Economics Bulletin

Volume 30, Issue 2

Market coverage and the nature of product differentiation: a note

Xavier Y Wauthy
CEREC, facultés Saint-Louis, Bruxelles and CORE, Université catholique Louvain

Abstract

In this note, we analyze the equilibrium outcomes of pricing games with product differentiation in relation with the extent of market coverage. It is a received idea in the IO literature that the horizontal and vertical models of product differentiation are almost formally equivalent. We show that this idea turns out to be wrong when the full market coverage assumption is relaxed. We then argue that there exist two fundamentally different classes of address-models of differentiation, although their difference is not perfectly captured by the standard horizontal/vertical typology.

I am grateful to Jean Gabszewicz for his constructive comments. This research has been conducted with the financial support of BELSPO IAP research program under contract IAP 06/09. I retain responsibility for any remaining errors.
Citation: Xavier Y Wauthy, (2010) "Market coverage and the nature of product differentiation: a note", *Economics Bulletin*, Vol. 30 no.2 pp. 1129-1136.
Submitted: Jan 04 2010. **Published:** May 03, 2010.

1 Introduction

It has been recognized for long that the introduction of product differentiation is instrumental in escaping from the Bertrand paradox. Address-models of product differentiation, as inspired by Hotelling (1929), are particularly suggestive. They make it explicit that, in order to relax price competition, firms need a two-dimensional heterogeneity: products have to differ in at least one characteristic, but the population of consumers must exhibit heterogeneous tastes as well. The relationship between these two dimensions of heterogeneity is crucial in characterizing the nature of product differentiation.

It is commonplace nowadays to distinguish between models of horizontal and vertical differentiation. However, the extent to which this distinction really matters for equilibrium outcomes is not clear. Several recent papers suggest that the horizontal and vertical approaches of differentiation are to a large extent equivalent. In particular, Cremer and Thisse (1991) show that *"every model belonging to a very large class of Hotelling-type models (including all the commonly used specifications) is actually a special case of a vertical differentiation model."* This claim seems to be confirmed in Irmen and Thisse (1998): their analysis of products' characteristics choices in a multidimensional setting suggests that the difference between horizontal and vertical characteristics does not really matter.[1] As a matter of fact, IO textbooks also seem to have chosen their side. They focus indeed much more on similarities than on differences. For instance Tirole (1988) starts the analysis of vertical differentiation by claiming that *"The study of vertical differentiation so closely resembles that of horizontal differentiation..."*. Shy (1996), in his chapter 12, builds the analysis of his vertical differentiation model as a particular case of the Hotelling model. Martin (2000) also emphasizes similarities between the two models. In a very recent contribution, Schmidt (2009) endorses a comparable point of view: building on a theoretical set-up which represents a model of either vertical or horizontal differentiation depending on the interpretation given to the variables, he establishes some policy recommendations which are invariant to the nature of differentiation he deals with.

Curiously enough then, vertical differentiation is also known to be a necessary condition for the finiteness property to hold. Generalizing the early arguments of Gabszewicz and Thisse (1979), Shaked and Sutton (1983) show that under vertical differentiation there may exist an upper bound to the number of firms which may co-exist in the market in the long run, even when entry cost is abritrarily small. This property is known as the finiteness property. Under this property, we may expect natural oligopolies to prevail, though exclusively under vertical differentiation. By contrast, the number of firms which may co-exist under horizontal differentiation tends to infinity when entry cost tends to zero. In view of the preceeding statements, this last result is surprising: if vertical and horizontal models of product differentiation are essentially two faces of the same coin, why is it that, in the long run, they possibly lead to radically different equilibrium market structures?

In this note, we argue that the similarities between the vertical and the horizontal approach, as emphasized for instance by Cremer and Thisse (1991), Schmidt (2009) and in most IO textbooks, are misleading. Those similarities are indeed formally established by relying on a two-stage model where firms choose first products' characteristics and then compete in prices. The model is solved exclusively for the parameters' constellations such

[1] The reader is referred to Cremer and Thisse (1991) and Irmen and Thisse (1998) for additional references on the distinction between horizontal and vertical differentiation.

that the market is fully covered. However, the extent of market coverage, i.e. whether full coverage prevails or not in equilibrium, should be endogenous to any two-stage model of differentiation. Once partial market coverage is considered, a key difference emerges between the prototype models of differentiation. In one class of models, akin to the concept of variety differentiation, firms may actually get rid of price competition through product differentiation whereas this is never possible under the other class, akin to the concept of quality differentiation.

2 An Example

Schmidt (2009) considers a population of consumers indexed by their type x. Types are uniformly distributed in the $[0,1]$ interval with a density equal to 1. Each type x is characterized by an indirect utility function $W(x,q,p) = u(x,q) - p$. He consumes at most one unit of product with attribute $q \geq 0$.

Definition 1 *A market is said to satisfy full coverage if and only if, at prevailing prices all types x buy one product.*

There are $n \geq 2$ products with attribute $q_i \geq 0$, $i = 1, ..., n$. We assume that $q_i \geq q_j$ if and only if $i \geq j$.

Let us then characterize the type of differentiation relying on the definition proposed in Schmidt (2009):

Definition 2 *A vertically (horizontally) differentiated market is a market where consumers have an identical (differing) preference ordering over the feasible product attributes.*

There exist different versions of the definition in the literature but the present one is quite representative. In particular, it is perfectly in line with the more restrictive definition put forward in Gabszewicz and Thisse (1986) or Tirole (1988) according to which vertical differentiation prevails when consumers agree on the ordering of products when they are sold at the same price. This definition also captures the intuitive parallel which is made between vertical and quality differentiation on the one hand and horizontal and variety differentiation on the other hand.

As recalled in Schmidt (2009), it is also often convenient to rely on the following complementary definition

Definition 3 *The preferred product version of type x, $\tilde{q}(x)$, is the version that yields the highest surplus to type x, when all feasible versions are offered at their marginal cost $c(q)$. Formally, $\tilde{q}(x) \in Argmax\ u(x,q) - c(q)$*

Relying on this second definition, it is clear that in a model where differentiation relies on quality levels, the preferred product version should be the same for all consumers if quality is not costly, i.e. if $c(q) = c \geq 0$. By contrast, we expect this preferred product to be specific to each x under variety differentiation. Notice that this second definition is also instrumental in checking whether the finiteness property holds or not. To put it simply,[2]

[2] The interested reader is referred to Shaked and Sutton (1983) for a detailed exposition.

the finiteness condition holds whenever the preferred product is the same for all consumers in the market. This condition is more demanding that vertical differentiation as defined above. As originally argued in Cremer and Thisse (1991) and recalled in Schmidt (2009), if the finiteness property does not hold, it can be shown that in equilibrium the choice of the products' characteristics and the prices are identical in the vertical differentiation model with marginal cost being quadratic in quality and the Hotelling model with quadratic transportation costs. Let us consider the following example which describes the two prototype models of product differentiation in the literature:[3]

$$\text{Model V: } V(\cdot) = v + q + xq - p \text{ with } n = 2 \text{ and } c(q) = \frac{q^2}{2}.$$

Assuming that v is large enough to ensure full market coverage, the unique subgame perfect equilibrium is easily established. A little more work is required to establish the equivalence with a Hotelling model under quadratic transportation costs, but not much![4] The corresponding specification of the model for this last case is:

$$\text{Model H :} U(\cdot) = v + t + \frac{(x-t)^2}{4} - p \text{ with } n = 2 \text{ and } t = 2q - 1.$$

The basic intuition for understanding why this equivalence prevails is the following. By assuming that v is arbitrarily large, we ensure that all consumers buy one unit either of good 1 or 2. Firms therefore compete for market shares and products attributes matter only for deciding which firm gets which side of the market. Only the price differential matters for the consumers and it is sufficient to identify the position of the indifferent consumer to define firms' demands. Let us then denote this consumer by $\tilde{x}(p_1, p_2)$ and assume wlog that $q_1 < q_2$. It is immediate to see that $D_1(p_1, p_2) = \tilde{x}(\cdot)$ and $D_2(p_1, p_2) = 1 - \tilde{x}(\cdot)$ in either model V or H. In such a case, the vertical or horizontal nature of the differentiation is formally irrelevant. In the vertical interpretation, the level of the marginal cost being quadratic in quality, the low quality firm can always secure a positive market share on the left of the interval whereas in the Hotelling interpretation, the firm located on the left side always secures a positive market share at the left extreme of the interval. In other words, the behaviour of firms' demands at the price competition stage are the same under vertical and horizontal differentiation. Moreover, the quadratic cost assumption, be it on transportation cost or quality cost, ensures that this is the case for all possible relevant price subgames.

We question now the robustness of this result to the full market coverage assumption. To which extent should we expect to obtain a comparable equivalence should parameter v not be arbitrarily large? The answer is almost immediate: we should not expect the previous equivalence to hold anymore! By definition, if the market is not fully covered, prices are such that for at least one consumers' type, the best available option is to refrain from consuming, i.e. $\exists\, x \neq 0$ such that $W(x, q_i, p_i) < 0$, $i = 1, 2$. The relevant question is then : where do these refraining consumers locate in the $[0, 1]$ interval?

Let us start with model V. According to the specification of $V(\cdot)$, all consumers agree on a ranking according to which the preferred characteristic is the largest q. Therefore, vertical differentiation prevails. But more importantly, the surplus function associated with this specification is strictly increasing in x. We may define by $\overline{x_i}$ the type x which

[3] The example is directly taken from Schmidt (2009)

[4] The interested reader is referred to Schmidt (2009) for a more general and detailed argument

satisfies $V(x, q_i, p_i) = 0$. Since $q_1 < q_2$, $\forall x < \overline{x_1}$, not buying is preferred to buy i. All types $x \geq \overline{x_1}$ buy one of the two products. The following property immediately follows:

Result 1 *In the case utility is defined by $V(\cdot) = v + q + xq - p$, for any feasible products' characteristics, if the market is not fully covered, non buying consumers are located in a unique sub-interval of $[0, 1]$. Moreover, firms' market shares are necessarily connected by an indifferent consumer.*

Firms' demands are defined by $D_2(p_1, p_2) = 1 - \tilde{x}$, $D_1(p_1, p_2) = \tilde{x} - \overline{x_1}$. Because firms' market shares are connected, firms compete with each other even though the market is not covered. This result must be contrasted with the specification of demand in a non-covered market in model H. In model H, it is clear that the surplus function is not monotonic in type x. More precisely, the sign of $\frac{\partial U(\cdot)}{\partial x}$ depends on $x - t$. An immediate consequence is the following result:

Result 2 *In the case utility is defined by $U(\cdot) = v + t + \frac{(x-t)^2}{4} - p$, if the market is not fully covered, the position of non buying consumers in the $[0, 1]$ interval depends on the specification of the products' characteristics t_i. Non-buying consumers may be located in disconnected intervals and firms' market shares need not be connected by an indifferent consumer.*

Suppose in particular that $t_1 = -\frac{1}{4}$ and $t_2 = \frac{5}{4}$, which define the equilibrium values in the unique subgame perfect equilibrium under full coverage. Clearly enough, if v is such that the market is not covered for some relevant price levels, the non-buying consumers will be located in the middle of the $[0, 1]$ interval. Market shares are not connected anymore, which implies that firms do not directly compete with each other but rather behave like local monopolists. Summing up, we observe that, in sharp contrast with the fully covered market case, the specification of demand functions at the price competition stage will most often differ fundamentally depending on whether model V or model H applies. As a consequence, payoffs function differ and the formal equivalence of equilibrium outcomes disappears. To sum up, we may claim:

Proposition 1 *The equivalence result established in Cremer and Thisse (1991) is not robust to the introduction of non-covered market configurations.*

This proposition is certainly not surprising. It nevertheless recalls that the similarities between vertical and horizontal models of product differentiation are over-emphasized.

3 Comments

In the previous section, we have shown that full market coverage was critical in establishing the formal equivalence between some classes of vertical and horizontal models. We investigate now in more depth what is further revealed by the analysis of non-covered market configuration. The scope for relaxing price competition depends on the way heterogeneity in products' attributes is combined with population' heterogeneity. By differentiating their products, firms actually decide of a particular sharing of consumers among firms and between buyers and non-buyers. In this perspective, a key concern is the possible existence of a hierachy *among consumers, as established by the firms.* In order to better assess the nature of product differentiation, it may be useful to put the standard approach on its head: instead of asking whether consumers are unanimous or not in their ranking of products' characteristics (as in Definition 2), one may ask whether *firms* are unanimous or not in *their* ranking of consumers' types. Consider the following definition of a "preferred consumer", which is a mirror image of Definition 3:

Definition 4 *The preferred consumer of a firm which sells characteristic q is the consumer that benefits from the highest gross surplus in the population when consuming product q. Formally $\tilde{x}(q) \in Argmax\ u(x,q) - c(q)$.*[5]

Relying on this definition, one may wish to consider the preference ordering that firms would establish over the set of consumers, depending on their own characteristics. In model V, the *preferred consumer* is unique, i.e. his type does not depend on the precise characteristics chosen by the firms whereas it always does in model H. This mere fact induces a very different structure of competition between the firms. Whenever there exists a unique preferred consumer, as in model V, market shares are necessarily connected at the price competition stage, whatever the products' choice made in the first stage and irrespective of the extent of market coverage. Firms remain direct competitors. By contrast, in model H, firms define their own ranking of consumers by deciding on their characteristics. It may then happen that by differentiating their products, the firms induce a non-covered market configuration where they end up not competing with each other because their markets are disconnected. To put it differently, product differentiation induces heterogeneous rankings of consumers in model H whereas it preserves the homogeneous ranking in model V.

Does the nature of differentiation as defined by Definition 2 have systematic implications for the diversity of preferred consumers as defined with Definition 4? We may easily think of vertically differentiated markets in the sense of Definition 2 in which preferred consumers differ depending on the products' characteristics chosen by the firms. Consider for instance the following location model with linear transportation costs: consumers are uniformly distributed in an interval $[a^-, a^+] \in [0,1]$. There are two feasible locations: $\{0,1\}$. Depending on whether $\frac{1}{2} \in [a^-, a^+]$ or not, we may either have horizontal or vertical differentiation in the sense of Definition 2, but it will always be the case that a firm locating at 0 has type a^- as its preferred consumer whereas a^+ is the preferred consumer of a firm located at 1. In the same way, it is easy to build a horizontal differentiation model in the sense of Definition 2 but in which preferred consumers do not depend on products' specifications. This is for instance the case in the following set-up: the consumers are splitted in $[0, a_1]$ and $[a_2, 1]$ intervals, with $a_1 < \frac{1}{2}$ and $a_2 > \frac{1}{2}$. The space of products' possible characteristics is defined as $[b_1, b_2]$ with $a_1 < b_1 < \frac{1}{2} < b_2 < a_2$.

Does it matter then to know whether the primitives of the model, i.e. the specification of consumers preferences and the distribution of types on the one hand, the space of products' attributes on the other hand, allow for different preferred consumers or not? If the location of the preferred consumer depends on product characteristics, product differentiation is apt to create localized competition (in the limit, it even destroys competition). This preserves the ability of each firm to sell to consumers with the highest surplus for

[5] We add $c(q)$ so as to enhance the similarity with Definition 3 but it should be clear that cost actually do not matter here.

their products. If the converse prevails, only one firm will end up selling to the highest surplus consumers, all the other ones will be confined to selling to "second-rate" consumers. It is our belief that this distinction is a relevant one in various strategic contexts. Consider for instance the order of moves at the product selection stage. Does it matter to move first? If the preferred consumer is unique, then playing first ensures that a firm can sell to its preferred consumer. Timing matter here. Another case in point is when firms have several strategic tools at their disposal. Is product differentiation robust to the presence of these additional commitment tools? In Boccard and Wauthy (2009) it is shown that within the standard vertical differentiation model popularized by Tirole (1988), there exists no subgame perfect equilibrium in which firms differentiate their products by quality when they are allowed to commit to limited capacity levels. The intuition underlying this result is simple: since capacity constraints already limit drastically price competition, firms are induced to both select the quality level which maximizes industry welfare, i.e. they focus on the same set of preferred consumers, for whom the surplus is maximized by selecting the best available quality. A comparable no-differentiation result would never obtain in a Hotelling model with capacity commitment. In the Hotelling model, each consumer has his ideal product type, which actually implies that by choosing different characteristics firms choose different preferred consumers. In such a case, maximizing industry surplus requires product differentiation for sure. The reason is not such much related to the distinction usually made between horizontal and vertical differentiation but rather to the existence, or non-existence, of a unique preferred consumer, as induced by the model primitives.

References

[1] Boccard, N. and X.Y. Wauthy (2010) "Equilibrium vertical differentiation in a Bertrand model with capacity precommitment" *International Journal of Industrial Organization* forthcoming.

[2] Cremer, H. and J.-F. Thisse (1991) "Location models of horizontal differentiation: a special case of vertical differentiation models" *The Journal of Industrial Economics* 39-4, 383-390.

[3] Gabszewicz, J. and J.-F. Thisse (1979), "Price competition, quality and income disparities" *Journal of Economic Theory* 20, 310- 359.

[4] Gabszewicz, J. and J.-F. Thisse (1986) "On the nature of competition with differentiated products" *The Economic Journal* 96, 160-172.

[5] Hotelling, H. (1929) "Stability in competition" *The Economic Journal* 39, 41- 57.

[6] Irmen, A. and J.-F. Thisse (1998), "Competition in multicharacteristics spaces: Hotelling was almost right" *Journal of Economic Theory* 78-1, 76-102.

[7] Martin, S. (2000) *Advanced Industrial Organization*, Blackwell

[8] Shaked, A. and J. Sutton (1983) "Natural Oligopolies" *Econometrica,* 51, 1469-1484

[9] Schmidt, R. (2009) "Welfare in differentiated oligopolies with more than two firms" *International Journal of Industrial Organization* 27, 501-507

[10] Shy, O. (1996) *Industrial Organization*, MIT Press

[11] Tirole, J. (1988) *The theory of industrial organization*, MIT Press.