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Measuring the Intertemporal Elasticity of Substitution for Consumption: Some Evidence from Japan

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Abstract

The purpose of this paper is to present improved estimates of the intertemporal elasticity of substitution (IES) for Japan assuming a constant relative risk aversion (CRRA) utility function. The estimates of the IES we obtain range from 0.2 to 0.5 when we use quarterly consumption data and the Continuous Updating Estimator (CUE). We find that the IES is weakly identified when we employ the two-step GMM estimator, while the CUE can identify the IES. Moreover, we also find that using consumption data of different frequencies leads to quite different estimates of the IES.

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1 Introduction

In general, it is expected that a household will change its intertemporal consumption allocation in response to changes in real assets returns. This relationship can be explained by the intertemporal elasticity of substitution (IES) or relative risk aversion (RRA). Many empirical studies still have an interest in the appropriate value of IES (see, for example, Oshio and Kobayashi (2009)). Therefore, measuring a consumer's IES (RRA) is one of the most important economic problems economists face. Estimates of the IES (RRA) are often based on the constant relative risk aversion (CRRA) utility function. Although there is a consensus that the estimates of the IES (RRA) based on the CRRA utility function cannot adequately explain movements in U.S., the same statement cannot be made for Japanese data. For example, Hamori (1992, 1996) and Baba (2000) use Japanese monthly or quarterly consumption data, and conclude that estimates of the IES (RRA) using the Generalized Method of Moments (GMM) estimator perform well empirically, while Nakano and Saito (1998) show that using Japanese semiannual consumption data the estimate of the IES (RRA) is not significantly different from zero.

In the context of this previous empirical work on estimating the IES (RRA), the main purpose of this paper is to resolve those two opposite stances in Japan. In particular, there are two issues we consider in obtaining estimates of the IES (RRA) based on the CRRA utility function. First, we investigate whether or not the frequency of consumption data used affects estimates of the IES (RRA). This is because there is a possibility that consumption data of a different frequency may produce different empirical results (for example, Nakano and Saito (1998) and Baba (2000)). Thus, we use not only quarterly consumption data, but also semiannual consumption data when we estimate the IES (RRA).

Secondly, we take into account the fact that the IES (RRA) is often estimated using a two-step GMM estimator. However, it is widely known that the two-step GMM estimator have weak instrument problems and poor small sample property problems. As for those problems of the two-step GMM estimator, Yogo (2008) uses the Continuous Updating Estimator (CUE) proposed by Hansen, Heaton, and Yaron (1996) because Newey and Smith (2004) show that this estimator performs better in finite samples than the two-step GMM estimator. Thus, we employ not only the two-step GMM estimator, but also the CUE.

The estimates of the IES we obtain range from 0.2 to 0.5 when we use quarterly consumption data and employ the CUE. We find that the IES is weakly identified when we employ the two-step GMM estimator, while the CUE can identify the IES. The main reason for this differing result may be that the CUE utilize more information of the dataset than the two-step GMM estimator. Therefore, these results suggest that the estimates of the IES in Japanese earlier studies may be not efficient and we can improve on the estimates of the IES by using the CUE. Moreover, we obtain the same results as Nakano and Saito (1998) that the estimate of IES is not significantly different from zero regardless of the empirical method when we use semiannual consumption data. These results suggest that consumption data of different frequency leads to quite different estimates of the IES. However, we conclude that one plausible reason for these different results is that the larger sample size available with quarterly data improves the efficiency of estimator.

This paper is organized as follows. In section 2, we describe our model and empirical method. In section 3, we explain the data that our study uses. In section 4, we present our empirical results. In the final section, we conclude with some remarks.

2 Model and Empirical Method

In this section, we explain the model and empirical method used in this paper. The representative consumer at time 0 is assumed to choose his/her life-time consumption and asset holdings to maximize his/her expected utility subject to the budget constraint. The consumer's optimization problem is summarized as follows:

$$\text{Max } E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma} - 1}{1-\gamma} \right], \quad 0 < \beta < 1, \quad 0 < \gamma, \tag{1}$$

$$\text{s.t. } C_t + p_t A_t = [p_t + d_t] A_{t-1} + Y_t, \tag{2}$$

where C_t is real per capita consumption at time t, p_t is the price of the asset at time t, d_t is the dividend of the asset at time t, A_t is the amount of the per capita asset held at time t, Y_t is real per capita labor income at time t, β is the subjective time discount factor, γ is the relative risk aversion (RRA), and $E_t[\cdot]$ is the expectation operator conditional on the information available at time t. In equation (1), we assume that the utility function is of the CRRA class. In CRRA utility functions, the IES is the reciprocal of the coefficient of RRA.

By solving the above utility maximization problem, we can derive the following Euler equation:

$$E_t[\beta(\frac{C_{t+1}}{C_t})^{-\gamma}(1 + r_{t+1}) - 1] = 0, \tag{3}$$

where r_{t+1} is the real return of the asset at time $t+1$, which is defined as

$$r_{t+1} = \frac{p_{t+1} + d_{t+1}}{p_t} - 1. \tag{4}$$

In order to estimate the parameters in the Euler equation, we employ the GMM estimator proposed by Hansen (1982). Define the error term $\mathbf{u}_{t+1}(\theta)$ as

$$\mathbf{u}_{t+1}(\theta) = \beta(\frac{C_{t+1}}{C_t})^{-\gamma}(1 + r_{t+1}) - 1,$$

where $\theta = (\beta, \gamma)'$. Let $\mathbf{z}_t$ be an $R \times 1$ vector of instrumental variables known at time t, and define an $R \times 1$ vector $\mathbf{g}_t(\theta)$ as

$$\mathbf{g}_t(\theta) = \mathbf{u}_{t+1}(\theta)\mathbf{z}_t, \tag{5}$$

Then, the Euler equation implies

$$E[\mathbf{g}_t(\theta)] = \mathbf{0}, \tag{6}$$

where $E[\cdot]$ is the unconditional expectations operator. Moreover, if we define $\overline{\mathbf{g}_T(\theta)}$ as

$$\overline{\mathbf{g}_T(\theta)} := \frac{1}{T} \sum_{t=1}^{T} \mathbf{g}_t(\theta), \tag{7}$$

where T is the sample size, then the GMM estimator of θ, $\hat{\theta}_{GMM}$, minimizes the quadratic form:

$$\hat{\theta}_{GMM} = \arg\min_{\theta} \overline{\mathbf{g}_T(\theta)}' \mathbf{W}_T \overline{\mathbf{g}_T(\theta)}, \tag{8}$$

where $\mathbf{W}_T$ is an $R \times R$ weighting matrix, which is assumed to be positive definite for any finite T.

We can obtain the most efficient GMM estimator by choosing the weighting matrix $\mathbf{W}_T = \mathbf{S}^{-1}$, where $\mathbf{S}^{-1}$ is the inverse of the asymptotic covariance matrix of $T^{1/2}\overline{\mathbf{g}_T(\theta)}$. However, because we cannot observe the true value of θ, we cannot know $\mathbf{S}^{-1}$ either. Therefore, we need to adopt a two-step GMM. In order to estimate $\mathbf{S}$, we use the estimator proposed by Newey and West (1987). Following Newey and West (1994), for the lag selection criteria of the Newey-West estimator, we use $int(4(T/100)^{2/9})$, where $int(\cdot)$ is the integer part of the argument.

Furthermore, in order to analyze the goodness-of-fit of the model, we adopt Hansen's (1982) J test of overidentifying restrictions. Under the null hypothesis that equation (6) is true, T times the minimized value of equation (8) is asymptotically distributed as χ^2_{R-K}, where K is the number of parameters (that is, two in our case).

However, it is widely known that the two-step GMM estimator has poor small sample properties. For example, Newey and Smith (2004) show that the CUE performs better in finite samples than the standard two-step GMM estimator. Thus, we also employ the CUE in addition to the two-step GMM estimator when we estimate the IES. The CUE minimizes

$$\hat{\theta}_{CUE} = \arg\min_{\theta} \overline{\mathbf{g}_T(\theta)\prime}[\mathbf{S}(\theta)]^{-1}\overline{\mathbf{g}_T(\theta)}, \tag{9}$$

where $[\mathbf{S}(\theta)]^{-1} = \mathbf{W}_T$. For the CUE, the weighting matrix is continuously updated as θ changes in the minimization process. The CUE $\hat{\theta}_{CUE}$ has the same asymptotic distribution as the two-step GMM estimator.

3 Data

We construct three datasets labeled "Dataset 1" through "Dataset 3" in this paper that differ in their choices of consumption. In all datasets long-term government bond is treated as the asset in equation (2), and its total asset return (LGB) is obtained from Ibbotson Associates. To compute the inflation rate, we use the total consumption deflator published in the *Annual Report on National Accounts*. "Dataset 1", quarterly data is used, and for per capita consumption, we use "Total consumption (Benchmark year is 2000)" divided by population which is reported in the *Annual Report on National Accounts* in Japan. This per capita consumption data is seasonally adjusted using the X-12 ARIMA procedure. As instrumental variables, we use the lagged values of the real return on the asset, the consumption growth rate, and the growth rate of the deflator. The real asset return and real consumption series are both computed using the total consumption deflator. The sample period is from 1980Q2 to 2008Q4.

"Dataset 2" differs from "Dataset 1" only in the frequency of the data used. "Dataset 2" uses semiannual data which is also used in Nakano and Saito (1998) when they estimate the IES. "Dataset 3" differs from "Datasets 1" only in the measure of consumption used. "Dataset 3" uses quaterly data, but uses "Nondurable goods plus services" as consumption data.

For both the two-step GMM and CUE, all variables that appear in the moment conditions should be stationary. To check whether the variables satisfy stationarity, we use the Augmented Dickey and Fuller (1981) (ADF) test. Table 1 provides some descriptive statistics and the results of the ADF tests. For all the variables, the ADF test rejects the null hypothesis that the variable contains a unit root at conventional significance levels. Although not reported, the Phillips and Peron (1988) test leads to similar results.

(Table 1 around here)

4 Empirical Results

Table 2 presents the empirical results using "Dataset 1" through "Dataset 3". Table 2 shows that the CUE estimates of β and γ are statistically significant at conventional levels only when we use "Dataset 1" or "Dataset 3". The estimates of β range from 0.9895 to 0.9990, which are economically realistic; and the estimates of γ range from 1.9082 to 5.9981, implying estimates of the IES in the range 0.1667 to 0.5240. The p values for the J test are large enough that we cannot reject the null that the moment conditions hold. We have confirmed that the CUE estimation results are robust to changes of the initial starting values. When we employ the two-step GMM, the estimates of β range from 0.9870 to 0.9923, which are economically realistic. The estimates of γ are not signficantly different from zero regardless of the type of the dataset. In constrast to the CUE, we have confirmed that the two-step GMM estimation results are not robust to changes of these initial values. Then, we must explain the reason why the two-step GMM estimation does not work.

(Table 2 around here)

We can consider one plausible reason is that γ is weakly identified. To see this point, Figures 1 and 2 provide three dimensional plots of the two-step GMM and CUE objective functions (see, Hall (2005), for a similar example).

(Figures 1 and 2 around here)

Figure 1 suggests that the two-step GMM objective function be flat in the direction of γ. On the other hand, in Figure 2 we can not ascertain whether the CUE is flat in the direction of γ. Then, in Figures 3 and 4, two dimensional plots of the two-step GMM and CUE objective functions as β and γ are varied to investigate whether these parameters are weakly identified. We find that β is clearly identified regardless of the empirical method. Menanwhile, we also find that the two-step GMM objective function is flat in the direction of γ, but the CUE objective function is not.[1] This figure supports our view that γ is weakly identified when we employ the two-step GMM estimator, while the CUE can identify γ. These results appear to be similar to Stock and Wright (2000). They show that if even one parameter is weakly identified, all other parameters do not satisfy the consistency and the asymptotic normality on two-step GMM estimation. Moreover, the main reason for these differing results may be that the CUE utilize more information of the dataset than the two-step GMM estimator.[2] Therefore, this suggests that the estimates of the IES in earlier studies be not correct and we can improve the estimates of the IES using the CUE.

(Figures 3 and 4 around here)

[1] We also attempt to test whether this result is changed when we adapt the different instrument variables such as "per capita real investment growth rate". The two-step GMM objective function is still found to be flat in the direction of γ.

[2] Newey and Smith (2004) show that the CUE is one special case of the Generalized Empirical Likelihood (GEL) estimator proposed by them.

Table 2 also shows that the estimates of IES (RRA) are not significantly different from zero regardless of empirical method when we use "Dataset 2". These results are the same as Nakano and Saito (1998). This suggests that consumption data of different frequency leads to different estimates of the IES. One plausible reason for these differing results is that the use of quarterly data increases the sample size leading to more efficient estimates.

5 Concluding Remarks

This paper has estimated the IES for Japan. We have specified the long-term government bond as the asset that the representative consumer invests in, and estimated the Euler equation using the two-step GMM estimator and CUE. The estimates of the IES we obtain range from 0.2 to 0.5 when we use quarterly consumption data and the CUE. We find that the IES is weakly identified when we employ the two-step GMM estimator, while the CUE can identify the IES. Moreover, we also find that using consumption data of different frequencies leads to quite different estimates of the IES.

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Table 1: Descriptive Statistics and Unit Root Tests

D/S	Variable	Mean	SD	Min	Max	ADF	CV
	CG_t	1.0033	0.0107	0.9569	1.0321	-10.8501	
1	r_t	0.0153	0.0367	-0.1072	0.1157	-9.7831	-3.9900
	π_t	1.0018	0.0062	0.9916	1.0301	-7.1070	
	CG_t	1.0069	0.0142	0.9668	1.0392	-5.9480	
2	r_t	0.0129	0.0378	-0.1059	0.0768	-5.0237	-3.1500
	π_t	1.0038	0.0100	0.9878	1.0444	-3.2865	
	CG_t	1.0027	0.0093	0.9759	1.0293	-11.5397	
3	r_t	0.0142	0.0366	-0.1094	0.1132	-9.7101	-3.9900
	π_t	1.0029	0.0058	0.9907	1.0333	-7.4078	

"D/S" denotes the dataset, "CG_t" denotes the gross real per capita consumption growth, "r_t" denotes the real return on long-term government bonds, "π_t" denotes the inflation rate, "SD" denotes the standard deviation, and "ADF" denotes the Augmented Dicky-Fuller (ADF) test statistic. In computing the ADF test, we assume a model with a time trend and a constant. "CV" for "Dataset 1" and "Dataset 3" denote the critical values at the 1% significance level for the ADF test for each sample size. "CV" for "Dataset 2" also denotes the critical values at the 10% significance level for the ADF test for sample size. The null hypothesis that each variable has a unit root is rejected at the 1% or 10% significance level, respectively.

Table 2: Empirical Results of two-step GMM and CUE

E/M	D/S	*Lag*	$\hat{\beta}$	$SE(\hat{\beta})$	$\hat{\gamma}$	$SE(\hat{\gamma})$	p-value	*DF*	*N*
		1	0.9870	0.0043	0.8046	0.9886	0.1293	2	114
	1	2	0.9923	0.0037	2.4930	0.7290	0.3856	5	113
		3	0.9908	0.0034	2.0854	0.6475	0.5440	8	112
		1	0.9837	0.0063	-0.5468	0.7467	0.8471	2	56
GMM	2	2	0.9834	0.0051	-0.7591	0.6910	0.9917	5	55
		3	0.9846	0.0032	-0.3710	0.3132	0.4542	8	54
		1	0.9908	0.0050	1.8440	1.4922	0.2809	2	114
	3	2	0.9920	0.0034	2.1630	1.0020	0.1380	5	113
		3	0.9917	0.0033	2.1049	0.8279	0.3543	8	112
		1	0.9895	0.0043	1.9082	0.9284	0.1158	2	114
	1	2	0.9928	0.0039	2.8379	0.8056	0.3941	5	113
		3	0.9931	0.0041	3.0717	0.8407	0.5457	8	112
		1	0.9837	0.0063	-0.5585	0.7457	0.8476	2	56
CUE	2	2	0.9837	0.0052	-0.7218	0.6934	0.9895	5	55
		3	0.9887	0.0041	0.3822	0.3812	0.5203	8	54
		1	0.9967	0.0049	4.5588	1.7642	0.1450	2	114
	3	2	0.9990	0.0051	5.8717	2.0190	0.2765	5	113
		3	0.9990	0.0051	5.9981	1.8190	0.5417	8	112

"E/M" denotes the empirical method, "D/S" denotes the dataset used, "*Lag*" denotes the number of lags of the instruments used, "$\hat{\beta}$" denotes the estimate of the subjective discount rate, "$\hat{\gamma}$" denotes the estimate of the relative risk aversion (RRA), "$SE(\cdot)$" denotes the Newey-West adjusted standard error of "$\hat{\beta}$" or "$\hat{\gamma}$", respectively, "p-value" denotes the p-value for Hansen's (1982) J test statistics, "*DF*" denotes the degrees of freedom for the J test, and "*N*" denotes the sample size. To compute the two-step GMM and CUE estimates, R version 2.10.1 was used. In our study, we assume that utility function is of the CRRA class. Therefore, the intertemporal elasticity of substitution (IES) is the reciprocal of the coefficient of relative risk aversion (RRA). The starting values of the parameters set equal to $\beta = 1$, $\gamma = 1$.

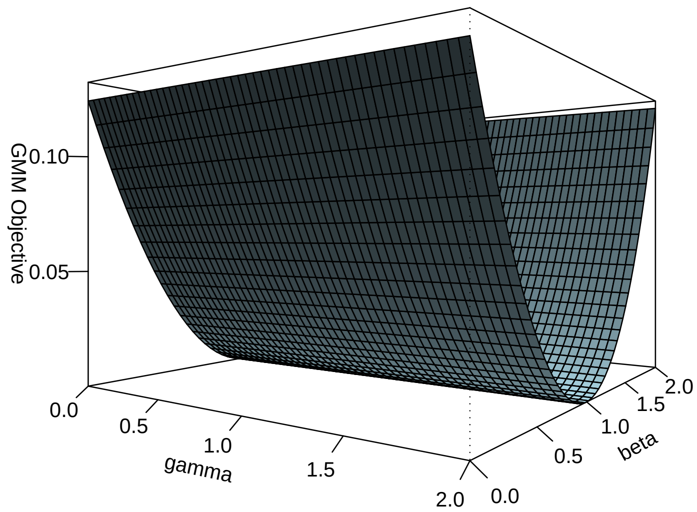

Figure 1: GMM Objective Function for Dataset1 with Lag=1

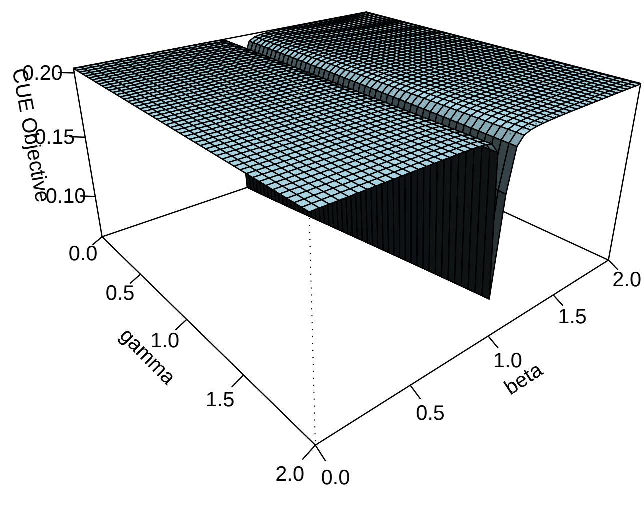

Figure 2: CUE Objective Function for Dataset1 with Lag=1

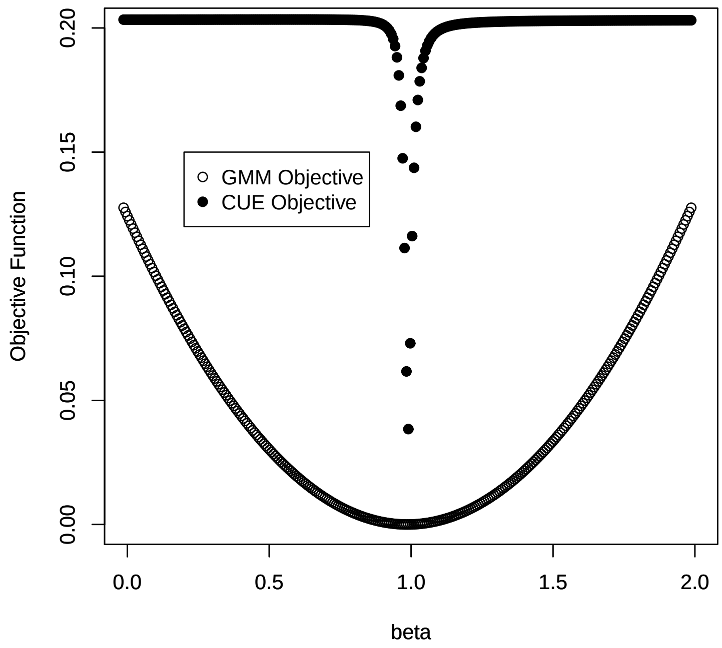

Figure 3: Objective Function for Dataset1 with Lag=1 (γ is fixed)

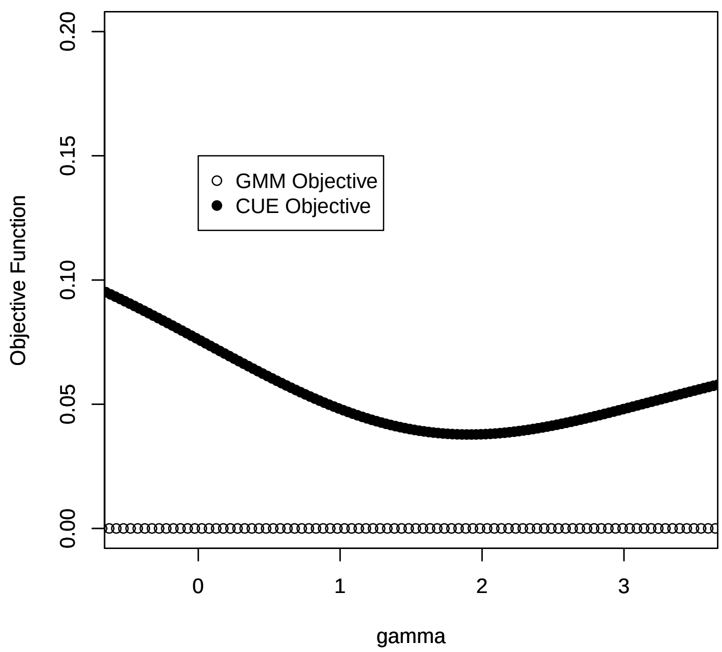

Figure 4: Objective Function for Dataset1 with Lag=1 (β is fixed)