Abstract
We study a loan market equilibrium in which some borrowers are optimistic and banks face imperfect competition. We show that the presence of optimistic borrowers reduces the interest rate paid by safe borrowers and increases the interest rate paid by risky borrowers. But it has no net impact on the banks' profits.


1 Introduction

People tend to be unrealistically optimistic about the future. They expect others to be victims of misfortune, but not themselves (see, e.g., Weinstein, 1980). Supporting this view, Cooper, Woo, and Dunkelberg (1988) find that many entrepreneurs overestimate their probability of success. In this note, we study a loan market equilibrium in which some borrowers are optimistic and banks face imperfect competition in loan market.

The model is an application of Salop’s (1979) circular road model in loan market. Three types of borrowers—safe, risky, and optimistic—are located uniformly around a unit circle, and incur a transportation cost when traveling to banks. A safe borrower is endowed with a safe project, and a risky borrower is endowed with a risky project. An optimistic borrower is endowed with either a safe or a risky project, but every optimistic borrower believes that his project is safe. Banks compete for borrowers by announcing the interest rate for each type of projects. Borrowers then travel to banks to apply for loans. After that banks screen each loan applicant to determine the applicable interest rate.

We show that the presence of optimistic borrowers reduces the interest rate paid by safe borrowers, and increases the interest rate paid by risky borrowers. The intuition is as follows. When deciding which bank to apply for a loan, optimistic borrowers only care about how much a bank would charge for a safe project, because every optimistic borrower believes that his project is safe. To attract optimistic borrowers, banks reduce the announced interest rate for safe projects. Once borrowers have applied for loans and learned the applicable interest rate, it is, however, too late to shop, because they have incurred the transportation cost. To profit from optimistic borrowers, banks increase the announced interest rate for risky projects. We also show that the presence of optimistic borrowers has no net impact on the banks’ profits.

Our paper is closely related to Hyytinen (2003). Using an elegant simple model, he shows that borrower optimism reduces both equilibrium interest rate and banks’ profits. Our model differs from his in two dimensions. First, in Hyytinen’s model, all the borrowers are optimistic. In contrast, in our model, while some borrowers are optimistic, others are rational. Assuming heterogeneous borrowers allows us to examine the impact of optimistic borrowers on rational borrowers. Second, in Hyytinen’s model, a borrower knows the applicable interest rate before he applies for a loan. In contrast, in our model, a borrower learns the applicable interest rate after he has applied for a loan and the bank has screened his creditworthiness. We believe that each model resembles the loan application process in some markets. For example, in the prime mortgage market, lenders reveal the applicable interest rate for free, without requiring potential borrowers to first submit loan application. Hyytinen’s model resembles this process. In contrast, in the subprime mortgage market featuring risk-based pricing, a borrower must often pay several hundred dollars in application and appraisal fees and wait until closing to discover the actual interest rate (see, e.g., McCoy 2007, and Willis 2006). Our model resembles this process. Thus, both models are useful for understanding the impact of optimistic borrowers on loan market equilibrium.

Our paper joins the emerging behavioral industrial organization literature (see, e.g., DellaVigna, 2009; Ellison, 2006). Papers in this literature note that some consumers are
biased, and examine the interaction between biased consumers and rational, profit-maximizing firms. Our paper contributes to this literature by examining how rational lenders respond to optimistic borrowers.

2 The model

Consider a universally risk-neutral economy in which a continuum of borrowers with unit mass are located uniformly around a unit circle. There are also \( n > 2 \) banks located symmetrically around the circle.

Each borrower is endowed with a project that requires an initial outlay of $1. A project can be either one of two types: safe or risky. A safe project generates a cash flow \( R > 1 \) with probability 1. A risky project generates a cash flow \( R \) with probability \( 0 < p < 1 \), and 0 with probability \( 1 - p \). Both types of projects are creditworthy.

There are three types of borrowers: safe, risky, and optimistic. A proportion \( 0 < \alpha < 1 \) of borrowers are safe, a proportion \( 0 < \beta < 1 \) of borrowers are risky, and the remaining borrowers are optimistic. A safe borrower believes, correctly, that he is endowed with a safe project. A risky borrower believes, correctly, that he is endowed with a risky project. A proportion \( 0 < \theta < 1 \) of optimistic borrowers are endowed with safe projects, and the remaining optimistic borrowers are endowed with risky projects. However, every optimistic borrower believes that his project is safe.\(^1\)

Borrowers have no initial wealth and must seek financing from banks. Traveling to banks is costly: borrowers incur a transportation cost of \( \tau \) per unit of length.\(^2\) Banks cannot observe the type of each borrower, but they know the proportion of each type, and the proportion of optimistic borrowers who have safe projects. Banks face a perfectly elastic supply of capital at a gross interest rate equal to \( \rho \).

The timing of events is as follows. First, banks simultaneously announce a pair \((R_s, R_r)\), where \( R_s \) is the interest rate for safe projects, and \( R_r \) is the interest rate for risky projects. Then, borrowers observe the announced interest rates and travel to the bank from which they would like to apply for a loan. Finally, banks screen each loan applicant to determine the applicable interest rate. We assume that banks can perfectly screen. This assumption simplifies the calculations, but still conveys the full intuition.

3 Loan market equilibrium

Following Hyytinen (2003), we restrict attention to full-scale competition, uniform pricing, and symmetric Nash equilibrium.\(^3\) We also assume that \( \tau \) is small enough to

\(^1\)This assumption is motivated by the well-documented “above average” effect: Most people believe that they are more likely than their peers to experience positive events, and less likely than their peers to experience negative events (see, among others, Weinstein, 1980).

\(^2\)Degryse and Ongena (2005) find that transportation costs are important for small borrowers.

\(^3\)Full-scale competition exists when the transportation cost \( \tau \) is sufficiently small. See Villas-Boas and Schmidt-Mohr (1999) for a complete characterization of the evolution of the equilibrium strategies with
ensure that in equilibrium the entire market is served.\textsuperscript{4}

Consider the decision problem of a representative bank \(i\). Suppose that bank \(i\) announces a pair of interest rates \((R_{is}, R_{ir})\), and all other banks announce \((R_s, R_r)\). A safe borrower located at distance \(x\) from bank \(i\) and distance \((1/n - x)\) from bank \(i+1\) will be indifferent between going to either bank if

\[
R - R_{is} - \tau x = R - R_s - \tau (1/n - x),
\]
or

\[
x = \frac{1}{2n} - \frac{R_{is} - R_s}{2\tau}.
\]

Taking into account the symmetric market area between bank \(i\) and bank \(i-1\), and the assumption that the proportion of safe borrower in the entire population of borrowers is \(\alpha\), gives the following demand for loans of bank \(i\) from safe borrowers:

\[
D_{is} = 2x\alpha = \alpha - \alpha \frac{R_{is} - R_s}{\tau}.
\]

Similarly, the demand for loans from risky borrowers is given by

\[
D_{ir} = \frac{\beta}{n} - \frac{\beta p (R_{ir} - R_r)}{\tau}.
\]

And the demand for loans from optimistic borrowers is given by

\[
D_{io} = \frac{1 - \alpha - \beta}{n} - (1 - \alpha - \beta) \frac{(R_{is} - R_s)}{\tau}.
\]

Denote bank \(i\)'s profits by \(\pi_i\). Recall that a proportion \(\theta\) of optimistic borrowers are endowed with safe projects, and the remaining optimistic borrowers are endowed with risky projects. Thus bank \(i\)'s profits can be written as:

\[
\pi_i = (R_{is} - \rho) (D_{is} + \theta D_{io}) + (p R_{ir} - \rho) [D_{ir} + (1 - \theta) D_{io}]. \tag{1}
\]

Maximizing \(\pi_i\) with respect to \(R_{is}\) yields the following first-order condition:

\[
\frac{\tau}{n} - 2R_{is} + R_s + \rho - \frac{(1 - \alpha - \beta) (1 - \theta)}{\alpha + (1 - \alpha - \beta) \theta} (p R_{ir} - \rho) = 0. \tag{2}
\]

Maximizing \(\pi_i\) with respect to \(R_{ir}\) yields the following first-order condition:

\[
[\beta + (1 - \alpha - \beta) (1 - \theta)] \frac{\tau}{n} - \beta p (R_{ir} - R_r) - (1 - \alpha - \beta) (1 - \theta) (R_{is} - R_s) - \beta (p R_{ir} - \rho) = 0. \tag{3}
\]

\textsuperscript{4}Entrepreneurs are very much a self-selected group. The assumption that every borrower is served in equilibrium constrains us from analyzing this issue. See de Meza and Southey (1996) for an interesting analysis.
Solving (2) and (3) simultaneously, and noting that in a symmetric Nash equilibrium \(R_{is} = R_s\) and \(R_{ir} = R_r\), yields the following results:

\[
R_s = \left\{1 - \frac{(1 - \alpha - \beta)(1 - \theta)(\beta + (1 - \alpha - \beta)(1 - \theta))}{\beta[\alpha + (1 - \alpha - \beta)\theta]}\right\} \frac{\tau}{n} + \rho, \tag{4}
\]

\[
R_r = \left[1 + \frac{(1 - \alpha - \beta)(1 - \theta)}{\beta}\right] \frac{\tau}{np} + \frac{\rho}{p}. \tag{5}
\]

Recall that the proportion of optimistic borrowers is \(1 - \alpha - \beta\). It is easy to verify that \(dR_s/d(1 - \alpha - \beta) < 0\) and \(dR_r/d(1 - \alpha - \beta) > 0\). We have thus obtained the following result.

**Proposition 1.** In a symmetric Nash equilibrium, the interest rate paid by safe borrowers decreases with the proportion of optimistic borrowers, and the interest rate paid by risky borrowers increases with the proportion of optimistic borrowers.

To understand the intuition behind proposition 1, note that when an optimistic borrower chooses which bank to apply for a loan, he only cares about the interest rate for safe projects, because every optimistic borrower believes that he has a safe project. This provides an incentive for banks to reduce the announced interest rate for safe projects. After an optimistic borrower arrives at a bank and learns whether his project is safe or risky, it is too late for him to shop, because he has already incurred the transportation cost. This provides an incentive for banks to increase the announced interest rate for risky borrowers. Thus, the presence of optimistic borrowers benefits safe borrowers, but hurts risky borrowers.

Substituting \(R_{is} = R_s\) given by (4) and \(R_{ir} = R_r\) given by (5) into (1), and simplifying terms, yields the banks’ profits in equilibrium:

\[
\pi = \frac{\tau}{n^2}.
\]

**Proposition 2.** In a symmetric Nash equilibrium, the presence of optimistic borrowers has no net impact on the banks’ profits.

The presence of optimistic borrowers increases the profits that banks can obtain from risky borrowers, but decreases the profits that banks can obtain from safe borrowers. In equilibrium these two effects cancel out. As a result, the banks’ profits are not affected by the presence of optimistic borrowers.

## 4 Discussion

We have presented a banking model with optimistic borrowers and imperfect competition. We show that the presence of optimistic borrowers affects the banks’ interest
rate decisions. Specifically, banks reduce the interest rate for safe borrowers, but increase the interest rate for risky borrowers.

In our model, the beliefs of borrowers are assumed to be exogenous. An important question is: Do banks have incentives to influence the beliefs of borrowers? In a recent paper, Inderst and Ottaviani (2010) develop a model in which an intermediary agent advises customers on which products to purchase, based on the match between customer needs and product features. Inderst and Ottaviani show that product providers have an incentive to pay commissions to the intermediary agent in order to increase the sales of their products. Jackson and Burlingame (2007) find evidence that some mortgage brokers advised borrowers to take out more expensive mortgages. Indeed, an important lesson emerging from the recent subprime mortgage crisis is that many subprime borrowers took out mortgages that they could not afford. Examining whether and how banks influence the beliefs of borrowers is an important question for future research.
References


