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A Comment on "The consequences of the minimum wage when other wages are bargained over"

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Abstract

Cahuc, Saint-Martin, and Zylberberg (2001) show numerically that a minimum wage hike can increase both skilled and unskilled employment in a right-to-manage wage bargaining setting. This comment demonstrates that this result crucially depends on an implicitly unrealistic choice for the skilled workers' alternative wage.

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1 Introduction

Several empirical studies (see e.g. Card and Krueger (1995) for the US, Dolado et al. (1996) for Europe, and Metcalf (2007) for the UK) find that a minimum wage either has no adverse effects on employment or, in some cases, even positive effects. These findings led, in turn, to the development of theoretical models, which question the idea of an inverse relationship between the minimum wage and employment. Most of these contributions build on refinements of the monopsony model, e.g. through the introduction of efficiency wages (Rebitzer and Taylor, 1995) or search unemployment (Manning, 1995; Burdett and Mortensen, 1998; Flinn, 2006).

Cahuc, Saint-Martin, and Zylberberg (2001) (henceforth CSZ) argue that wage bargaining provides another channel for a possibly positive effect of a minimum wage on employment. More precisely, they analyse the impact of a binding minimum wage in a situation where unions dominated by skilled workers set wages. For the special cases of a CES technology or insider unions, they show analytically that the relationship between the minimum wage, the bargained wage and employment of skilled and unskilled workers depends on the magnitude of the elasticity of substitution between skilled and unskilled labour. In order to demonstrate the empirical relevance of these theoretical results under more general assumptions, they present a numerical example using a translog cost function and a non-insider union. They find that a minimum wage hike can increase both skilled and unskilled employment for reasonable parameter settings. This comment demonstrates that this result crucially depends on an implicitly unrealistic choice for the skilled workers' alternative wage. Using more realistic alternative wages, the intriguing result of rising employment even for the unskilled despite a minimum wage increase vanishes.

2 The CSZ model

For the sake of self-containment and in order to motivate our discussion of the numerical example, we first give a short presentation of the CSZ model. The model considers a monopolist firm producing a single good with two types of labour, i.e. skilled labour l_q and unskilled labour l_n. Capital is kept implicit. The firm unilaterally sets employment levels after the wage bargain, following the 'right-to-manage' assumption.

The monopolist firm is assumed to be endowed with a production function $F(l_q, l_n)$, which is twice continuously differentiable, increasing, concave and homogeneous of degree $\alpha \in]0,1[$ with respect to skilled and unskilled work. Homogeneity implies that the cost function $C(\mathbf{w},y) = \min_{l_q,l_n} \sum_{i=q,n} w_i l_i$ s.t. $F(l_q, l_n) \geq y$ given wages $\mathbf{w} = (w_q, w_n)$ and output y can be written as $C(\mathbf{w},y) = W(\mathbf{w}) L$ with $L \equiv y^{1/\alpha}$, where $W(\mathbf{w})$ can be interpreted as a wage index which is increasing, concave, and homogeneous of degree one with respect to $\mathbf{w}$. The partial wage elasticities, equal to the cost share of the respective labour type, are defined as

$$\eta_i \equiv \frac{\partial W}{\partial w_i}\frac{w_i}{W} = \frac{w_i l_i}{WL},\ i = q, n, \tag{1}$$

where Shepard's Lemma is used on the right hand side. Furthermore, it is assumed that the monopolist faces a demand with constant price elasticity with absolute value $e > 1$. The profit Π is therefore $\Pi = py - WL$, with the price of the good $p = Ay^{-1/e}$ and shift demand parameter $A > 0$. Profit maximisation with respect to L gives $L = (\nu W/(\alpha A))^{-\nu/(\nu-\alpha)}$, where $\nu = e/(e-1) > 1$ is the mark-up. Labour demands are given by $l_i = c\eta_i W^{-\alpha/(\nu-\alpha)}/w_i$, $i = q, n$ with $c \equiv (\nu/(A\alpha))^{-\nu/(\nu-\alpha)}$, while optimal profits amount to $\Pi^* = (1-\alpha/\nu)\, A^{\nu/(\nu-\alpha)} (\nu W/\alpha)^{-\alpha/(\nu-\alpha)}$.

The wage bargain is modelled by the generalised Nash solution. Only skilled workers are represented by a trade union. Nonetheless, it is assumed that both wages, w_q and w_n, are bargained over. The skilled workers' contributions to the Nash program is given by $l_q^\chi \cdot (\upsilon(w_q) - \upsilon(\overline{w}_q))$. The parameter $\chi \in [0, 1]$ measures the weight of employment in the trade union's objective function, and $\upsilon(w_q)$ is the utility reached by a skilled worker if there is no strike. Utility $\upsilon(\cdot)$ is increasing, concave and homogeneous. In case of a strike, no one gets paid and the represented workers receive the reservation wage $\overline{w}_q$. The firm contributes its optimal profit to the Nash program. The Nash program can therefore be written as

$$\max_{w_q, w_n} N = \left[l_q^\chi \cdot (\upsilon(w_q) - \upsilon(\overline{w}_q))\right]^\gamma [\Pi^*]^{1-\gamma} + \lambda (w_n - \overline{w}_n), \tag{2}$$

where γ measures the trade union's bargaining power $(0 < \gamma < 1)$, $\overline{w}_n$ represents the minimum wage, and $\lambda \geq 0$ is the Kuhn-Tucker multiplier associated with the restriction $w_n \geq \overline{w}_n$. CSZ argue that the minimum wage is binding, if and only if $\chi \cdot (\nu/(\nu-\alpha) - \sigma(s)) + (1/\gamma - 1)/(\nu/\alpha - 1) > 0$. Since this condition is met for the range of empirical estimates of χ, ν, α, and the elasticity of substitution between skilled and unskilled workers, $\sigma \equiv (dl/ds) \cdot s/l$, where $l \equiv l_q/l_n$, $s \equiv w_n/w_q$, the constraint $w_n \geq \overline{w}_n$ will be binding, implying that the unskilled wage is exogenous and equal to the minimum wage. Thus, only the first order condition $\partial N/\partial w_q = 0$ is relevant for the simulation. It is given by

$$0 = V(w_q) - \mu\pi_q(s) + \chi\varepsilon_q(s), \tag{3}$$

with $\mu \equiv \frac{\gamma-1}{\gamma}$. Note that $V(w_q) \equiv \upsilon'(w_q)\, w_q/(\upsilon(w_q) - \upsilon(\overline{w}_q))$ denotes the skilled wage elasticity of the utility difference $\upsilon(w_q) - \upsilon(\overline{w}_q)$. Defining the partial skilled wage elasticity of profits $\pi_q(s) \equiv \frac{\partial \Pi^*}{\partial w_q}\frac{w_q}{\Pi^*} = -\alpha/(\nu-\alpha)\,\eta_q(s)$ and the partial skilled wage elasticity of skilled labour demand, $\varepsilon_q(s) \equiv \frac{\partial l_q}{\partial w_q}\frac{w_q}{l_q}$, (3) implies that the bargained wage equalises the utility elasticity $V(w_q)$ to the sum of the weighted (by $-\mu$) skilled wage elasticity of profits and the weighted (by χ) skilled wage elasticity of the demand for skilled workers.

Given this model setup, CSZ derive the following results for the special cases of a CES technology (σ constant) or for an insider union ($\chi = 0$):

$$\begin{aligned} &\partial w_q / \partial \overline{w}_n \lesseqqgtr 0 \iff \sigma \gtreqqless 1, \\ &\partial \left(\overline{w}_n / w_q \right) / \partial \overline{w}_n > 0, \\ &\partial l_i / \partial \overline{w}_n \begin{cases} < 0 \text{ if } \sigma \leq 1 \\ \gtreqless 0 \text{ if } \sigma > 1 \end{cases}, \ i = q, n, \end{aligned} \tag{4}$$

i.e. an increase in the minimum wage $\overline{w}_n$ will result in a lower bargained w_q, if the elasticity of substitution $\sigma > 1$ (vice versa, if $\sigma < 1$), while the relative wage $\overline{w}_n / w_q$ will always increase after a minimum wage hike. Labour demands for both skill types will fall, if $\sigma \leq 1$. For $\sigma > 1$ the employment reaction is theoretically ambiguous. If the fall in the skilled wage is strong enough, a minimum wage increase can result in higher employment for both skill types.

3 The CSZ numerical example revisited

Using a numerical example, CSZ try to demonstrate that their theoretical results not only hold for the special cases of a CES technology or an insider union. Therefore, they relax both assumptions by employing a translog cost function, which exhibits a non-constant elasticity of substitution, while at the same time assuming that employment is a part of the union's objective function during the bargain ($\chi > 0$).

As in CSZ, we assume that workers' preferences are given by $v(w_q) = w_q^\rho$, where $\rho \in]0, 1[$ denotes the degree of risk aversion. Using this specification, the first order condition (3) can be solved for w_q,

$$w_q = \left(1 - \frac{\rho}{\mu \eta_q(s) - \chi \varepsilon_q(s)} \right)^{-1/\rho} \overline{w}_q. \tag{5}$$

Assuming a translog cost function amounts to specifying the wage index as

$$\ln W = \ln w_q + (1 - a) \ln s - \frac{b}{2} (\ln s)^2, \tag{6}$$

where a and b are technology parameters to be calibrated. With the wage index (6), equation (1) implies $\eta_q(s) = a + b \ln s$. Also, the elasticity of substitution is given by

$$\sigma(s) = 1 + \frac{b}{\eta_q(s) \left(1 - \eta_q(s) \right)}. \tag{7}$$

In order to replicate the results in CSZ, we use the same calibration for the parameters of the model. Thus, we choose the elasticity of substitution $\sigma = 1.2$, the trade union bargaining power $\gamma = 0.4$, the mark-up $\nu = 1.3$, and we assume the technology to be homogenous of degree $\alpha = 0.65$. The benchmark relative wage is set to $s = 0.5$, while the proportion of unskilled workers is $l_n/(l_q + l_n) = 0.67$.[1] The skilled workers' reservation wage $\overline{w}_q$ as well as the demand shift parameter A can both be set to an arbitrary value, since they only determine the level of wages and employment, but not the variation with respect to s. As in CSZ, we choose unity for both parameters. With these parameter values, we can calibrate the technology parameters $a = 0.53$ and $b = 0.05$ using (7) and the relation of skilled to unskilled workers,

$$\frac{l_q}{l_n} = \frac{s\eta_q(s)}{1 - \eta_q(s)}. \tag{8}$$

Finally, a choice for the weight of employment in the trade union's utility function, χ, is needed in order to simulate the reaction of the bargained wage w_q with respect to a change in the relative wage s.[2] Since there is practically no empirical information on the magnitude of χ, CSZ arbitrarily set a relatively low value, $\chi = 0.15$, while remarking that the positive relationship between an increase in the minimum wage and employment can also be found for higher values of χ, if the mark-up increases as well.

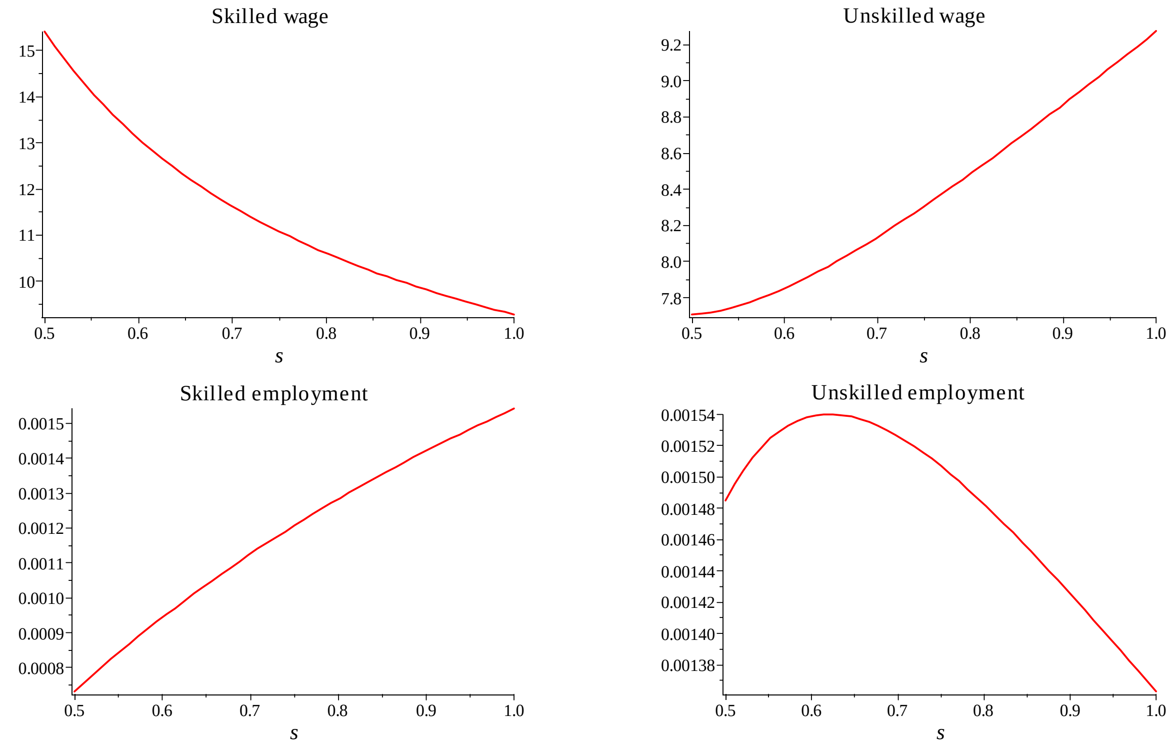
Figure 1 reproduces the CSZ wage and employment reactions to a change in the relative wage s, giving the *levels* instead of the changes relative to the benchmark. As can be seen from the upper left panel, the CSZ calibration results in a skilled wage of 15.4 in the benchmark ($s = 0.5$). Since the benchmark reservation wage is $\overline{w}_q = 1$, this implies a ratio of the alternative wage to the bargained wage (henceforth "wage ratio") of $\overline{w}_q/w_q = 1/15.4 = 0.065$, which is unrealistically low. Even under the extreme assumption that workers necessarily become unemployed in the case of disagreement,

[1] Apparently, there is a typo on p. 348 in CSZ. The sentence "The corresponding proportion of *skilled* workers in total employment is about 2/3" (emphasis by author) should read "...proportion of *unskilled* workers...", since 2/3 is approximately the correct share for unskilled workers using the definitions of skilled and unskilled labour in Sneessens and Shadman-Metha (1995). The simulation results of CSZ are obviously based on the correct proportion, $l_n/(l_q + l_n) = 0.67$, since we are able to replicate their results using this setting. Thus, CSZ assume 2/3 of all workers to receive the minimum wage in the benchmark. Nevertheless, this rather high share of minimum wage receivers is not crucial for the model's ability to produce positive employment effects after an increase in the minimum wage. For example, by slightly increasing the bargaining power to $\gamma = 0.5$ and at the same time reducing the weight of employment in the trade union's utility function to $\chi = 0.08$, the positive employment effect for both types of labour can be recovered using $l_n/(l_q + l_n) = 0.33$ in the benchmark.

[2] Employing the relative wage s as the exogenous parameter instead of directly using the minimum wage $\overline{w}_n$ simplifies the simulation, since the expression (5) can be used to compute w_q for a given s. Using $\overline{w}_n$ would require solving the first order condition (3) for each value of $\overline{w}_n$ numerically for w_q. Following the latter approach does not change the simulation results qualitatively, but quantitatively we find small differences to the former approach.

workers would still receive unemployment benefits and/or other transfers. For example, the OECD (2004) reports gross replacement ratios for 21 countries in 2003 varying between 0.08 and 0.53, with an (unweighted) average of 0.3, and thus markedly higher than the benchmark wage ratio assumed in CSZ. Allowing for the possibility to obtain a job elsewhere will typically lead to an expected wage ratio at least an order of magnitude higher than in the CSZ calibration.[3]

Figure 1: Reaction of wages and employment when the relative wage s rises



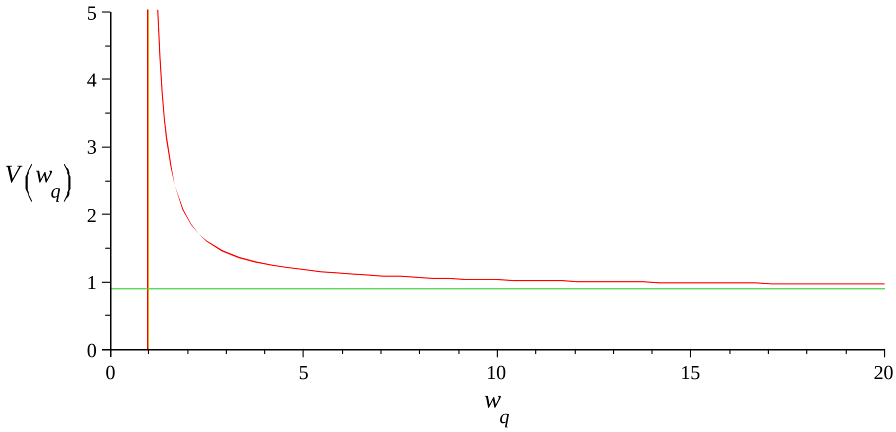
Replication of CSZ simulation, variables in levels.

A closer inspection of the first order condition (3) reveals why the choice of the benchmark wage ratio is crucial for a positive employment effect of a minimum wage to occur. Given the specification for workers' preferences $\upsilon(w_q) = w_q^\rho$, from the definition of $V(w_q)$ we get $V(w_q) = \rho w_q^\rho / \left(w_q^\rho - \overline{w}_q^\rho\right)$. Thus, the elasticity $V(w_q)$ is strictly decreasing in w_q, where $\lim_{w_q \to \overline{w}_q} V(w_q) = \infty$ and $\lim_{w_q \to \infty} V(w_q) = \rho$. Figure 2 plots $V(w_q)$ for the CSZ values $\overline{w}_q = 1$ and $\rho = 0.9$. As mentioned above, the CSZ calibration sets the benchmark value of w_q to 15.4. At that wage, the slope of $V(w_q)$ is small,

[3]For example, assuming that a) unemployment benefits are indexed to the gross wage with a replacement rate of 0.3, b) the unemployment rate amounts to $u = 0.1$, and c) skilled workers can obtain the same wage elsewhere, the expected wage ratio is given by $(u \cdot 0.3 \cdot w_q + (1-u) \cdot w_q)/w_q = 0.93$.

since the elasticity is already close to its lower limit $\rho = 0.9$ $(V(15.4) = 0.98)$. Starting from this benchmark and using the CSZ calibration, an increase in the minimum wage, which is reflected in an increase in the relative wage s, leads to an increase of the term $\mu\pi_q(s) - \chi\varepsilon_q(s)$ in the first order condition. In order for (3) to be satisfied, this must be accompanied by an increase in $V(w_q)$ by the same amount. Because of the small slope of $V(w_q)$ at the benchmark value for w_q, this increase in $V(w_q)$ requires a relatively strong fall in the skilled wage.[4] For the CSZ calibration, the drop in w_q proves to be sufficiently high to produce positive employment effects for the skilled as well as for the unskilled workers. Put differently, the simulated positive employment reactions in CSZ hinge crucially on an unrealistically low wage elasticity of trade union utility in the initial situation.

Figure 2: Plot of the trade unions' elasticity of utility $V(w_q)$.



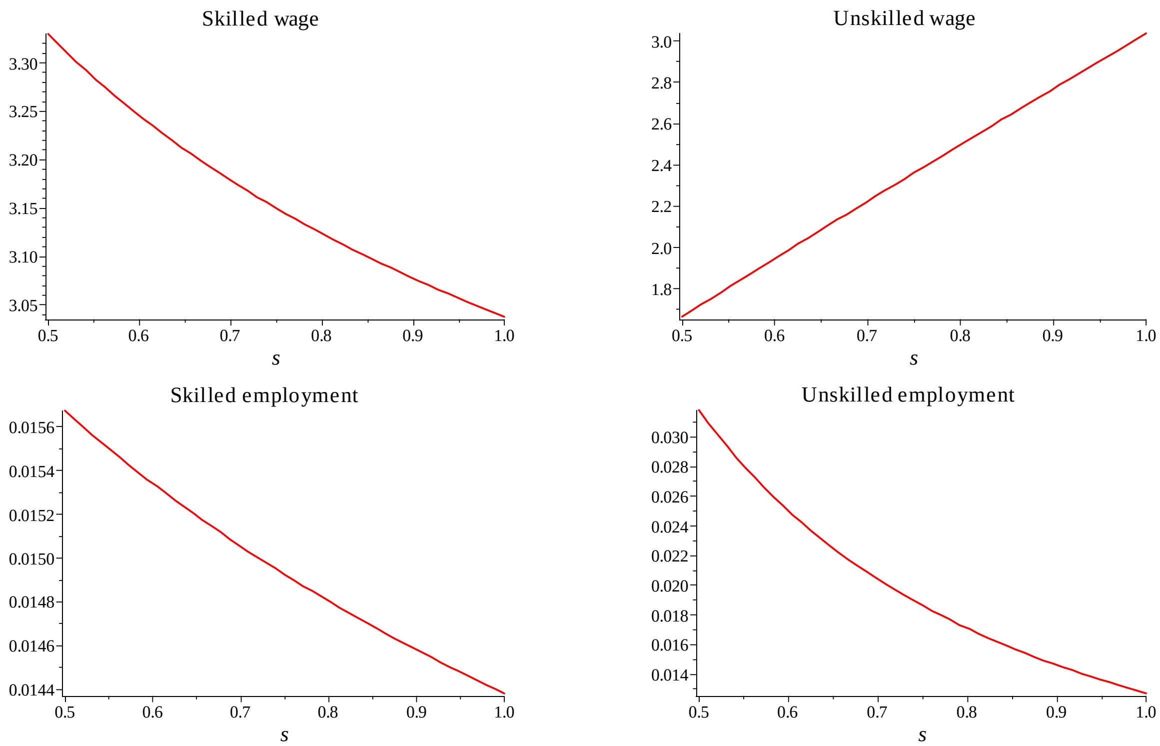
Note that there is no compelling reason for choosing χ arbitrarily, since the first order condition (3) can be employed to calibrate the parameter. Since the reservation wage $\overline{w}_q$ is the numeraire in the model, the benchmark value for w_q should be chosen in a way to reflect empirical wage ratios. Employing the average gross replacement rate of 0.3 in 2003 from OECD (2004) gives a benchmark value of $w_q = 3.33$. Given the other CSZ parameter settings, this implies a benchmark value of $\chi = 0.38$. Using this calibration, the demand for both types of work decreases as a result of an increase in the minimum wage (see Figure 3).[5] This holds a fortiori, if we consider the possibility that skilled workers can obtain a comparable wage elsewhere, since this will result in

[4]Inverting $u = V(w_q)$, we get $w_q = V^{-1}(u)$. The elasticity of this inverse function with respect to u is given by $\xi_{w_q}(u) \equiv \left(dV^{-1}(u)/du\right)\left(u/V^{-1}(u)\right)$. In the benchmark, $\xi_{w_q}(0.98) = -12.5$.

[5]Calibrating χ has no effect on the technology parameters a and b, since χ does not occur in the equations (7) and (8), which are used for calibrating a and b. Also, second order conditions for a maximum of (2) are satisfied at all values of s using $\chi = 0.38$ for the benchmark calibration.

We tried to recover the positive employment effects reported in CSZ by varying the bargaining power γ, the mark-up ν, the skilled workers degree of risk aversion ρ, and the benchmark elasticity an expected wage ratio close to one and – because of the steepness of $V(w_q)$ close to $\overline{w}_q$ – to a correspondingly small decrease in w_q.

Figure 3: Reaction of wages and employment when the relative wage s rises



Simulation based on OECD (2004) gross replacement rates, variables in levels.

4 Concluding remarks

We have shown that the model proposed in Cahuc, Saint-Martin, and Zylberberg (2001) is not able to produce a positive employment effect for an increase in the minimum wage, especially for unskilled workers, if more realistic settings for the ratio of the skilled alternative wage to the bargained skilled wage are used in the calibration. The question remains whether it is possible to change some elements of the CSZ model in a way which allows for positive employment effects to occur using reasonable parameter settings. We leave this question to future research.

of substitution σ, reflecting the wide range of estimates for these parameters. While variations in γ, ν, and ρ do not change the employment reactions qualitatively, a c.p. increase in σ to approximately 1.3 and higher leads to an increase in the demand for skilled labour – at the cost of an even stronger fall in the demand for unskilled labour and overall employment.

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