How productive is optimism? the Impact of ambiguity on the "big push"

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Abstract

The paper finds that sufficient ambiguity leads to the uniqueness of equilibrium in macroeconomic coordination games. The results have a Keynesian flavour: sufficient optimism gives rise to a Pareto-optimal equilibrium; and sufficient pessimism results in a Pareto-inferior equilibrium. This analysis is applied to a "Big Push" model from the economic growth literature.

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1 Introduction

Keynes (1937) argued that uncertainty plays a vital role in macroeconomics. Investment is subject to waves of expectations driven by optimism and pessimism. Sudden shifts in psychological forces can produce booms and slumps. The description of the 2001 financial crisis in Argentina in DeLong (2005), could serve as a modern illustration:

“When it [the currency board] collapsed, Argentina’s consolidated debt-to-GDP ratio was about 50%. That is not an unsustainable debt load. And the Argentinian government was managing to run a primary surplus. If there had been confidence in Argentina’s fiscal future – confidence that no financial crisis was on the horizon– then interest rates would have been much lower, and the primary surplus would have generated only a moderate general deficit. With low interest rates, Argentina’s prospects for growth would have been relatively good. With good growth prospects and a relatively moderate overall government budget deficit, there would be no reason to fear that fiscal policy is unsustainable. Only the fact that a crisis was expected pushed interest rates up to the level where investment was strangled, growth impossible, the overall budget deficit large, and a crisis inevitable”.

We propose one way to model Keynes’ ideas by studying the influence of ambiguity on macroeconomics. It is plausible that ambiguity would affect macroeconomic coordination since it is intrinsically difficult to assign a probability of success when success requires simultaneous investment across many sectors. Consider, for instance, how many investors made incorrect predictions about the dotcom boom of the 1990’s.

The “Big Push” concerns the development of an economy from a low activity state to
a higher level equilibrium.\footnote{See Rosenstein-Rodan (1943) and Murphy, Shleifer & Vishny (1989)} Threshold effects and non-linearity imply that simultaneous industrialisation of many sectors can be self-sustaining even if no sector can profitably invest on its own.

Our model is a macroeconomic coordination game with strategic complementarity. In other words an increase in one player’s strategy will give others an incentive to increase their strategy as well, see Cooper & John (1988). Strategic complementarities can give rise to Pareto-ranked multiple equilibria. In the “Big Push” model, there is one equilibrium where no firm industrialises and another where all industrialise. We find that sufficient ambiguity will lead to a unique equilibrium. Optimism can lead an economy to a Pareto-superior equilibrium, while pessimism causes it to become “stuck” in an inefficient state.

Ambiguity is defined as situations where beliefs cannot be quantified by subjective probabilities without doubt. In the “Big Push” model, whether or not industrialisation is profitable depends on whether enough other firms industrialise. Firms have beliefs about the likelihood of industrialisation but may not feel confident in them. This can be caused by either limited access to information or because there is no reliable information (Frisch & Baron (1988)). The level of confidence may cause a firm to either increase or decrease their belief in success therefore influencing their investment behaviour\footnote{Empirical evidence suggests that individuals can be either ambiguity averse or ambiguity loving. (see Ellsberg (1961), Heath & Tversky (1991) ).}.

\section{Ambiguity}

Here, we explain our model of ambiguity in games. Consider a symmetric game $\Gamma = \langle I, (S_i, u_i)_{i \in I} \rangle$ with two players $I = \{1, 2\}$, where player $i$ has a closed, bounded and convex strategy set $S_i \subseteq \mathbb{R}$. Both players have the same payoff function $u_i(s_i, s_{-i})$.\footnote{For consideration of asymmetric equilibrium under ambiguity, see Eichberger & Kelsey (2000).}
Players are assumed to follow the axioms of Chateauneuf, Eichberger & Grant (2007), (henceforth CEG), which is a special case of Choquet Expected Utility (CEU), see Schmeidler (1989). These imply that the beliefs of player $i$ may be represented by a neo-additive capacity on $S_{-i}$.

**Definition 2.1** A neo-additive capacity, is a function $\nu : \mathcal{P}(S_{-i}) \rightarrow \mathbb{R}$ defined by,

$\nu(\emptyset) = 0, \nu(S_{-i}) = 1$ and $\nu(E) = \delta \alpha + (1 - \delta) \pi(E)$ for $\emptyset \subseteq E \subset S_{-i}$, where $0 \leq \delta \leq 1, 0 \leq \alpha \leq 1$ and $\pi$ is an additive probability on $S_{-i}$.

CEG represent preferences as a weighted average of the minimum, the maximum and the mean of $u_i$.

**Definition 2.2** The Choquet expected value of $u_i(s_i, s_{-i})$ with respect to a neo-additive capacity $v$ is defined as: $V_i(s_i) = \int u_i(s_i, s_{-i}) \, dv(s_{-i}) = \delta (1 - \alpha) \cdot \underline{u}_i(s_i, s_{-i}) + \alpha \cdot \overline{u}_i(s_i, s_{-i}) + (1 - \delta) E_{\pi} u_i(s_i, s_{-i})$, where $\underline{u}_i(s_i, s_{-i}) = \min_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$, $\overline{u}_i(a(s)) = \max_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$, and $E_{\pi} u_i(s_i, s_{-i})$ denotes the expected value of utility with respect to $\pi$.

These preferences may be interpreted as representing a situation where agents have beliefs represented by probabilities $\pi$. However they may have some doubts about these beliefs represented by $\delta$. Agents respond to ambiguity in part with optimism (represented by $\alpha$) by over-weighting the best outcome and in part with pessimism (represented by $1 - \alpha$).

We define the support of a neo-additive capacity to be equal to the support of the additive belief on which it is based.\(^4\)

**Definition 2.3** The support, $\text{supp} \, v$, of neo-additive capacity $v = \delta \alpha + (1 - \delta) \pi$ is defined by $\text{supp} \, v = \text{supp} \, \pi$.

\(^4\)For a justification of this support notion see Eichberger & Kelsey (2002).
We use an equilibrium concept which takes $\alpha$ and $\delta$ as exogenous.\(^5\)

**Definition 2.4** An equilibrium under ambiguity (EUA) is a belief system $(v_i^*, ..., v_I^*)$ where $v_i^*$ is a neo-additive capacity on $S_{-i}$, if for all $i \in I$ supp $v_i^* \subseteq \times_{j \neq i} R_j \left( v_j^* \right)$, where $R_i \left( v_i \right) = \left\{ \hat{s}_i \in S_i : \int u_i \left( \hat{s}_i, s_{-i} \right) dv_i \left( s_{-i} \right) \geq \int u_i \left( s_i, s_{-i} \right) dv_i \left( s_{-i} \right), \forall s_i \in S_i \right\}$ denotes the best response of player $i$ given his/her beliefs $v_i$. An equilibrium is pure if supp $v_i^*$ contains a single strategy profile, otherwise we say that it is mixed.

We confine our attention to symmetric pure equilibria. A standard Nash equilibrium can be regarded as a special case where there is no ambiguity i.e. $\delta = 0$.

### 3 The Big Push

We consider a “Big Push” model with one time period and a continuum of production sectors, indexed by $i \in [0, 1]$.\(^6\) There are 2 types of firms in each sector: a competitive fringe with constant returns to scale converting one unit of input into one unit of output and a monopolist able to access two technologies, a low and a high productivity technology with marginal costs $\delta_h$ and $\delta_l$ respectively, where $\delta_h > \delta_l > 1$. The low technology has no fixed costs and represents ‘non-industrialisation’. In contrast, the high technology represents ‘industrialisation’ and incurs a fixed cost, $I$, which is assumed to be the expenditure on intermediate goods. There is a representative consumer who owns all claims to labour income, profits, and sales of the intermediate goods. (S)he is a price taker in consumption and in the competitive labour market and supplies inelastically $N$ units of labour. Labour demand in each sector is denoted by $n_i$. Taking the wage rate as the numeraire, the total wage bill is $\int_0^1 n_i = N$. Labour is the only variable cost of

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\(^5\)For further discussion of the equilibrium concept see Dow & Werlang (1994) and Eichberger & Kelsey (2000).

\(^6\)The model is adapted from Shleifer & Vishny (1988) and Fatas & Metrick (1997)
production.

Production in sector \( i \) is denoted by \( q_i \) and the price of output by \( p_i \). Expenditure is \( \mu I \) when a fraction \( \mu \) of firms adopt the high technology. Denoting production profit as \( \Pi \), total income (expenditure) can be written as \( Y = N + \Pi + \mu I \). Assuming utility is Cobb-Douglas, \( U = \exp \left[ \int_0^1 \ln q_i \, di \right] \), and the budget constraint is \( \int_0^1 p_i q_i \, di = Y \), (s)he will spend equal amounts on each commodity. Thus revenue \( p_i q_i \) is constant and equal to \( Y \) in each market.

Profit is maximized at \( p_i = 1 \), where \( q_i = Y \). The maximum profits of firms adopting high and low technology are respectively, \( \pi^h_i = a_h Y \) and \( \pi^l_i = a_h Y - I \), where \( 0 < a_h := \frac{\delta_h - 1}{\delta_h} \) and \( 0 < a_l := \frac{\delta_l - 1}{\delta_l} < 1 \).

If a fraction \( \mu \) of firms industrialise, we have \( Y = \frac{N}{1-\mu a_h -(1-\mu)a_l} \), and firm \( i \) will industrialise if,

\[
\pi^h_i - \pi^l_i = (a_h - a_l) \cdot \frac{N}{1-\mu a_h -(1-\mu)a_l} - I > 0. \tag{1}
\]

Equation (1) shows that strategic complementarity multiple equilibria are present. Formally, for a fixed cost of industrialisation \( I \), and \( (a_h - a_l) \cdot \frac{N}{1-a_l} |_{\mu=0} < I < (a_h - a_l) \cdot \frac{N}{1-a_h} |_{\mu=1} \), we have at least two Pareto-ranked equilibria.\(^7\)

### 3.1 The Role of Ambiguity

If firms view the decisions of others as ambiguous, they will perceive their own demand curve to be ambiguous. Demand is highest (resp. lowest) when all (resp. no) firms are industrialised, that is, \( Y^h = \frac{N}{1-a_h} \) and \( \mu = 1 \), (resp. \( Y^l = \frac{N}{1-a_l} \) and \( \mu = 0 \)). Therefore, the

\(^7\)The following two inequalities hold at the same time: \( \pi^h_i (1) - \pi^l_i (1) = (a_h - a_l) \cdot \frac{N}{1-a_h} |_{\mu=1} - I > 0 \) and \( \pi^h_i (0) - \pi^l_i (0) = (a_h - a_l) \cdot \frac{N}{1-a_l} |_{\mu=0} - I < 0 \). Meanwhile, \( \pi^h_i (1) - \pi^l_i (0) = \frac{a_h N}{1-a_h} |_{\mu=1} - I - \frac{a_l N}{1-a_l} |_{\mu=0} = (a_h - a_l) \cdot \frac{N}{1-a_h} |_{\mu=1} - I > 0 \) and \( \pi^h_i (0) - \pi^l_i (1) = \frac{a_h N}{1-a_h} |_{\mu=0} - I - \frac{a_l N}{1-a_l} |_{\mu=1} = (a_h - a_l) \cdot \frac{N}{1-a_h} |_{\mu=0} - I < 0 \).
(Choquet) expected value of demand faced by a firm is

\[ V(Y) = \delta (1-\alpha) \cdot \frac{N}{1-a_l} + \delta \alpha \cdot \frac{N}{1-a_h} + (1 - \delta) \frac{N}{1-\mu a_h - (1-\mu)a_l}. \]  

(2)

The firm will industrialise if:

\[ \pi^h_i - \pi^l_i = (a_h - a_l) \cdot \left[ \delta (1-\alpha) \frac{N}{1-a_l} + \delta \alpha \frac{N}{1-a_h} + (1 - \delta) \frac{N}{1-\mu a_h - (1-\mu)a_l} \right] - I > 0. \]  

(3)

Next we show that sufficient ambiguity will generically lead to an unique equilibrium.\(^8\)

**Proposition 3.1** If there is sufficient ambiguity, i.e., \( \delta \) is close to 1, there exists \( \bar{\varepsilon} \), such that if firms are sufficiently pessimistic, i.e., \( 1 - \alpha > \bar{\varepsilon} \), (resp. optimistic i.e., \( \alpha > 1 - \bar{\varepsilon} \)), then non-industrialisation (resp. industrialisation) will be the unique equilibrium.

**Proof.** Suppose, \( \delta \to 1 \), then equation (3) becomes,

\[ \pi^h_i - \pi^l_i = \left[ (a_h - a_l) \cdot \frac{N}{1-a_l} - I \right] + \alpha (a_h - a_l) \left[ \frac{N}{1-a_h} - \frac{N}{1-a_l} \right]. \]

1. If non-industrialisation is the unique best response for firm \( i \), \( \pi^h_i - \pi^l_i < 0 \). This happens when, \( 1 - \alpha > \bar{\varepsilon} = \frac{(a_h - a_l) \cdot \frac{N}{1-a_l} - I}{(a_h - a_l) \cdot \frac{N}{1-a_h} - \frac{N}{1-a_l}} \).

2. Suppose firm \( i \) is optimistic, the equilibrium is industrialisation and \( \pi^h_i - \pi^l_i > 0 \). This happens when \( \alpha > 1 - \bar{\varepsilon} = \frac{I - (a_h - a_l) \cdot \frac{N}{1-a_l}}{(a_h - a_l) \cdot \frac{N}{1-a_h} - \frac{N}{1-a_l}} \).

Given sufficient ambiguity, beliefs have the power to determine the equilibrium outcomes. Pessimism can disarm well intended policy efforts, make ‘good’ look like ‘bad’ and ‘bad’ seem ‘worse’. Hence the economy is trapped in a low level equilibrium. On the other hand, optimism can amplify the effects of policies, create and maintain a favourable envi-

\(^8\)By generically we mean except when \( \alpha = \frac{I - (a_h - a_l) \cdot \frac{N}{1-a_l}}{(a_h - a_l) \cdot \frac{N}{1-a_h} - \frac{N}{1-a_l}} \).
Figure 1: The curve $\Delta\pi$ depicts the increased profit from industrialisation corresponding to different values of $\mu$. Optimism shifts the curve $\Delta\pi$ upward to $\Delta\pi^\alpha$, the perceived scale of industrialisation now is $\mu_2$ instead of $\mu$, which indicates the positive value of $\Delta\pi$ in the case of industrialisation; when pessimism prevails, the curve $\Delta\pi$ shifts downward to $\Delta\pi^{1-\alpha}$, the perceived scale of industrialisation is $\mu_1$ which indicates the negative value of $\Delta\pi$.

Environment which further raises optimism. The economy takes off to reach a self-sustained “economics of euphoria”.

We may rewrite the decision rule,

$$\pi_i^h - \pi_i^l = (a_h - a_l) \left[ \delta (1 - \alpha) \left( \frac{N}{1 - a_l} - \frac{N}{1 - \mu a_h - (1 - \mu) a_l} \right) \right] + (a_h - a_l) \left[ \delta \alpha \left( \frac{N}{1 - a_h} - \frac{N}{1 - \mu a_h - (1 - \mu) a_l} \right) + \frac{N}{1 - \mu a_h - (1 - \mu) a_l} \right] - I. \ (4)$$

Now suppose $\mu$ is at the marginal level $\hat{\mu}$ which makes $\pi_i^h - \pi_i^l = (a_h - a_l) \frac{N}{1 - \mu a_h - (1 - \mu) a_l} - I = 0$. If firms become more pessimistic and for simplicity we assume that $\alpha < \frac{1}{2}$, the negative term dominates the positive one and we have $(\pi_i^h - \pi_i^l) |_{\mu = \hat{\mu}} < 0$. Thus, firm $i$ does not industrialise. Conversely, if $\alpha > 1/2$, we have $(\pi_i^h - \pi_i^l) |_{\mu = \hat{\mu}} > 0$, which implies that firm $i$ industrialises. Intuitively, optimism makes the anticipated scale of industrialisation larger, which encourages firms to industrialise. This can be shown in figure 1.

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9The threshold value of $\alpha$ to capture pessimism is determined by the value of the fixed cost $I$ in the interval of $\left[(a_h - a_l) \cdot \frac{N}{1 - a_h}, (a_h - a_l) \cdot \frac{N}{1 - a_l}\right]$. 
4 Conclusion

Our findings suggest that in a “Big Push” model, when ambiguity is high the equilibrium is unique. There are other studies on the selection of multiple equilibria in coordination games, for instance the global game approach, Morris & Shin (2003) and sunspot equil- librium theory, Shell & Cass (1983). There is a basic difference between this paper and the global game literature. While the global game approach is applied to games with incomplete information, we investigate coordination games with complete information.

References


