. Introduction

Recent advances in evaluation theory and methods (Rosenbaum, 1995, Heckman *et al.*, 1998, Imbens, 2004, Rubin, 2006, Inbens *et al.*, 2009) together with the progressive availability of experimental and non experimental data have greatly improved the evaluator tool-kit. While most of program evaluations until the 1990s relied on the comparison of outcomes with and without programs (incidence evaluations), today most evaluations make an effort to consider more in detail behavioral effects (impact evaluations).

Impact evaluations of social programs rely on the comparison of a *treated* group (the group subject to the program) and a *control* group (a comparable group of people not subject to the program). Assuming the two groups identical in all relevant respects except treatment, the simple difference in mean outcomes between the two groups is considered as a proper estimator of the impact of the program.

Mean values are statistically convenient moments to represent distributions but they are not necessarily the best instruments to illustrate and compare distributions. Stochastic dominance theory (Quirk and Saposnik, 1962, Whitmore, 1970) emerged as an alternative tool to summary statistics for comparing distributions in contexts as diverse as financial risk theory (Rothschild and Stiglitz, 1970), choice theory (Hadar and Russell, 1969) or the study of income distribution (Kolm, 1969, Atkinson, 1970) and poverty (Atkinson, 1987, Foster and Shorrocks, 1988).

Research on program evaluation and causal effects seems to have made little use of the stochastic dominance literature. For example, the 2006 book "Matched Sampling for Causal Effects" by Rubin (which collects all major articles by Rubin and co-authors on the subject) does not discuss stochastic dominance theory. None of the articles in the 2008 handbook of development economics[1] (which is entirely dedicated to program evaluations) mentions stochastic dominance theory.[2] And the 2009 article "Recent Developments in the Econometrics of Program Evaluation" on the Journal of Economic Literature by Inbens and Wooldridge does not cite the literature on stochastic dominance. Research on program evaluation has not ignored stochastic dominance theory (see for example Imbens and Rubin, 1997, Abadie, 2002 and Abadie *et al.* 2002) but much of the program evaluation literature remains focused on means differences.

By contrast, the literature on stochastic dominance focused on the conditions to compare distributions from a theoretical and empirical perspective and paid less attention to the questions of population groups comparability, behavioral effects and the missing data problem central in program evaluation theory. The early literature in this field developed the theoretical foundations for dominance comparisons (Quirck and Saposnik, 1962, Hadar and Russell, 1969, Whitmore, 1970) and for applying dominance analysis to the study of poverty and inequality (Atkinson, 1970, 1987, Foster and Shorrocks, 1988). These studies prepared the ground for further theoretical and methodological contributions accompanied by empirical applications to public policies such as the works by Yitzhaki and others on Dalton improving tax reforms with applications to commodities subsidies (Yitzhaki and Thirsk, 1990, Yitzhaki and Slemrod 1991, Mayshar and Yitzhaki, 1995, Yitzhaki and Lewis, 1996) or the works by Duclos, Makdissi and Wodon on the

[1] Schultz and Strauss (2008)

[2] Stochastic dominance is only quoted once in one of the chapters ("Household Formation and Marriage Markets in Rural Area") on page 231.

comparison of income and consumption distributions (Makdissi and Wodon, 2002, Duclos and Makdissi, 2005, Duclos *et al.* 2008). These works typically focused on distributional impacts and did not consider explicitly behavioral changes induced by participation to the programs. In essence, program evaluation theory and stochastic dominance theory have crossed each other only occasionally while they may both gain from increased mutual contamination.

This paper provides a simple example of how advances in stochastic dominance theory could contribute to the evaluation of anti-poverty programs. Dominance theory need not to be constrained to poverty and may well be applied more generally to welfare. However, in this paper, we wish to illustrate the use of stochastic dominance theory in the context of the evaluation of anti-poverty programs exploiting the fact that one class of additive poverty indexes (the FGT poverty indexes)[3] can be expressed in terms of means of poverty gaps providing a convenient link to single means difference estimations. Based on poverty orderings and FGT measures, we propose a treatment effect curve and illustrate the use and advantages of this curve with simulated data.

In section 2, we describe the main problem of program evaluations and some of the possible solutions proposed by the literature. In section 3, we turn to stochastic dominance theory and illustrate advances related to poverty orderings. Section 4 shows how poverty dominance theory could be used for the evaluation of anti-poverty programs through the construction of a treatment effect curve and section 5 illustrates the use of these tools with artificial data.

2. Single difference for causal effects

Let y_i be the potential outcome of a program for person i and let y_{i1} and y_{i0} be the actual outcomes of the program if a person is treated or non treated by the program. Then the the gain from treatment of person i is defined as

$$\Delta_i = y_{i1} - y_{i0} \tag{1}$$

and the average gain for treatment for a population is

$$\Delta = E(y_1) - E(y_0) \tag{2}$$

The problem of identifying Δ above is that we cannot observe the same person or the same group of people in two states at the same time. In the equations above, when we observe treatment y_1 we cannot observe non treatment y_0 and, vice-versa, when we observe y_0 we cannot observe y_1 simultaneously. We have a missing data problem, the central knot in program evaluation theory.

This problem can be addressed in several ways. If the evaluation is prepared *ex-ante*, before the program is launched, we can design a random experiment and extract from the population a random sample to be treated. We can then estimate the treatment effect cross-section in the post-treatment period by taking the means difference between the random treated group and the rest of the population. Given that the random selection of the treated should deliver a representative group of the population, the simple mean

[3] See further for a description of these measures

difference between the treated group and the rest of the population should provide a reliable estimate of the treatment effect. Program evaluation theory is not limited to random experiments and a wealth of evaluation methods have developed to respond to various data and evaluation problems that typically arise in the practice of evaluation.[4] In this paper, we focus exclusively on the ideal setting of a random evaluation and consider only the situation of two perfectly comparable population groups in the post-treatment phase.

Consider now y as an outcome measure of welfare such as income. While the pre-treatment income distributions of the treated and the non treated should center on the same mean and have a similar shape in virtue of the random selection of the treated, the post-treatment distributions may be very different in virtue of the effects determined by the administration of the program on the treated.[5] In particular, the impact of treatment may vary across individuals. Some individuals may have benefitted from the program more than others and the distributions of incomes for the treated and non treated groups in the post-treatment period may have very different shapes in addition to different means. A difference of summary statistics such as the difference in means values provides an average treatment effect but does not provide information on the distribution of the treatment effect. Yet, we may be interested in finding not only *if* the program works but also *who* benefits from the program and *how*. This problem is similar to the problem of comparing means or other moments of a distribution instead of comparing the whole distributions and is a problem treated by stochastic dominance theory.

3. Stochastic dominance and poverty orderings

Let $\mathcal{F}$ be a set of probability density functions of a random variable y defined on a closed interval $[0; y*]$ (where y is non negative and $y*$ is the upper bound) and let $f_A(y)$ and $f_B(y)$ denote two probability density functions belonging to this set. Call also $F_A(y)$ and $F_B(y)$ the respective Cumulative Distribution Functions (CDFs). In the stochastic dominance jargon, distribution B is said to dominate distribution A stochastically at first order if $F_A(y) \geq F_B(y)$ for all $y \in [0; y*]$. This is what is called the *first-order* condition for stochastic dominance. It is then said that distribution B dominates distribution A stochastically at second order if $\int_0^{y*} F_A(y) - F_B(y) \geq 0$ for all $y \in [0; y*]$. This is what is called the *second-order* condition for stochastic dominance. Note that first order stochastic dominance implies second order stochastic dominance but not vice-versa, which means that the second order condition is weaker than the first order condition. Higher order conditions can be established iteratively by taking succesive integrals of the primary density function as follows

$$F^0(y) = f(z) \tag{3}$$

$$F^1(y) = \int_0^y F^0(z)dz \tag{4}$$

$$F^2(y) = \int_0^y F^1(z)dz \tag{5}$$

....

[4]See Schultz and Strauss (2008) for a recent review of the state of the art in this field.

[5]This is the more so if the program has also spill-over effects on the non-treated.

$$F^s(y) = \int_0^y F^{s-1}(z)dz \tag{6}$$

for all $y \in [0; y*]$.[6]

Consequently, dominance conditions of order s for distributions A and B are simply defined as $F_A^s(z) \geq F_B^s(z)\ \forall z$, which reads "Distribution B dominates distribution A at the s^{th} degree for all values of z".

In welfare economics, the first works on stochastic dominance relate to the study of inequality. Kolm (1969) and Atkinson (1970) showed independently how the comparison of two Lorenz curves with equal mean is equivalent to test for second order stochastic dominance condition. This work was later extended to the study of poverty and builds on findings previously emerged on the relation between different poverty measures.

Foster *et al.* (1984) had established that three of the most popular measures of poverty - the headcount, poverty gap and severity of poverty indices - belonged to the same family of measures described as follows

$$P(z;0) = 1/z^0 \int_0^n \pi(z - \tilde{y}_i)^0 dF(y) \tag{7}$$

$$P(z;1) = 1/z^1 \int_0^n \pi(z - \tilde{y}_i)^1 dF(y) \tag{8}$$

$$P(z;2) = 1/z^2 \int_0^n \pi(z - \tilde{y}_i)^2 dF(y) \tag{9}$$

....

$$P(z;\alpha) = 1/z^\alpha \int_0^n \pi(z - \tilde{y}_i)^\alpha dF(y) \tag{10}$$

where $\tilde{y}_i = y_i$ if $y_i < z$ and $\tilde{y}_i = z$ if $y_i \geq z$, z is the poverty line, y_i is income, $z - y_i$ is the poverty gap, π is a disutility function of the poverty gap with $\pi' > 0$ (the disutility of poverty increases with the poverty gap) and α is a poverty aversion parameter (the greater is α the greater is the weight attributed to the poorest among the poor). Indices belonging to this family became known as the FGT poverty indices from the initials of the authors of the seminal article (Foster, Greer and Thorbecke). In equation [10], for $\alpha = 0$ we have the poverty headcount index, for $\alpha = 1$ the poverty gap index and for $\alpha = 2$ the severity of poverty index.

Foster and Shorrocks (1988) then showed that the poverty aversion parameter α had a straightforward connection with the stochastic dominance ordering conditions. In fact, by estimating a poverty index $P(z;\alpha)$ across a range of reasonable poverty lines $[z^-; z^+]$, it is possible to construct a poverty dominance curve of $s = \alpha + 1$ degree, and, by comparing two poverty dominance curves for distribution A and B, it is possible to establish a poverty ordering of $s = \alpha + 1$ degree. More generally, the poverty ordering condition is defined as

$$P_A(\alpha; z) - P_B(\alpha; z) \geq 0\ \forall\ z \in [z^-; z^+] \tag{11}$$

Poverty orderings of first, second and higher degree can therefore be established by comparing poverty curves constructed by estimating FGT poverty measures over a range of

[6] Note that for third degree dominance both equations (3) and (6) must be satisfied (Whitmore, 1970).

poverty lines $[z^-; z^+]$. For first degree poverty dominance this is equivalent to comparing the CDFs of distributions A and B within a $[z^-; z^+]$ range of poverty lines. For second degree poverty dominance, this is equivalent to comparing the space under the two CDFs for the same range of poverty lines, which, in turn, is equivalent to comparing the Lorenz curves for two distributions with equal means or the generalized Lorenz curve for distributions with unequal means (Atkinson, 1987).[7] These are convenient properties if we wish to evaluate the impact of an anti-poverty program on poverty estimated across a range of poverty lines.

4. Program evaluation and the treatment effect curve

We now come back to program evaluation theory and the single difference estimator discussed in section two. Recall that a single difference estimation is defined as the difference in mean values of two distributions of outcomes for two groups of comparable individuals. Let y be a non-decreasing and right-continuous income variable and let y_A and y_B be two distributions of incomes related to two groups of comparable individuals where A refers to the treated group and B refers to the non-treated group in the post-treatment phase. With group A and B identical in all relevant respects except treatment, the average treatment effect is defined as $E(y_A) - E(y_B)$.

Consider now to use the FGT poverty measures $P(z; \alpha)$ as outcome indicators. We wish to know, for example, if an anti-poverty program such as a conditional cash-transfer has managed to reduce poverty. From section [3] we know that the headcount index is $P(z; 0)$, which is equivalent to saying that the headcount index is the average number of poor of an indicator variable where '1' equals poor and '0' equals non-poor. We also know that the average poverty gap index is $P(z; 1)$ and that the average poverty gap squared (the severity of poverty) index is $P(z; 2)$. More generally, the FGT poverty measures are the average of the poverty gap function to the power of α and over F(y) where y is censored at z. Considering that the expected value of a random variable y is its mean and is described as $E(y)$, we can define the gain from treatment for the treated and for the FGT poverty measures as

$$\Delta(z; \alpha) = E(z - y_A)^\alpha - E(z - y_B)^\alpha \tag{12}$$

Following from section [3], it is then easy to see that we can construct Treatment Effect Curves (TECs) of $s = \alpha + 1$ degree by changing the poverty line z across a range of reasonable poverty lines $[z^-; z^+]$. For example, by comparing the two CDFs and the two generalized Lorenz curves for distributions y_A and y_B we can establish poverty dominance of first and second degree. This is a useful complement to simple differences in mean outcomes.

[7] Note, however, that for poverty aversion parameters higher than one and, more generally, for poverty measures that are increasing and convex, second order dominance should be assessed using the rotated generalized Lorenz rather than the generalized Lorenz curve (Yitzhaki, 1999, see also Spencer and Fisher, 1992).

5. An example

An example on simulated data illustrates the use of the TECs. We generated two random samples of income ($A; B$) with the same mean (5; 5) but different variance (1; 2), skewness (2; 0) and kurtosis (9; 3). Distribution A represents the group of treated individuals and distribution B the group of non-treated individuals. We look at first degree dominance by comparing the two CDFs and at second degree dominance by comparing the two Lorenz curves. As we focus on poverty, we plotted the curves for all reasonable poverty lines z by excluding the top and bottom 10% of y values. This is equivalent to test for poverty dominance by estimating the FGT poverty measures over a range of poverty lines. By taking the difference between the two CDFs and the two Lorenz curves we then plotted the TECs for stochastic dominance of first and second degree.

Figure 1, left panel, shows the CDFs and the TEC for first degree dominance. The single means difference for the two distributions is zero by construction so that a standard single difference program evaluation would attribute a zero impact to the program. The comparison of the two CDFs and the TEC shows however that distribution A dominates distribution B at the first degree in the lower part of the distribution (among poorer people) and is dominated by B in the upper part of the distribution (among richer people). 'Dominate' in this case means that distribution A shows higher incomes than B at equal shares of population. Where A dominates B, A has less poverty than B. Note that distributions A and B cross at $y = 4.33$. This means that for poverty lines $z < 4.33$ the impact of the program is always positive with a net decline in poverty for the treated group as compared to the non-treated group. Therefore, the simulated anti-poverty program has a beneficial effect on the poorest. This lesson would be missed with standard single difference program evaluations.

Figure 1, right panel, compares the Lorenz curves for the two distributions and plots the TEC to test for second degree dominance.[8] In this case, the two curves do not cross and we can conclude with confidence that distribution A dominates distribution B for any of the poverty lines considered. With the weaker second degree condition, the impact of the simulated program is always positive. This is also a lesson that would be missed with standard single means difference estimates.

[8]Note that the Lorenz curves are used in place of generalized Lorenz or rotated generalized Lorenz curves because the means of the two distributions are equal.

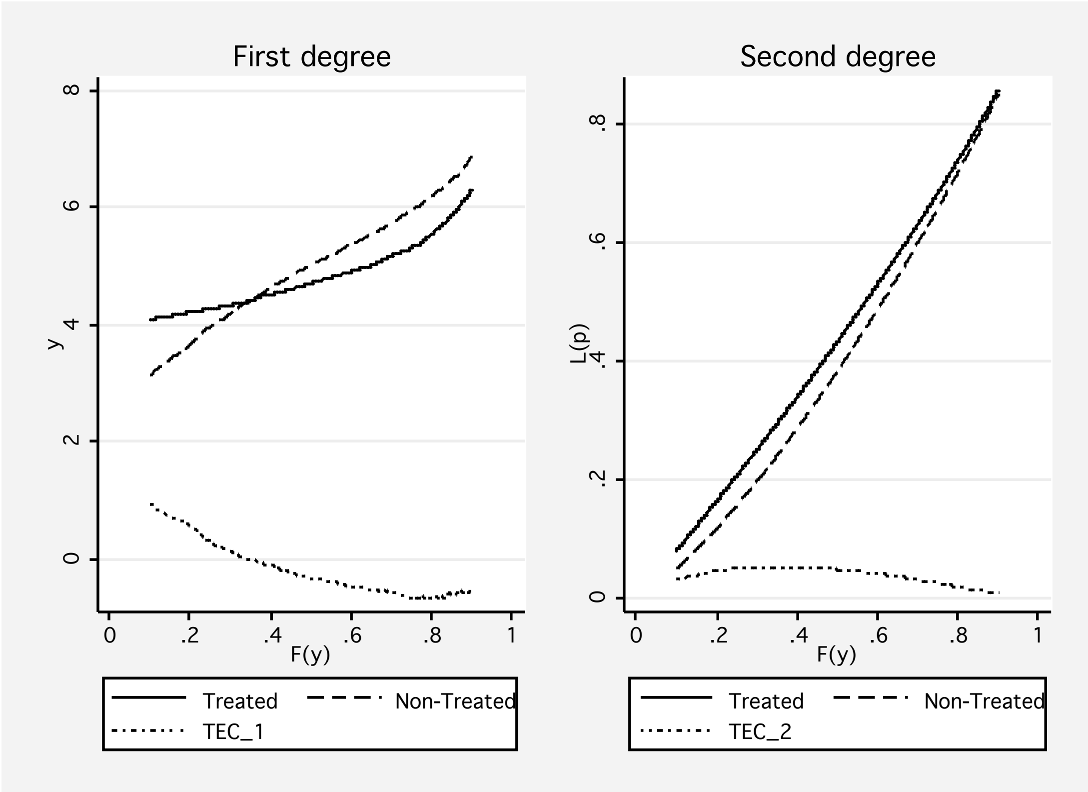

Figure 1: Treatment Dominance

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