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Stability under learning of equilibria in financial markets with supply information

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Abstract

In a recent paper Ganguli/Yang (2009) demonstrate, that there can exist multiple equilibria in a financial market model a' la Grossman/Stiglitz (1980) if traders possess private information regarding the supply of the risky asset. The additional equilibria differ in some important respects from the usual equilibrium of the Grossman-Stiglitz type which still exists in this model. This note shows that these additional equilibria are always unstable under eductive learning (cf. Guesnerie (2002)) and adaptive learning via least-squares estimation (cf. Marcet/Sargent (1988) or Evans/Honkapohja (2001)). Regarding the original Grossman-Stiglitz type equilibrium, the stability results are less clear cut, since this equilibrium might be unstable under eductive learning while it is always stable under adaptive learning.

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1 Introduction

In a recent paper Ganguli and Yang (2009) demonstrate, that there can exist multiple equilibria in a financial market model á la Grossman and Stiglitz (1980) if traders possess private information regarding the supply of the risky asset. The informational properties of the additional equilibria differ from the usual Grossman–Stiglitz like equilibrium which still exists in this model.

As usual in case of multiple equilibria, the question arises whether or not there exists a plausible selection device which implies that traders indeed coordinate on these additional equilibria. One important selection device asks whether or not a specific equilibrium is stable under learning. Discussing this briefly, Ganguli and Yang (2009) note that the static setup of their model doesn't allow for such an analysis as learning processes are inherently dynamic.

This, however, is not entirely correct. Not only do there exist concepts of learning that are applicable to static models. It is moreover possible to put the model of Ganguli and Yang (2009) into a framework which makes it possible to analyse real-time adaptive learning processes. Using the concepts of 'eductive learning' tracing back to Guesnerie (2002) and adaptive learning via least–squares estimation following Marcet and Sargent (1988) or Evans and Honkapohja (2001) it is shown that the additional equilibria described by Ganguli and Yang (2009) are always unstable under learning. Thus, the strict use of stability under these two learning procedures as a selection device would always eliminate these additional equilibria. From a more general perspective, instability of the additional equilibria under these two types of learning procedures gives at least rise to some doubts regarding their plausibility, because we can not take it for granted that traders will coordinate on these equilibria or learn to form corresponding expectations. Regarding the original Grossman–Stiglitz type equilibrium, we get no clear cut stability results, since this equilibrium might be unstable under eductive learning while it is always stable under adaptive learning.

Before proceeding with the analysis, a remark is necessary. The following analysis assumes that the amount of private information on the side of the traders is exogenously given, whereas Ganguli and Yang (2009) analyze a model where traders buy this information at a cost.[1] However, as will be argued in Section 4, the results regarding instability under learning derived from the model with exogenous private information carry over to this case.

2 A financial market model with supply information

There is a continuum of traders $i \in I = [0,1]$ and each trader is endowed with $\bar{x}$ units of the riskless asset and $\bar{z}(i)$ units of a risky asset. The riskless asset yields 1 unit, the risky asset β units of a single consumption good, where β is unknown and drawn from a normal

[1] With respect to the acquisition of private information they in fact analyze two different versions of the model.

distribution with mean $\overline{\beta}$ and precision τ. Traders possess private information regarding the return of the risky asset, but since aggregate supply of the stock is stochastic too, the REE price of the asset will not be fully revealing. Each trader observes a private signal $s(i) = \beta + u(i)$ that informs about β. Here $u(i)$ is for all i an independent and normally distributed random variable with zero mean and precision τ_u. The endowment of a trader with the risky asset is given by $\overline{z}(i) = \overline{z} + \varepsilon + \eta(i)$, where $\eta(i)$ is an idiosyncratic shock, which is normally distributed with zero mean and precision τ_η. The common shock ε to the aggregate supply of the stock is also normally distributed with zero mean and precision τ_ε.[2]

Using the riskless asset as numeraire and with p denoting the price of the risky asset as well as z_i denoting the demand of the risky asset of trader i, his final wealth $W_{1,i}$ is:

$$W(i) = \overline{x} + p\,\overline{z}(i) + z(i)\,[\beta - p]$$

Each trader maximizes the expected utility of his final wealth $W(i)$, where the utility function exhibits constant absolute risk aversion $0 < \gamma < \infty$ for all $i \in I$. A trader's asset demand $z(i)$ in this model is conditioned on his private signal $s(i)$ regarding the asset return, his information regarding the aggregate supply of the stock contained in $\overline{z}(i)$ as well as the current asset price p. Optimal asset demand of trader i then results as:

$$z(i)^* = \frac{1}{\gamma\,\mathrm{Var}[\beta\,|\,s(i), p, \overline{z}(i)]}\left[\mathrm{E}[\beta\,|\,s(i), p, \overline{z}(i)] - p\right]$$

From the assumptions made above it then follows that the model exhibits a linear rational expectations equilibrium.[3] In particular this means:

Proposition 1 *If* $\frac{\tau_u \tau_\eta}{\gamma^2} < \frac{1}{4}$ *then there exist two rational expectations equilibria in which asset demand* $z^*(i)$ *of trader i observing the signal* $s(i)$*, his endowment* $\overline{z}(i)$ *and the current price p is given by the linear function* $z(i)^* = \delta_0^* + \delta_1^* s(i) + \delta_2^* p + \delta_3^* \overline{z}(i)$*, where*

$$\delta_0^* = \frac{(1-\delta_3^*)\left[(1-\delta_3^*)\,\overline{\beta}\,\tau + \delta_1^* \tau_\varepsilon \overline{z}\right]}{(1-\delta_3^*)^2\gamma + \delta_1^*(\tau_\eta + \tau_\varepsilon)} \tag{1a}$$

$$\delta_1^* = \frac{\tau_u}{\gamma} \tag{1b}$$

$$\delta_2^* = -\frac{\delta_1^{*2}(\tau_\varepsilon + \tau_\eta) + (1-\delta_3^*)^2(\tau + \tau_u)}{(1-\delta_3^*)^2\gamma + \delta_1^*(\tau_\eta + \tau_\varepsilon)} \tag{1c}$$

$$\delta_3^* = \frac{\delta_1^* \tau_\eta}{\gamma(1-\delta_3^*)} \tag{1d}$$

Proof. See Proposition 1 of Ganguli and Yang (2009). □

[2] As the model used here is one, where all traders possess private information, this model is closer in spirit to Diamond and Verrecchia (1981) than to Grossman and Stiglitz (1980). However, with respect to the stability analysis this difference is of minor importance.

[3] A usual question is whether there exist nonlinear equilibria besides the linear equilibria described below. See Vives (1993) for a suitable set of additional assumptions that allow to prove uniqueness of linear equilibria.

Multiple equilibria arise from the quadratic equation (1d). If $\frac{\tau_u \tau_\eta}{\gamma^2} < \frac{1}{4}$ this equation exhibits two real solutions, henceforth denoted $\delta^*_{3,I}$ and $\delta^*_{3,II}$:

$$\delta^*_{3,I} = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\tau_u \tau_\eta}{\gamma^2}}, \qquad \delta^*_{3,II} = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\tau_u \tau_\eta}{\gamma^2}} \tag{2a}$$

As (1b) reveals, δ^*_1 is unique across these equilibria, while δ^*_0 and δ^*_2 are not. Thus, if $\frac{\tau_u \tau_\eta}{\gamma^2} < \frac{1}{4}$ we end up with two rational expectations equilibria characterized by $\Delta_I = (\delta^*_{0,I}, \delta^*_1, \delta^*_{2,I}, \delta^*_{3,I})$ and $\Delta_{II} = (\delta^*_{0,II}, \delta^*_1, \delta^*_{2,II}, \delta^*_{3,II})$. Δ_I is denoted the Grossman–Stiglitz like equilibrium as the properties of this REE are similar to the one of the Grossman and Stiglitz (1980) model.

2.1 The T–map

In what follows, the analysis of learning processes, either eductive or adaptive, will be conducted with the help of the so called T–map. This T–map describes how parameters of a linear decision rule followed by the agents change with the passage of (virtual or real) time due to learning. This T–map is extensively used in the analysis of adaptive learning processes following the approaches of Marcet and Sargent (1988) and Evans and Honkapohja (2001). In the present context, this T–map turns out to coincide with the best response mapping defined in the following Proposition.

Proposition 2 *If asset demand $z(i)$ of all traders i is linear in $s(i)$, p and $\overline{z}(i)$ and given by $z(i) = \delta_0 + \delta_1 s(i) + \delta_2 p + \delta_3 \overline{z}(i)$, the best response of any trader $j \in I$ is also a linear function $z(j)^* = \delta'_0 + \delta'_1 s(j) + \delta'_2 p + \delta'_3 \overline{z}(j)$, where*

$$\delta'_0 = \frac{(1-\delta_3)^2 \overline{\beta} \tau - \delta_1 \left[\delta_0 (\tau_\varepsilon + \tau_\eta) - (1-\delta_3) \tau_\varepsilon \overline{z}\right]}{\gamma (1-\delta_3)^2} \tag{3a}$$

$$\delta'_1 = \frac{\tau_u}{\gamma} \tag{3b}$$

$$\delta'_2 = \frac{(1-\delta_3)^2 (\tau + \tau_u) + \delta_1 (\delta_1 + \delta_2)(\tau_\varepsilon + \tau_\eta)}{\gamma (1-\delta_3)^2} \tag{3c}$$

$$\delta'_3 = \frac{\delta_1 \tau_\eta}{\gamma (1-\delta_3)} \tag{3d}$$

Proof. See Appendix. □

With $\delta' = (\delta'_0 \ldots, \delta'_3)'$ and $\delta = (\delta_0, \ldots, \delta_3)'$ equations (3a)–(3d) give rise to the so called T-map which is central to the analysis of learning processes:

$$\delta' = T_\delta(\delta) \tag{4}$$

Obviously, the above described REE Δ_I and Δ_{II} are fixed points of this T–map.

2.2 Eductive learning

The concept of a strongly rational expectations equilibrium (SREE) asks, whether a specific REE can be 'educed' by agents assuming nothing more than individual rationality and common knowledge.[4] The idea is that agents will not follow strategies that are not best responses to other agent's strategies. Thus, in a way analogous to the concept of a rationalizable Nash–equilibrium, non–best responses can be eliminated from the agent's strategy sets. A REE is eductively stable or a SREE, whenever the REE is the unique outcome of this process. Guesnerie (2002) provides a comprehensive description of this concept and the reader is referred to this reference for details.

Regarding the proof of eductive stability, the essential point is that this proof obviously depends on the properties of the best response mapping. A REE is eductively stable if and only if this REE turns out to be a locally stable stationary point of the best response mapping. As this best response mapping coincides with the T–map, eductive stability requires that all eigenvalues of the Jacobian of the T–map (4) evaluated at the specific REE are less than one in absolute value. Now, from (3a)–(3d) and using (1b) we get that the eigenvalues $\lambda_1, \ldots, \lambda_4$ of the T–map are given by:

$$\lambda_1 = 0, \quad \lambda_2 = \frac{\tau_u \tau_\eta}{(1-\delta_3^*)^2 \gamma^2}, \quad \lambda_3 = \lambda_4 = -\lambda_2 - \frac{\tau_u \tau_\varepsilon}{(1-\delta_3^*)^2 \gamma^2} \tag{5}$$

(1d) implies $(1-\delta_3^*) = \frac{\tau_u \tau_\eta}{\gamma^2 \delta_3^*}$ and so λ_2 becomes $\lambda_2 = \frac{\gamma^2 \delta_3^{*2}}{\tau_u \tau_\eta}$. Together with (2a) we then get that λ_2 is non negative and always greater than one in case of the Δ_{II}–REE and always smaller than one in case of the Δ_I–REE. While this implies that the Δ_{II}–REE is never a SREE, it does not imply that the Δ_I–REE is always eductively stable. As (5) reveals, $0 < \lambda_2 < 1$ doesn't rule out that the remaining two eigenvalues λ_3 and λ_4 are smaller than -1. This simply repeats an already known result (cf. Desgranges and Heinemann (2003)) according to which the unique REE of the original Grossman–Stiglitz model is not always a SREE. Some computations show that a sufficient condition for eductive stability of the Grossman–Stiglitz like Δ_I–REE is $\tau_u < \tau_\eta$.

3 Stability under adaptive learning

In order to analyze the stability of the two above described REE under adaptive learning it is necessary to embed the hitherto static model into a dynamic framework such it is at all possible to analyze real time learning processes. Thus, from now on it is assumed that the just described static model is repeated over a long horizon. In each period t, two ex ante unobserved random variables $\bar{z}_t$ and β_t realize and traders observe their private signals $s(i)_t = \beta_t + u(i)_t$ as well as $\bar{z}(i)_t = \bar{z}_t + \varepsilon_t + \eta(i)_t$. Individual asset demand depends on an estimator $\hat{\beta}(i)_t$ of the unknown asset as well as an estimator for its variance $\mathrm{Var}[\hat{\beta}](i)_t$ based on data available up to time t. At the end of every period, agents then revise their estimates

[4] The terms 'strongly rational expectations equilibrium' and 'eductively stable equilibrium' can be used interchangeable.

$\hat{\beta}(i)$ and $\mathrm{Var}[\hat{\beta}](i)$ in the light of new data, consisting of the endogenous variable p_t and their private signals $s(i)_t$ and $\overline{z}(i)_t$ as well as the ex post observed realizations $\overline{z}_t$ and β_t. The recursive estimation is done using recursive least squares.

Estimation of the equation

$$\beta = \alpha_0 + \alpha_1\, s(i) + \alpha_2\, p + \alpha_3\, \overline{z}(i)\,,$$

by trader i using data up to time t then leads to an estimator $\hat{\beta}(i)_{t+1}$ for β given by $\hat{\beta}(i)_{t+1} = y(i)'_{t+1}\hat{\alpha}(i)_{t+1}$, where $y(i)_t = (1, s(i)_t, p_t, \overline{z}(i)_t)'$, $\alpha(i)_t = (\alpha(i)_{0,t}, \ldots, \alpha(i)_{3,t})'$ and

$$\hat{\alpha}(i)_{t+1} = \hat{\alpha}(i)_t + \frac{1}{t} R(i)_t^{-1} y(i)_t \left(\beta_t - y(i)'_t\, \hat{\alpha}(i)_t\right) \tag{6a}$$

$$R(i)_{t+1} = R(i)_t + \frac{1}{t} \left(y(i)_t\, y(i)'_t - R(i)_t\right) \tag{6b}$$

An estimator $\hat{v}(i)$ for the variance results as

$$\hat{v}(i)_{t+1} = \hat{v}(i)_t + \frac{1}{t} \left([\beta_t - y(i)'_t\hat{\alpha}(i)_t]^2 - v(i)_t\right) \tag{6c}$$

Given these estimates, asset demand of trader i in period t is given by:

$$z(i)_t = \frac{\hat{\beta}(i)_t - p_t}{\gamma\, \mathrm{Var}[\hat{\beta}](i)_t} = \frac{1}{\gamma \hat{v}(i)_t} \left(\hat{\alpha}(i)_{0,t} + \hat{\alpha}(i)_{1,t}\, s(i)_t + (\hat{\alpha}(i)_{2,t} - 1)\, p_t + \hat{\alpha}(i)_{3,t}\, \overline{z}(i)_t\right), \tag{7}$$

Equation (7) is again linear in $s(i)$, p and $\overline{z}(i)$ and the question now is, whether adaptive learning implies that the coefficients of this linear demand function converge against their REE counterparts Δ_I or Δ_{II}. With respect to this, it turns out that the asymptotic properties of the adaptive learning process are again characterized by the properties of the above described T–map (see Heinemann (2009) for details). Using the stochastic approximation tools described by Evans and Honkapohja (2001), it can be shown (see Appendix A.2 for details) that the asymptotic dynamics of the learning algorithm are governed by a system of ordinary differential equations, which is given by:

$$\begin{pmatrix} \dot{\alpha} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} T_\alpha(\alpha, v) - \alpha \\ T_v(\alpha, v) - v \end{pmatrix} \tag{8}$$

Thus, a REE of the model is stable under adaptive learning whenever the eigenvalues of Jacobian of (T_α, T_v) evaluated at an REE are smaller than one (implying that the eigenvalues of the map (8) are negative).

Now, the eigenvalues of the Jacobian of $(T_\alpha(\alpha, v), T_v(\alpha, v))$ evaluated at an REE coincide with the respective eigenvalues of the Jacobian of $T_\delta(\delta)$ (see again Appendix A.2 for details). Therefore, as the above discussion of eductive stability revealed, the Δ_{II}–REE cannot be stable under adaptive learning as this equilibrium implies that one eigenvalue (λ_2 from (5)) is greater than one. On the other hand, the above described results imply that the Δ_I–REE is always stable under adaptive learning.

4 Summary and discussion

The aim of the paper was to show that it is possible to analyze the properties of multiple equilibria existing in the financial market model of Ganguli and Yang (2009) under learning. This analysis revealed that the additional equilibria which arise in their model due the existence of supply shocks are unstable under eductive as well as adaptive learning. If ever, the original Grossman–Stiglitz type REE turns out to be stable under learning as this equilibrium is always stable under adaptive learning and potentially stable under eductive learning.

As the model analyzed in the present paper is one where — contrary to the analysis by Ganguli and Yang (2009) — private information is exogenously given, it remains to discuss, whether the endogenization of the decision to acquire information can lead to any changes of the stability results. Ganguli and Yang (2009) discuss two versions of their basic model. While in the first version only private information regarding the unknown asset return β is acquired endogenously, the second version additionally assumes private information acquisition regarding the aggregate supply of the stock $\bar{z}$. With respect to the equilibria that arise taking as given the private acquisition of information (what they call 'financial market equilibria'), both versions lead to identical conclusions.

With regard to the acquisition of information, the crucial point is that the decision of a trader to acquire information will be based on the costs as well as the expected benefits of private information acquisition. Therefore, this decision is made in prospect of a specific REE.[5] As a consequence, any REE which already turns out to be unstable under learning with exogenously given information will also be unstable when acquisition of information is endogenous. In a formal analysis, endogenous acquisition of information goes along with an additional condition for stability under learning which may or may not be stronger than those derived here for the case with exogenous information. While this will not alter the properties of a REE which is already unstable in case of exogenous information, a REE which is stable with exogenous information might still become unstable in case of endogenous information.[6]

Altogether, this implies that an REE which is unstable under learning with exogenous information must be also unstable under learning when the decision to acquire information is endogenous. This argument applies to the above described Δ_{II}–REE and thus to the additional REE equilibria Ganguli and Yang (2009) obtain in both versions of their model. These equilibria are therefore unstable under eductive learning and least–squares learning with exogenous as well as endogenous private information. Things are a bit different for the Δ_I–REE which might be stable under learning. Here endogenous acquisition of in-

[5] The two–stage procedure adopted e.g. by Verrecchia (1982) to compute REE with endogenous acquisition of information illustrates this very clearly.

[6] With respect to eductive learning, this is demonstrated by Desgranges and Heinemann (2003) in a model similar to the financial market model of Grossman and Stiglitz (1980). They show that eductive stability with exogenous information is a necessary condition for eductive stability with endogenous acquisition of information as the latter leads to additional and possibly stronger conditions for eductive stability.

formation can in fact give rise to stronger stability conditions. It is, however, beyond the scope of this paper — and therefore an open question that might merit further research — to derive the set of conditions that govern stability of the Δ_I–REE under learning in case of endogenous information acquisition in a financial market model with supply information á la Ganguli and Yang (2009).

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A Appendix

A.1 Best response mapping

Given $z(i) = \delta_0 + \delta_1 s(i) + \delta_2 p + \delta_3 \overline{z}(i)$ for all $i \in I$, we have $p = \frac{(\bar{z}+\varepsilon)(1-\delta_3)-\delta_0-\delta_1\beta}{\delta_2}$. With $y(j) = (s(j), p, \overline{z}(j))'$ and $\overline{y} = (\overline{\beta}, \overline{p}, \overline{z})'$ it then follows:

$$\mathrm{E}[\beta \,|\, p, s(j), \overline{z}(j)] = \overline{\beta} - \overline{y}' M_{yy}^{-1} M_{\beta y}' + y(j)' M_{yy}^{-1} M_{\beta y}'$$

$$\mathrm{Var}[\beta \,|\, p, s(j), \overline{z}(j)] = \frac{1}{\tau} - M_{\beta y}' M_{yy}^{-1} M_{\beta y}$$

Here $M_{yy} = \mathrm{E}[y(j)y(j)']$ and $M_{\beta y} = \mathrm{E}[y(j)\beta]$ and the respective moments appearing in the matrix M_{yy} and vector $M_{\beta y}$ are functions of $\delta_0, \ldots, \delta_3$. It then follows that optimal asset demand $z^*(j) = \frac{\mathrm{E}[\beta \mid p, s(j), \bar{z}(j)] - p}{\gamma \mathrm{Var}[\beta \mid p, s(j), \bar{z}(j)]}$ of a trader j is a linear function of $s(j)$, p and $\bar{z}(j))$ the coefficients of which depend on $\delta_0, \ldots, \delta_3$ too. Computing the respective moments substituting these into the asset demand function then gives the best response mapping.

A.2 Asymptotic Properties of Least–Squares Learning

Using stochastic approximation tools described by Evans and Honkapohja (2001), it follows that with respect to $\hat{\alpha}(i)$ and $\hat{v}(i)$ the asymptotic dynamics of the learning process (6a)–(6c) are governed by ordinary differential equations which in the present context are given as follows:

$$\begin{aligned}
\dot{\alpha}(i) &= \mathrm{E}\left[R(i)^{-1} y(i) \left(\beta - y(i)' \alpha(i)\right)\right] = \left(\mathrm{E}\left[y(i)y(i)'\right]\right)^{-1} \mathrm{E}\left[y(i)\beta\right] - \alpha(i) \\
&= M_{yy}^{-1} M_{\beta y} - \alpha(i) && \text{(A.9a)} \\
\dot{v}(i) &= \mathrm{E}\left[(\beta - y'(i)\alpha_i)^2 - v(i)\right] = \mathrm{E}[\beta^2] - \mathrm{E}\left[y(i)\beta\right]' \left(\mathrm{E}\left[y(i)y'(i)\right]\right)^{-1} \mathrm{E}\left[y(i)\beta\right] - v(i) \\
&= \frac{1}{\tau} - M_{\beta y}' M_{yy}^{-1} M_{\beta y} - v(i) && \text{(A.9b)}
\end{aligned}$$

The moments appearing in the matrix M_{yy} and the vector $M_{\beta y}$ are functions of the parameters $\alpha_0, \ldots, \alpha_3$ and v of the other traders' demand functions. Thus, (A.9a) and (A.9b) define two dynamic equations $\dot{\alpha}(i) = T_\alpha(\alpha, v) - \alpha(i)$ and $\dot{v}(i) = T_v(\alpha, v) - v(i)$. Now, all traders learn in an identical way from individual data which is drawn from identical distributions. Due to this symmetry, we can drop the individual subscripts when studying the asymptotic behavior of the learning process such that we end up with the following dynamic system:

$$\begin{pmatrix} \dot{\alpha} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} T_\alpha(\alpha, v) - \alpha \\ T_v(\alpha, v) - v \end{pmatrix} \tag{A.10}$$

A REE is a stable stationary point of this system, whenever the eigenvalues of the Jacobian of (T_α, T_v) evaluated at the REE are smaller than one. Computing the respective derivatives and using the fact that in a REE we must have $\frac{\alpha_0^*}{\gamma v^*} = \delta_0^*$, $\frac{\alpha_2^* - 1}{\gamma v^*} = \delta_2^*$ and $\frac{\alpha_3^*}{\gamma v^*} = \delta_3^*$ as well as

$$v^* = \mathrm{Var}[\beta \mid p, s(i), \bar{z}(i)] = \frac{(\delta_3^* - 1)^2}{(\delta_3^* - 1)^2(\tau + \tau_u) + \delta_1^{*2}(\tau_\varepsilon + \tau_\eta)}$$

then reveals after some manipulation that the eigenvalues of the Jacobian of (T_α, T_v) coincide with the eigenvalues of T_δ