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A stochastic model of the provision of guided tours to tourists

Amitrajeet A Batabyal
Department of Economics, Rochester Institute of Technology

Hamid Beladi
Department of Economics, University of Texas at San Antonio

Abstract

In this paper, we analyze a discrete-time Markov chain theoretic model of the provision of guided tours to tourists by a private firm. Specifically, we first determine the equilibrium probability distribution of the number of tourists in the guided tour providing firm's waiting room just before this firm begins to match tour guides with tourists. Second, we compute the long run expected number of tourists in this firm's waiting room. Finally, we ascertain the long run expected delay per tourist in the firm's waiting room.

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1. Introduction

Researchers studying the economics of tourism now recognize that a salient decision problem confronting a firm that is in the business of providing guided tours to tourists is a *scheduling* problem. The scheduling problem we have in mind concerns the frequency with which guided tours to a particular tourist attraction ought to be provided. Relative to the peak season, the magnitude of the stochastic demand that faces a guided tour providing firm is typically much lower. Even so, it is important to comprehend that this basic scheduling problem exists in both the off-peak and in the peak tourist seasons.

Researchers have now begun to *empirically* study seasonality in the context of tourism and the provision of guided tours to tourists. Focusing on Australia, Lim and McAleer (2000) have studied deterministic and probabilistic seasonality and the extent to which these two kinds of seasonality explain variations in the international tourist arrival time series. Koenig and Bischoff (2003) have compared the seasonal nature of tourism in Wales and Scotland and have discussed alternate ways of addressing this seasonality problem. Andriotis (2005) has studied the extent to which the diversification of the product mix, a change of the customer mix, and assertive pricing can reduce problems arising from the seasonality in the demand for tourism in Crete. Sung (2008) has shown that the temporary plan for snowmobile riders and guided tour providers in Yellowstone National Park has increased the producer surplus of these guided tour providers. Finally, Baez Montenegro *et al.* (2009) have used a hypothetical guided walking tour construct to estimate the value of historical sites in Valdivia, Chile.

The studies discussed in the previous paragraph have certainly increased our understanding of the many empirical features of both seasonal tourism and the provision of guided tours to tourists. Even so, with the exception of three recent papers that have analyzed the provision of guided tours to tourists but specifically during the *off-peak* season, the existing literature has paid virtually no attention to the many *theoretical* aspects of the provision of guided tours to tourists. Using an exogenously given decision rule, Batabyal (2009) has derived certain long run metrics that are germane in the context of the provision of transport to tourists. Batabyal and Yoo (2009) have conducted a probabilistic analysis of the provision of guided tours to a single class of tourists. Finally, Batabyal (2010) has extended this Batabyal and Yoo (2009) analysis by examining the guided tour provision question when the firm providing the guided tours is faced with two distinct classes of tourists.

Our objective in this paper is to generalize the analysis in these three previous papers by constructing and analyzing a discrete-time Markov chain theoretic model of the provision of guided tours to tourists that is relevant in both the off-peak and in the peak seasons. In this regard, we first determine the equilibrium or limiting probability distribution of the number of tourists in a guided tour providing firm’s waiting room just before this firm begins to match tour guides with tourists. Second, we compute the long run expected number of tourists in this firm’s waiting room. Finally, we ascertain the long run expected delay per tourist in the firm’s waiting room.

The rest of this paper is organized as follows. Section 2.1 describes a stochastic model that captures, from the perspective of a private firm, the general features of off-peak and peak season guided tours to city attractions and to scenic locations such as fiords and lakes. Section 2.2 derives the limiting probability distribution mentioned in the previous paragraph. Section 2.3 calculates the previous paragraph's long run average number of tourists. Section 2.4 computes the long run mean delay per tourist discussed in the preceding paragraph. Section 3 concludes and then discusses ways in which the research described in this paper might be extended.

2. The Theoretical Framework

2.1. Preliminaries

Consider a private firm that provides guided tours to tourists interested in visiting a particular location during either the off-peak season or the peak season. For concreteness, we shall think of this location as the Taj Mahal in Agra, India, but, without loss of generality, our analysis holds for other locations as well.

Tourists arrive at this private firm's facility in accordance with a stationary Poisson process with rate $\lambda > 0$.[1] Arriving tourists are initially seated in a waiting room before they are matched with tour guides. This waiting room is inspected every T time periods and only at these specific inspection epochs are waiting tourists matched with available tour guides. Our guided tour providing firm has g tour guides on its payroll.[2]

This firm seeks to provide its customers (the tourists) with a personalized and high quality guided tour experience. To this end, the firm under study ensures that each tour guide is matched with a single tourist. The times taken to complete the individualized guided tours are assumed to be independent *random* variables. However, because the location—such as the Taj Mahal—for which the guided tours are sought is the same for all the arriving tourists, we suppose that these independent random variables have a common exponential distribution function with mean $1/\mu$. With this description of the basic setup in place, we are now in a position to use discrete-time Markov chain analysis to determine the equilibrium probability distribution of the number of tourists in our guided tour providing firm's waiting room just before this firm begins to match tour guides with tourists. By "discrete-time Markov chain analysis" we mean that our analysis of the research questions delineated in the penultimate paragraph of section 1 will involve the utilization of the theory of discrete-time Markov chains as discussed in standard texts such as Ross (2003) and Tijms (2003). Full details about the ways in which we use this theory in the context of our problem are provided in sections 2.2-2.4 below.

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[1] The Poisson process has been used previously in the literature—see Martin-Cejas (2006), Batabyal (2007), and Batabyal and Beladi (2008)—to model and analyze problems in the economics of tourism. See Ross (2003, pp. 288-348) or Tijms (2003, pp. 1-32) for textbook treatments of the Poisson process.

[2] The models employed by Batabyal and Yoo (2009) and Batabyal (2010) are special cases of the model in this paper in the sense that $g=1$ in these previous two papers.

2.2. The equilibrium probability distribution

We begin by letting X_n denote the number of tourists in our guided tour providing firm's facility at the *nth* inspection epoch. Recall that this firm's waiting room is inspected every T time periods and only at these specific inspection epochs are waiting tourists matched with available tour guides. Therefore, the *nth* inspection epoch is the *nth* time period at which an inspection is conducted. Because the Poisson process and the exponential distribution possess the memoryless property,[3] it follows that $\{X_n\}$ is a discrete-time Markov chain with infinite state space $I=\{0,1,2,...\}$. Let

$$a_k(i)=\binom{i_g}{k}(1-e^{-\mu T})^k e^{-\mu T(i_g-k)},\ k=0,1,...,i_g, \tag{1}$$

denote the probability that k guided tours have been completed during a given time slot that begins with i tourists present in our firm's facility. Here, $i_g=\min(i,\ g)$. With this specification in place, we deduce that the one-step transition probabilities of the discrete time Markov chain $\{X_n\}$ under study are given by

$$p_{ij}=\sum_{k=0}^{i_g} a_k(i)e^{-\lambda T}\frac{(\lambda T)^{j-i+k}}{(j-i+k)!},\ i\geq 0,\ j\geq i-i_g. \tag{2}$$

Denote the equilibrium or limiting probabilities of interest by π_j. Then, we want to show that the infinite system of linear equations for the limiting probabilities, i.e., the $\pi_j^{/}s$, can be reduced to a finite system of linear equations using the geometric tail method described in Tijms (2003, pp. 111-116). To see this clearly, let us write the pertinent equilibrium linear equations as

$$\pi_j=\sum_{i=0}^{g-1}\pi_i p_{ij}+\sum_{i=g}^{j+g}\pi_i p_{ij},\ j=0,1,... \tag{3}$$

Let z be a real-valued variable with $|z|\leq 1$. Then, multiplying both sides of (3) with z^j and summing over j, we find, after interchanging the order of summation, that

$$\sum_{j=0}^{\infty}\pi_j z^j=N(z)+\sum_{i=g}^{\infty}\pi_i z^{i-g}\sum_{j=i-g}^{\infty}p_{ij}z^{j-i+g}, \tag{4}$$

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For the stationary Poisson process, the times between successive arrivals is exponentially distributed and the exponential distribution has the memoryless property. What this means is the following. Let *Y* be an exponentially distributed random variable that represents the lifetime of a certain item. Then, if the residual life of this item has the same exponential distribution as the original lifetime, regardless of how long this item has already been in use then *Y* has the memoryless property. See Ross (2003, pp. 272-279) or Tijms (2003, pp. 2-5) for textbook treatments of the exponential distribution's memoryless property.

where $N(z) = \Sigma_{j=0}^{\infty} z^j \Sigma_{i=0}^{g-1} \pi_i p_{ij}$.

Now, if we use the form—see (2) above—for the p_{ij} for $i \geq g$, then tedious but straightforward algebraic computations show that

$$\sum_{j=i-g}^{\infty} p_{ij} z^{j-i+g} = \sum_{k=0}^{g} \binom{g}{k} (1-e^{\mu T})^k e^{-\mu T(g-k)} B_{i,k}(z), \tag{5}$$

where

$$B_{i,k}(z) = e^{-\lambda T} (\lambda T)^{k-g} e^{\lambda T z}. \tag{6}$$

To proceed further, it will be necessary to use Newton's generalized binomial theorem[4] or what is sometimes also known as Newton's binomium. Using this theorem, the left-hand-side (LHS) of (5) can be written as

$$\sum_{j=i-g}^{\infty} p_{ij} z^{j-i+g} = e^{-\lambda T(1-z)} [1 - e^{-\mu T} + \frac{e^{-\mu T}}{\lambda T}]^g. \tag{7}$$

Using the generating function of the equilibrium probabilities, i.e., the $\pi_j^{/}s$, (7) can also be written as

$$\Pi(z) = \frac{N(z)}{1 - z^{-g} e^{-\lambda(1-z)} [1 - e^{-\mu T} + e^{-\mu T}/\lambda T]^g}, \tag{8}$$

where $N(z)$ is as described in the text immediately after (4).

Let τ be the smallest root of the denominator of the $\Pi(z)$ function on the interval $(0,\infty)$. Then, because the conditions necessary for theorem C.1 in Tijms (2003, p. 453) to hold are satisfied, we can apply this theorem to our equilibrium probability determination problem and conclude that the equilibrium probability distribution of the number of tourists in our firm's waiting room just before this firm begins to match tour guides with tourists exists and that these equilibrium probabilities exhibit the so called geometric tail behavior. Mathematically, this means that these equilibrium probabilities have the form

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See Rudin (1976, p. 201) for more on this theorem.

$$\pi_j \sim \gamma \tau a^{-j} \ as \ j \to \infty \tag{9}$$

for some constant $\gamma>0$. In this context, the reader should note that for two functions of the variable x, $f(x)$ and $h(x)$, $f(x) \sim h(x)$ as $x \to \infty$ means that $\lim_{\{x \to \infty\}} f(x)/h(x) = 1$.

What we have just shown is that the relevant state probabilities of the discrete-time Markov chain $\{X_n\}$ that we are studying exhibit "geometric tail behavior." Therefore, we do not have to work with an infinite system of linear equations—recall that the state space of $\{X_n\}$ is infinite—but, instead, we can focus on a finite set of linear equilibrium equations. It should be clear to the reader that the equilibrium or limiting probabilities described in (9) can be used by our guided tour providing firm for planning purposes. In particular, these probabilities can be used to determine whether the inspection time variable T ought to be *altered* and also to ascertain whether our firm ought to *change* the number of tour guides it has on its payroll. We now proceed to the second task of this paper and that is to compute the long run expected number of tourists in our guided tour providing firm's waiting room.

2.3. Long run expected number of tourists

Let us denote the long run expectation we seek by L_t. Now, suppose that a cost at rate k is incurred by our private firm when there are k tourists in the waiting room for $k=0,1,2,...$ Then, the important point to note is that the expectation L_t is given by the long run *average cost* per unit time that is incurred by this firm. Recall that the tourist arrivals in the model of this paper occur in accordance with a stationary Poisson process with rate $\lambda>0$. Therefore, to compute the above mentioned long run average cost, we can make use theorem 1.1.3 in Tijms (2003, p. 6). Applying this theorem to our problem, we reason that the average cost incurred in a time slot given that j tourists are waiting at the beginning of this time slot is equal to

$$(j - j_g T) + \frac{1}{2}\lambda T^2. \tag{10}$$

Now, to compute the long run average cost per time slot, we shall use theorem 3.3.3 in Tijms (2003, p. 103). The application of this ergodic theorem to our problem tells us that the average cost we seek is given by

$$\sum_{j=0}^{\infty} [(j - j_g)T + \frac{1}{2}\lambda T^2]\pi_j = \sum_{j=g}^{\infty} (j-g)T\pi_j + \frac{1}{2}\lambda T^2. \tag{11}$$

Dividing the right-hand-side (RHS) of (11) by T gives us an expression for the long run average cost per unit time and, as discussed in the first paragraph of this section, this expression is also equal to the long run expected number of tourists in our guided tour providing firm's waiting room or L_t. Performing the division, we get

$$L_t = \sum_{j=g}^{\infty} (j-g)\pi_j + \frac{1}{2}\lambda T. \quad (12)$$

Inspecting (12), it is clear that the long run expected number of tourists in our firm's waiting room is an *increasing* function of the Poisson tourist arrival rate $\lambda > 0$ and the inspection time variable T. It is unlikely that the private firm under study will be able to control the rate $\lambda > 0$ at which tourists arrive at its facility. This tells us that if our guided tour providing firm would like to reduce crowding in its waiting room then it will want to *reduce* T or, equivalently, *increase* the frequency with which it inspects the waiting room before matching tour guides with tourists. We now proceed to the last task of this paper and this involves ascertaining the long run expected delay per tourist in our private firm's waiting room.

2.4. Long run expected delay per tourist

Let W_t denote the long run expectation we seek. To obtain this expectation, we shall make use of a well known result in queuing theory known as Little's formula.[5] Applied to our problem, this formula tells us that W_t is given by the long run expected number of tourists in our guided tour providing firm's waiting room or L_t divided by the Poisson tourist arrival rate $\lambda > 0$. Therefore, dividing the RHS of (12) by $\lambda > 0$ tells us that the long run expected delay per tourist in our private firm's waiting room is equal to

$$W_t = \frac{\sum_{j=g}^{\infty} (j-g)\pi_j}{\lambda} + \frac{1}{2}T. \quad (13)$$

Looking over (13), we see that *unlike* the expectation for L_t given in (12), the long run expected delay per tourist in our firm's waiting room is a *decreasing* function of the Poisson tourist arrival rate $\lambda > 0$. In contrast and *like* the expectation for L_t in (12), W_t is an *increasing* function of the inspection time variable T. Therefore, as in the case of the analysis presented in section 2.3, if the guided tour providing firm would like to reduce the sightseeing delay encountered by the waiting tourists then it ought to *lessen* T or *raise* the frequency with which it inspects the waiting room before matching tour guides with tourists.

3. Conclusions

In this paper, we analyzed a discrete-time Markov chain theoretic model of the provision of guided tours to tourists by a private firm. Specifically, we first determined the equilibrium probability distribution of the number of tourists in a guided tour providing firm's waiting room just before this firm began the task of matching tour guides with tourists. Then, we computed the long run expected number of tourists in the waiting room of the private firm under study. Finally, we ascertained the long run expected delay per tourist in this same firm's waiting room.

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[5] See Ross (2003, pp. 476-478) or Tijms (2003, pp. 50-52) for textbook treatments of Little's formula.

The analysis in this paper can be extended in a number of different directions. We now make two suggestions for extending the research described here. First, it would be useful to determine the extent to which one can obtain analytic results for a discrete-time Markov chain theoretic model of the provision of guided tours to tourists who arrive at the pertinent firm's facility in accordance with a non-stationary Poisson process with a time dependent intensity function, say, $\lambda(t)$. Second, following the discussion towards the end of sections 2.3 and 2.4, it would be informative to formulate and solve an optimization problem for a guided tour providing firm in which the decision variable T is chosen to optimize a criterion function of which the RHSs of (12) or (13) are a part. Studies of the provision of guided tours to tourists that incorporate these aspects of the problem into the analysis will provide further insights into questions in the economics of tourism that have both theoretical and practical ramifications.

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