Economics Bulletin

Volume 30, Issue 2

# Predictable Signals in Excess Returns: Evidence from Non-Gaussian State Space Models

Khurshid Kiani
*Nottingham Business School*

## Abstract

The present work investigates predictable components in size-based and value-weighted market portfolios excess returns from NYSE, AMEX, and NASDAQ stocks over US Treasury bills using various Gaussian and non-Gaussian versions of state space or unobserved components models. Our state space or unobserved components model improves on Conrad and Kaul (1988) by taking into account fat tails that are widely documented in the returns series. Statistical hypotheses tests show existence of predictable components in excess returns for most size-based portfolios (Cap-1 through Cap-9) even at percent level of significance. However, for value-weighted market and largest size-based portfolio (Cap-10) the hypothesis tests fail to reveal existence of any predictable component. The results for most size-based portfolios are in conformance with Conrad and Kaul (1988) except the value-weighted market excess returns as well as the largest size-based portfolio (Cap-10). Conrad and Kaul (1988) isolated time-varying expected returns in weekly size-based excess returns using the same methodology but in a Gaussian setting. However, our results on value-weighted market excess returns are in line with Bidarkota and McCulloch (2004) who investigated value-weighted market excess returns in CRSP data.

**Citation:** Khurshid Kiani, (2010) "Predictable Signals in Excess Returns: Evidence from Non-Gaussian State Space Models", *Economics Bulletin*, Vol. 30 no.2 pp. 1217-1232.
**Submitted:** Apr 03 2009. **Published:** May 06, 2010.

# 1. INTRODUCTION

The literature survey by Fama (1991) reveals that possible existence of predictable components in stock returns have been investigated extensively. This is because even a small level of predictability could lead to large economic gains through suitable trading strategies (Xu 2004). Investigation of the predictability in stock returns is imperative for portfolio allocation (Barberis 2000) and it has implications for the models of asset pricing (Cecchetti, Lam, and Mark 1990). While modeling stock returns predictability, researchers focused on the two aspects of the stock returns predictability i.e. non-normality and volatility persistence. For instance, Akagri and Booth (1988), Jansen and de Vries (1991), Buckel (1995), Mantegna and Stainley (1995), and McCulloch (1997) including others maintained that non-normality is prevalent in stock returns. Similarly, Nelson (1991), Danielson (1994) Pagan and Schwartz (1990), Diebold and Lopez (1995), and Goose and Kroner (1995) showed that volatility persistence exists in stock returns over time.

A number of studies that include Fama and Roll (1977), Cornew, Town, and Crowson (1984), and Nolan and Panorska (1997) introduced more generalized distributional models encompassing stable distribution to which normal distribution is a sub set. Empirical research demonstrates that models with such features are able to explain the behavior of economic and financial data more accurately. This enabled the empirical researchers to show that economic and financial data have leptokurtic distributions with fat tails. For example, Leitch and Paulson (1975), and Fielitz and Rozelle (1983) studied stock price behavior using stable distributions, Hall, Brorsen, and Irwin (1989) considered working on the distribution of futures prices, and McCulloch (1996a) worked with financial applications using stable distributions. However, normal distribution can be employed relatively more easily since many of its properties are well known. Therefore, despite a strong consensus on the presence of non-normality as well as volatility persistence in financial and economic time series data, models employed by most studies for stock returns predictions; do not encompass features that account for fat tails. Estimation inefficiencies would result when fat tails are not taken into account while modeling the stock excess return series. This could result in failure to detect predictability in stock returns even when it would exist.

Conrad and Kaul (1988) modeled weekly returns on size-based portfolios returns using an unobserved components or state space model wherein the stochastic shocks in both the observation as well as the state equation were assumed to be identically and independently distributed (i.i.d.) normal. Further, they assumed stock returns to be time-varying and predictable, and modeled them to evolve from a first order autoregressive process. However, it is now well known that stock returns are typically non-Gaussian (McCulloch 1996a). Therefore, Bidatkota and McCulloch (2004) employed non-Gaussian models with Paretian Stable distribution for testing persistence in stock returns, Kiani (2006) for stock returns predictability in emerging market, and Kiani (2007) for predictability in stock returns in transition economies. The Paretian stable distributions have also been used to model fat tails in stock returns by a number of earlier studies such as Buckel (1995), Mantegna and Stanley (1995), and McCulloch (1997). Therefore, to account for potentially non-Gaussian nature of data, in the present study, returns series are modeled within the framework of Paretian stable distribution.

There are a number of other parametric and non-parametric approaches that test predictability in the return series but most of these approaches do not consider encompassing features that account for fat tails in the models employed for predicting stock excess returns. For example McQueen and Thorely (1991) used Markov chains to test stock returns predictability. In Markov chains the outcome from the current period experiment is assumed to affect the outcome of the next period with some probability and so on. Similarly, in non-parametric approaches for example, the artificial neural networks appear to be a candidate for predicting possible existence of stock returns but neural networks are under heavy criticism for being black boxes (not having an explicit functional form) and overfitting issues associated with them, although, following Kiani (2005) overfitting could be mitigated by careful construction of a neural network architecture that enables neural networks to be suitable forecasting models for predicting stock excess returns. However, in the present study, the predictable components are estimated as model parameters in the state space model employed, which is expected to do a better job of extracting the signals of the predictable component from all the size-based portfolio excess return series as well the value-weighted market excess returns series employed.

The present study investigates possible existence of predictable components (if any) in monthly size-based and value weighted Center of Research of Security Prices (CRSP) in American Stock Exchange (AMEX), New York Stock Exchange (NYSE), and the National Association of Securities Dealers Automated Quotation (NASDAQ) stocks prices over the relevant risk free rates i.e. U.S. Treasury bill rates using Gaussian state space models due to Conrad and Kaul (1988), and their improved versions (non-Gaussian) that account for fat tails in the returns series. The errors in the non-Gaussian space state models are assumed to come from non-normal family, therefore, the recursive algorithm from Sorenson and Aspatch (1971) that was further modified by Kitawaga (1987) is employed for estimation of the non-Gaussian state space or unobserved component models.

The remaining study is organized as follows. Section 2 elaborates the econometric models used, and estimation results as well as hypotheses of interest are discussed in section3. Finally, section 4 incorporates conclusion.

## 2. ECONOMETRIC MODELS

The most general state space or unobserved component model i.e. model1, employed in this research, incorporates non-normality as well as predictable components. Restricted versions of the most general model that exclude predictable components and fat tails are also employed. The estimates from these restricted models are used for testing various hypotheses of interest. Similarly, the Gaussian versions of the state space model with and without predictable components are also estimated. The most general model and its restricted versions employed in this study are elaborated in the following sub-sections.

### 2.1. Model 1: Stable Model with Predictable Components

The following most general unobserved component model is employed to detect possible existence of variations that might persist in excess returns series. The model for forecasting mean returns is a state space or unobserved component model that encompasses features like non-normality and predicable component (if any). The most general model of this type also termed as model 1 is shown in the following two Equations.

$$r_t = x_t + \varepsilon_t, \quad \varepsilon_t \sim c_t z_{1t},\ z_{1t} \sim i.i.d \quad S(0,1) \tag{1}$$

$$(x_t - \mu) = \phi(x_{t-1} - \mu) + \eta_t,\ \eta_t \sim c_\eta c_t z_{2t},\ \eta_t \sim i.i.d.\ S(0,1) \tag{2}$$

where, $r_t$ is the observed excess return, $x_t$ is unobserved predictable component, and $z_{1t}$ and $z_{2t}$ are independent white noise processes. A random variable $X$ is said to have a symmetric stable distribution $S_\alpha(0,c)$ if its log-characteristic function can be expressed as $\ln[Exp(iXt)] = i\delta t - |c_t|^\alpha$. The parameter $\delta \in -(\infty,\infty)$ is the location parameter that shifts the distribution either to the left or to the right along the real line, $c > 0(c \in [0,\infty])$ is the scale parameter that contracts or expands the distribution, and $\alpha \in (0,2]$ is the characteristic exponent that governs the tail behavior. Smaller values of the characteristic exponent $\alpha$ indicate thicker tails. However, when the characteristic exponent ($\alpha$) equals to $2$ normal distribution prevails with a finite variance that equals to $2c^2$.

Stable distributions have thick tails which enhances the likelihood of the occurrences of the large shocks. Therefore, big market crashes as well as booms are expected more in this setup than in a Gaussian framework. Mandelbrot (1963) recommended the use of the stable distributions for modeling fat tails, and McCulloch (1996a) provided a comprehensive survey on the financial applications of the stable distributions.

Any time variation in (conditional) mean excess returns is because of the presence of the predictable component $x_t$ which is assumed to follow a simple AR (1) process. The unobserved component or state space model for predicting excess returns is a simple AR (1) process plus noise. It is related to the unobserved component mean-reverting model for stock prices due to Summers (1986). Mean reversion was also studied by Paresh and Prasad, (2007).

A version of the unobserved components model given in Equations 1 and 2 that incorporates time-varying volatility was estimated by Bidarkota and McCulloch (2004), but in the present study, attempts to estimate such conditionally heteroskedastic versions of the unobserved components or state space model with CRSP data by maximum likelihood estimation failed in five out of the eleven series employed. Therefore, further discussion on time-varying volatility is obviated throughout the text.

## 2.2. Model 2: Stable Model without Predictable Components

Model 2 is a stable model that does not include features to account for predictable components in the excess return series employed. This model is obtained restricting predictable components in excess return series ($\phi = 0$) in the most general model (i.e.model1). The resulting model 2 is shown in the following Equation.

$$r_t = x_t + \varepsilon_t, \quad \varepsilon_t \sim c_t z_t,\ z_t \sim i.i.d\ \ S(0,1) \tag{3}$$

As shown in the Equation 3 above, the error terms $\varepsilon_t$ and $\eta_t$ are not separately identified, therefore the signal to noise ratio ($c_\eta = 0$) is also not identified. As it will be clear later, this will cause difficulty in constructing various hypothesis of interest using likelihood ratio test statistics.

## 2.3. Model 3: Gaussian Model with Predictable Components

A Gaussian version of the unobservable component model encompassing predictable component that was also employed by Conrad and Kaul (1988) can be shown by the following Equations.

$$r_t = x_t + \varepsilon_t, \quad \varepsilon_t \sim \sqrt{2}c\, z_{1t}, \; z_{1t} \sim i.i.d \;\; N(0,1) \tag{4}$$

$$(x_t - \mu) = \phi(x_{t-1} - \mu) + \eta_t, \quad \eta_t \sim \sqrt{2}c\, z_{2t} \quad \eta_t \sim i.i.d. \; N(0,1) \tag{5}$$

Model 3 is a Gaussian model that includes features to account for predictable component in excess return series. This model is obtained by restricting characteristic exponent ($\alpha = 2$) in the most general model i.e. model 1 and recognizing that when $\alpha$ equals 2 the variance of a stable random variable $S_\alpha(0,c)$ would reduce to $2c^2$. Unlike model 2, model 3 has an observation Equation shown in Equation 4, and a state Equation that is presented in Equation 5. Therefore, contrary to model 2, the error terms $\varepsilon_t$, and $\eta_t$ are identified separately in this model.

### 2.4. Model 4: Gaussian Model without Predictable Components

The Gaussian version of the state space model for excess returns with no predictable component takes on the following form since the model is obtained restricting predictable component ($\phi = 0$) in model 3. This causes the state Equation to disappear from the model and the resulting model which encompasses an observation equation only can be shown in Equation 6.

$$r_t = x_t + \varepsilon_t, \quad \varepsilon_t \sim \sqrt{2}c\, z_t, \; z_t \sim i.i.d \;\; N(0,1) \tag{6}$$

Again like model 2, the error terms $\varepsilon_t$, and $\eta_t$ are not separately identified in model 4 as well which would cause difficulties in construction of various hypotheses of interest.

### 2.5. Estimation Issues

Sorenson and Alspach (1971) developed a filtering algorithm to estimate non-Gaussian state space models. This algorithm provides optimal filtering and predicting densities for any given distribution for errors and a framework for computing the log likelihood function. The closed form analytical expressions for the recursive equations for computing the filtering and predicting densities are generally intractable except in very special cases. For example when the above Equations (Equation 1 and 2) are linear with errors distributed normally, the integrals can be evaluated analytically and the algorithm reduces to well known Kalman Filter. However, when the errors are stably distributed, as is the case in this study, the integrals can not be analytically evaluated. These integrals can be numerically evaluated as is done in Bidarkota and McCulloch (2004) or alternatively using Monte Carlo integration techniques due to Durbin and Koopman (2000). The stable distributions can be evaluated by fast numerical approximations as in McCulloch (1996b). The present study employs this framework for analytical evaluation of the integrals.

## 3. EMPIRICAL RESULTS

The following sub-section elaborates data sources, estimation results as well as hypotheses of interests that include normality test, and test for predictable component in addition to discussions on the study results.

### 3.1 Data Sources

The present study employs ten size-based portfolios returns from NYSE, AMEX, and NASDAQ stock prices. The smallest size portfolio of stock returns is denoted by Cap-1 whereas the largest portfolio of stock returns is denoted by Cap-10. The portfolio excess returns for all the series are the differences between the portfolio prices over the relevant risk free rates i.e. U.S. Treasury bill rates. The market value-weighted CRSP excess returns[1] that are calculated from the value-weighted market portfolio returns from NYSE, AMEX, and NASDAQ stock prices over the U.S. Treasury bills are also included in the analysis. The portfolio returns are extracted from monthly CRSP data from January 1962 to December 2002.

### 3.2 Estimation Results

The parameter estimates for models 1 through 4 are shown respectively in Tables 1 to 4. In these Tables the estimated excess returns are expressed as percentages per annum. The maximum likelihood estimates (MLE) of Models 1 and 2 are presented respectively in Tables 1 and 2. All estimates reported in Tables 1 through 4 are rounded off to second decimal place. Estimates of the mean excess returns ($\mu$) for the value-weighted market portfolio excess returns is 9.37 percent per annum and for the size-based portfolios excess returns the estimates for $\mu$ ranges between 11.22 for Cap-8 to 14.59 for Cap-10. The estimate for the AR coefficient ($\phi$) for the value-weighted market excess returns is 0.18, and for size-based portfolio excess returns, the estimates for AR coefficient ($\phi$) ranges from 0.04 for Cap-2 to 0.91 for Cap-6. The estimate for the signal-to-noise ratio $(c_\eta)$ is 0.04 for the value-weighted market portfolio excess returns and for size-based portfolios the estimate of $c_\eta$ ranges from 0.00 for Cap-9 to 5.65 for Cap-1. The estimate for the characteristic exponent $\alpha$ for the value-weighted market excess returns is 1.88 whereas for size-based portfolios excess returns the value of the characteristic exponent $\alpha$ range between 1.5 for Cap-1 to 1.88 for Cap-8 showing non-Gaussian behavior in all the series. Likewise, the remaining parameter estimates included in the models show vide variation across various portfolios.

### 3.3. Hypotheses Tests

The chief hypothesis of this study is no predictable component versus the alternate hypothesis of predictable components in all the excess returns series employed. Additional null hypothesis of normality versus the alternative hypothesis of non-normality is also tested in non-Gaussian settings in all the series. Likewise, the hypothesis of no predictable components versus the alternative hypothesis of predictable components is also tested assuming that the errors for each of the series are normally distributed. These hypotheses tests are elaborated in the following sub-sections.

#### 3.3.1. Test for Normality

The test for normality is based on the null hypothesis of normality against the alternative hypothesis of non-normality in all the series. Model 3 is the null model for this test that is obtained by restricting non-normality ($\alpha = 2$) in the most general model i.e. model 1. The null hypotheses for this test for each of the series is tested using likelihood ratio (LR) test statistic which is calculated from the log likelihood estimates from model 1 and model 3. The LR test statistics for all the series are reported in Table 1. In this Table, row 1 in column 8 shows the LR test statistic for normality test for the value-weighted market excess returns. The LR test statistics for normality test for the size-based market excess returns for the smallest size-based

[1] Thanks are due to Prasad V. Bidarkota at Florida International University for providing data for this study.

portfolios (Cap-1) to the largest size-based portfolio (Cap-10) are also presented in this Table. The test statistics for Cap-1 through Cap-10 are shown respectively in rows 2 -11 in column 8. The LR test statistic for normality hypothesis has a non-standard distribution, since the null hypothesis lies on the boundary of the admissible values for the characteristic exponent $(\alpha)$, and hence, the standard regularity conditions are not satisfied. Hypotheses testing when the standard regularity conditions are not satisfied are given in Andrews (2001). Therefore, the critical values from simulations in McCulloch (1997) are employed for evaluating the test statistics for the normality tests. Using these critical values, normality is easily rejected in all the series. However, exclusion of predictable component $(\phi)$ from the non-Gaussian state space models does not alter the results for normality test.

3.4. Test for Predictable Components

The null hypothesis for this test is no predictable components in excess returns where, the alternative hypothesis is the existence of predictable component in the return series. The restricted model under the null hypothesis is obtained by setting $\phi = 0$ in Model1. In this case, the two shocks, $\varepsilon_t$ and $\eta_t$ are not separately identified. Therefore, the standard likelihood ratio (LR) test is not applicable because the signal to noise ratio $(c_\eta)$ is also not identified $(c_\eta = 0)$.

Hansen (1992) developed a bound for asymptotic distribution of a standardized likelihood ratio test statistic that is applicable in similar situations. However, the present work refrains using it because the use of Hansen's test may result in under rejection of the null hypothesis and a subsequent loss of power, since Hansen himself noted that his test provides a bound for asymptotic distribution as against the actual asymptotic distribution itself. Therefore, the estimates from the Gaussian version of the null and the alternative models are employed to generate small sample *p*-values from Monte Carlo simulations for each of the series because estimation of the alternative Model 1 in the present study is computationally very intensive.

The LR test statistic for the null hypothesis of no time-varying predictable components in excess returns for each of the series is reported in the last column of the Table1. In this Table, row 1 in column 7 shows likelihood ratio (LR) test statistic for no predictable component for the value-weighted market excess returns. Similarly, the LR test statistics for the ten size-based portfolios i.e. Cap-1 to Cap-10 are shown respectively in rows 2 -12 in column 7 of this Table. The small sample *p*-values that are obtained from Monte Carlo simulations for each of the series are reported beneath each LR test statistic in parentheses.

The predictable component shows one-step ahead forecast of future excess returns. Plotting $E(x_t \mid r_1, r_2, \ldots\ldots, r_t)$ along with its estimated standard errors for all the portfolios (plots not shown for brevity) it transpires that the plots for the value-weighted market excess returns (Value-w) are similar to those of the largest size-based portfolio excess returns (Cap-10). There is much variability for the remaining size-based portfolio excess returns (Cap-1- Cap-9) which increases the amplitudes of the positive or the negative peaks of the excess returns. This variability in amplitude of the excess returns increases with the decrease in the firm size and vice versa, and finally, such variations mitigate for the largest size firm's excess return series. However, the potential of elevated excess return in small firms is easily captured using the signal extraction approach using state space or unobserved component models.

The study results from the non-Gaussian state space models show statistically significant evidence of predictable components in excess returns in nine size-based portfolios i.e. Cap-1 through Cap-9 at all levels of significance. However, the null of no predictable component could not be rejected in the value-weighted market excess returns and the largest size-based portfolio i.e. Cap-10 even at 10 percent level of significance.

In addition to testing possible existence of predictable component in non-Gaussian setting, persistence of predictable component is also tested in Gaussian state space models. The LR test statistic calculated from the likelihood estimates from Gaussian model 1 and model 3 for all the series are reported in Table 3. In this Table, rows 1 in column 7 shows the LR test statistic for no-predictable components for the value-weighted market excess returns. The LR test statistics for the size-based portfolio excess returns for Cap-1 through Cap-10 are shown respectively in rows 2 -11 in the same column. Small sample *p*-values generated from Monte Carlo simulations for each of the series are placed beneath each test statistic in parenthesis.

The results from Gaussian state space models show statistically significant evidence of predictable components in size-based portfolios i.e. Cap-2, Cap-3, and Cap-4 at all levels of significance. However, the results for the remaining size-based portfolio excess returns and the value-weighted market excess returns do not show statistically significant predictable component even at 10 percent level of significance.

### 3.5. Discussions on Results

The study results from non-Gaussian state space or unobserved component models reveal statistically significant evidence of predictable components in all the size-based portfolio excess returns i.e. Cap-1 to Cap-9 in exception of the largest size-based portfolio excess returns i.e. Cap-10. A plausible reason for this variation could be an inverse relationship between the stock returns predictability and the firm size (Conrad and Kaul 1988). However, the study results for the value-weighted market excess returns do not show any evidence of persistent predictable signals in excess returns which is in line with Bidarkota and McCulloch (2004) who also studied predictable components in value-weighted market excess returns in CRSP data.

Using Gaussian unobserved component model the findings of the present work do not show any evidence of persistence predictable signal in value-weighted market excess returns as well as in seven out of ten size-based portfolio excess returns. Therefore, compared to the results reported in the preceding sub-sections that are obtained from the non-Gaussian state space models, these results changed significantly. A plausible reason for this variation can be attributed to exclusion of non-normality from the employed state space model although non-normality is widely documented in the literature (McCulloch 1996a). This can be one of reasons why predictions from the non-Gaussian state space models for the largest size-based excess returns are in sharp contrast with Conrad and Kaul (1988). Conrad and Kaul (1988) also used state space models with the assumptions that the errors follow a normal distribution. However, the present work employs non-Gaussian state space models with the assumptions that the errors follow a non-normal distribution with fat tails since the assumption of non-normality is widely documented in the wide body of the recent empirical literature.

## 4. CONCLUSION

This research employs non-Gaussian state space or unobserved components models for discovering possible existence of predictable components in 10 size-based as well as the value-weighted market excess returns in AMEX, NYSE, and NASDAQ monthly stock prices over the risk free rates i.e. the U.S. Treasury bills rates. The unobserved component model used encompasses non-normality and predictable component to foretell any variation that might be present in the stock excess return series. This model is an improvement over the models employed by previous studies since it includes features that account for fat tails in the return series which is in line with the developments in the recent empirical literature. In addition to non-Gaussian model, Gaussian state space models with and without predictable components are also employed prediction from which is used for testing various hypotheses of interest at Gaussian settings.

The study results reveal statistically significant evidence of predictable components in monthly excess return series for the most size-based (Cap-1 through Cap-9) portfolios excess returns. However, the results for the value-weighted market excess returns as well as the largest size-based portfolio excess returns (Cap-10) from NYSE, AMEX, and NASDAQ stocks over the U.S. Treasury bills are in sharp contrast. The findings for the nine size-based portfolios (Cap-1 through Cap-9) are in conformance with Conrad and Kaul (1988) in exception of the largest size-based portfolio (Cap10) the results for which are in sharp contrast.

Conrad and Kaul (1988) detected significant predictable components in weekly returns on size-based portfolios assuming that errors are normally distributed. Likewise, the results from the present work for the monthly value-weighted market excess returns are in line with Bidarkota and McCulloch (2004), who failed to reveal significant predictable component in value weighted market excess returns in CRSP data for NYSE, AMEX, and NASDAQ returns over U.S. Treasury bills. However, the non-Gaussian state space or unobserved component models encompassing features to account for fat tails in the return series employed in the present study are powerful computational models that are able to extract predictable signals or information on stock return predictability in nine out of the ten size-based portfolio excess return series employed.

## REFERENCES

Andrews, D.W.K. (2001) "Testing When a Parameter is on the Boundary of the Maintained Hypothesis" *Econometrica* **69**, 683-734.

Akgiray, V., and G. Booth (1988) "The Stable Law Model of Stock Returns" *Journal of Business Economics Statistics* **6**, 51-7.

Barberis, N. (2000) "Investing for the Long Run When Returns are Predictable" *Journal of Finance* **54**, 225-264.

Bidarkota, P.V., and J.H. McCulloch (2004) "Testing for Persistence in Stock Returns with GARCH-Stable Shocks" *Quantitative Finance* **4**, 256-265.

Buckle, D. J. (1995) “Bayesian Inferences for Stable Distribution” *Journal of American Statistical Association* **90**, 605-13.

Cecchetti, S.G., P-s. Lam, and N.C. Mark (1990) “Mean reversion in equilibrium asset prices” *American Economic Review* **80**, 398-418.

Conrad, J., and G. Kaul (1988) “Time-Variation in Expected Returns” *Journal of Business* **61**, 409-425.

Cornew, R. W., D. E. Town, and L. D. Crowson (1984) “Stable Distributions, Futures Prices and the Measurement of Trading Performance” *Journal of Futures Markets* **4**, 531-57.

Danielsson, J. (1994) Stochastic Volatility in Asset Prices Estimation with Simulated Maximum Likelihood” *Journal of Econometrics* **64**, 315-400.

Diebold, F. X., and J. A. Lopez (1995) Modeling Volatility Dynamics Macroeconomics: Developments, Tensions and Projects ed, K Hoover (Boston, MA: Kulwer-Academics).

Durbin, J., and S. J. Koopman (2000) “Time Series Analysis of Non-Gaussian Observations Based on State Space Models from Both Classical and Bayesian Perspectives” *Journal of The Royal Statistical Society Series B*, **62**, Part 1, 3-56.

Fama, E.F. (1991) “Efficient Capital Markets: II” *The Journal of Finance* **XLVI**, 1575-617.

Fama, E., and R. Roll (1971) “Parameter Estimates for Symmetric Stable Distributions” *Journal of American Statistical Association* **66**, 331-338.

Fielitz, B. D., and J. P. Rozelle (1983) “Stable Distributions and the Mixtures of Distributions Hypothesis for Common Stock Returns” *Journal of American Statistical Association* **78**, 28-36.

Goose, D., and K. F. Kroner (1995) “The Relationship Between GARCH and Symmetric Stable Processes, Finding the Source of Tails in Financial Data” *Journal of Empirical Finance* **2**, 225-51.

Hall, J. A., B. W. Brorsen, and S. H. Irwin (1989) “The Distribution of Futures Prices: A Test of the Stable Paretian and Mixture of Normal Hypotheses” *Journal of Financial and Quantitative Analysis* **21**, 105-116.

Hansen, B. E. (1992) “The Likelihood Ratio Test Under Nonstandard Conditions: Testing the Markov Switching Model of GNP” *Journal of Applied Econometrics* **7**, S61-S82.

Jensen, D., and C. de Vries (1991) “On the Frequency of Large Stock Returns: Putting Booms and Busts into Perspective” *Review of Economics and Statistics* **73**, 18-24.

Kiani, K. M. (2007) “Stock Returns Predictability in Transition Economies” *Transition*

*Studies Review* **14**, 93-104.

Kiani, K. M. (2006) “Predictability in Stock Returns in an Emerging Market: Evidence from KSE 100 Stock Price Index” *Pakistan Development Review* **45**, 368-381.

Kiani, K. M. (2005) “Detecting Business Cycle Asymmetries Using Artificial Neural Networks and Time Series Models” *Computational Economics* **26**, 65-89.

Kitagawa, G. (1987) “Non-Gaussian State Space Modeling of Nonstationary Time Series” *Journal of the American Statistical Association* **400**, 1032-63.

Leitch, R. A., and A. S. Paulson (1975) “Estimation of Stable Law Parameters: Stock Price Behavior Application” *Journal of American Statistical Association* **70**, 690-697.

Mandelbrot. B**.** (1963) “The Variation of Certain Speculative Prices” *The Journal of Business* **36**, 394-425.

Mantegna, R. N., and H. E. Stanley (1995) Scaling Behavior in the Dynamics of An Economic Index” *Nature* 376, 46-9.

McCulloch, J. H. (1997) “Measuring Tail Thickness in Order to Estimate the Stable Index $\alpha$ : A critique” *Journal of Business and Economic Statistics* **15**, 74-81.

_________ (1996a) “Financial Applications of Stable Distributions” in: G.S. Maddala and C.R. Rao (eds.), Handbook of Statistics, Vol.14 (Elsevier, Amsterdam).

_________ (1996b) “Numerical Approximation of the Symmetric Stable Distribution and Density” in R. Adler, R. Feldman, and M.S. Taqqu (eds.), A Practical Guide to Heavy Tails: Statistical Techniques for Analyzing Heavy Tailed Distributions, Boston: Birkhauser.

McQueen, G., and S. Thorley (1991) “Are Stock Returns Predictable?” *Journal of Finance* **66**, 239-63.

Narayan, Paresh and Arti Prasad, (2007) "Mean Reversion in Stock Prices: New Evidence from Panel Unit Root Tests for Seventeen European Countries." *Economics Bulletin* **3**,1-6.

Nelson, D.B. (1991) Conditional Heteroskedasticity in Asset Returns: A New Approach” *Econometrica* **59**, 347-70.

Nolan, J. P. (1997) “Numerical Computation of Stable Densities and Distribution Functions” *Communications in Statistics, Stochastic Models* **13**, 759-774.

Pagan, A. R., and G.W. Schwert (1990) Alternative Models for Conditional Stock Volatility” *Journal of Econometrics* **45**, 267-90.

Sorenson, H.W., and D.L. Alspach (1971) Recursive Bayesian Estimation Using Gaussian sums” *Automatica* **7**, 465-79.

Summers, L. H. (1986) Does the Stock Market Rationally Reflect Fundamental Values?" *Journal of Finance* **41**, 591-601.

Xu, Y. (2004) Small Levels of Predictability and Large Economic Gains" *Journal of Empirical Finance* **11**, 247-275.

**Table 1: Model 1 Estimates: Stable Models with Predictable Components**

| Portfolio | $\alpha$ | $\mu$ | $c$ | $c_\eta$ | $\phi$ | *Log Likelihood* | $LR(\alpha=2)$ | $LR(\phi=c_\eta=0)$ |
|---|---|---|---|---|---|---|---|---|
| | 1.88 | 9.37 | 35.11 | 0.04 | 0.17 | | | 0.34 |
| Value-w | (0.00) | (0.77) | (1.08) | (0.02) | (0.07) | -2716.69 | 18.08 | (0.79) |
| | 1.53 | 1.74 | 8.19 | 5.65 | 0.30 | | | 67.70 |
| Cap-1 | (0.05) | (4.63) | (4.72) | (3.08) | (0.04) | -2940.04 | 157.02 | (0.00) |
| | 1.57 | 12.45 | 9.73 | 4.49 | 0.04 | | | 38.86 |
| Cap-2 | (0.07) | (4.19) | (3.74) | (1.65) | (0.04) | -2889.26 | 81.44 | (0.00) |
| | 1.63 | 12.65 | 9.96 | 4.31 | 0.26 | | | 46.66 |
| Cap-3 | (0.23) | (4.19) | (4.80) | (1.99) | (0.04) | -2865.55 | 69.20 | (0.00) |
| | 1.73 | 13.13 | 19.29 | 1.86 | 0.34 | | | 38.86 |
| Cap-4 | (0.08) | (4.02) | (11.39) | (1.54) | (0.13) | -2854.63 | 59.04 | (0.00) |
| | 1.76 | 14.16 | 16.31 | 2.67 | 0.21 | | | 29.10 |
| Cap-5 | (0.07) | (3.67) | (2.75) | (0.43) | (0.04) | -2845.69 | 72.82 | (0.00) |
| | 1.77 | 14.22 | 11.05 | 3.91 | 0.91 | | | 28.62 |
| Cap-6 | (0.04) | (3.60) | (4.23) | (1.47) | (2.11) | -2841.72 | 60.16 | (0.00) |
| | 1.77 | 14.33 | 14.18 | 2.99 | 0.18 | | | 18.34 |
| Cap-7 | (0.20) | (3.36) | (4.13) | (0.85) | (0.04) | -2830.83 | 53.18 | (0.00) |
| | 1.83 | 14.59 | 3.36 | 0.21 | 0.15 | | | 16.28 |
| Cap-8 | (0.08) | (3.17) | (1.72) | (6.34) | (0.04) | -2802.57 | 47.84 | (0.00) |
| | 1.88 | 12.69 | 39.54 | 0.00 | 0.09 | | | 6.69 |
| Cap-9 | (0.06) | (0.72) | (1.48) | (0.00) | (0.00) | -2778.77 | 36.02 | (0.00) |
| | 1.87 | 11.22 | 33.77 | 0.08 | 0.98 | | | 1.56 |
| Cap-10 | (0.12) | (2.78) | (1.78) | (0.05) | (0.09) | -2704.38 | 17.98 | (0.29) |

**Notes on Table 1**

1. The following unobserved component or state space model with non-normality (stable model) is employed to estimate the results shown in this Table.

$$r_t = x_t + \varepsilon_t, \quad \varepsilon_t \sim c_t z_{1t}, \quad z_{1t} \sim iid\ S_\alpha(0,1) \qquad (1a)$$
$$(x_t - \mu) = \phi(x_{t-1} - \mu) + \eta_t, \quad \eta_t \sim c_\eta c_t z_{2t}, \quad z_{1t} \sim iid\ S_\alpha(0,1) \qquad (1b)$$

2. The model is estimated using value weighted market excess returns (Value-w), and ten size-based portfolio returns where the smallest size-based portfolio is denoted by Cap-1 and the largest size-based portfolio is termed as Cap-10. The model is estimated with eleven different excess return series where the variables estimated for each of the series are the characteristic exponent $\alpha$, mean excess returns μ, scale ratio $c$, signal-to-noise ratio $c_\eta$, and AR coefficient $\phi$ of the model that are shown respectively in columns 2 to 6, and the log likelihood estimates from the model are shown in column 7.
3. All estimates are rounded off to the second decimal place. and the Hessian-based standard errors for the parameter estimates are reported in parenthesis beneath each parameter estimate.
4. Column 8 show likelihood ratio (LR) test statistics for normality test $[LR(\alpha = 2)]$ which gives the value of the likelihood ratio test statistic for the null hypothesis of normality. The LR test statistics for this test are calculated from log likelihood estimates from model 1 and its restricted version (model 2 ) that restricts non-normality in it.
5. The small-sample critical value for testing null of normality at the 0.01 significance level for a sample size of 300 is reported to be 4.764 from simulations in McCulloch (1997).
6. The last column in the Table (column 9) shows the likelihood ratio (LR) test statistic for testing no-predictable components $[LR(\phi = c_\eta = 0)]$ in the excess return series. The $LR(\phi = c_\eta = 0)$ for this test gives the value of the likelihood ratio test statistic. It is a test for no predictable components in excess returns. Under this null, the distribution of the LR test statistic is non-standard. The LR test statistics for this test are calculated from log likelihood estimates from model 1 and its restricted version (model 2 ) that restricts predictable components ( $\phi$ ) in it.
7. The null of no-predictable component in the returns series is tested using *p*-values generated by estimating Gaussian versions of Models 1 and 2 with data simulated from the estimated Gaussian Model 2 are reported in parentheses.

**Table 2: Model 2 Estimates: Stable Models without Predictable Components**

| Portfolio | $\alpha$ | $\mu$ | $c$ | *Log Likelihood* |
|---|---|---|---|---|
| | 1.87 | 12.12 | 35.13 | |
| Value-w | (0.06) | (2.29) | (1.44) | -2973.89 |
| | 1.66 | 15.55 | 52.15 | |
| Cap-1 | (0.08) | (3.53) | (2.47) | -2793.89 |
| | 1.71 | 14.66 | 48.23 | |
| Cap-2 | (0.06) | (3.23) | (2.11) | -2919.83 |
| | 1.73 | 14.12 | 46.06 | |
| Cap-3 | (0.09) | (3.09) | (2.17) | -2891.88 |
| | 1.79 | 14.22 | 45.84 | |
| Cap-4 | (0.07) | (3.03) | (1.96) | -2874.06 |
| | 1.82 | 14.79 | 5.17 | |
| Cap-5 | (0.06) | (2.98) | (1.80) | -2860.24 |
| | 1.83 | 14.85 | 44.94 | |
| Cap-6 | (0.06) | (2.96) | (1.81) | -2855.16 |
| | 1.83 | 14.76 | 43.64 | |
| Cap-7 | (0.05) | (2.86) | (1.71) | -2840.02 |
| | 1.86 | 14.79 | 41.74 | |
| Cap-8 | (0.06) | (2.75) | (1.63) | -2810.71 |
| | 1.87 | 13.97 | 39.76 | |
| Cap-9 | (0.06) | (2.59) | (1.52) | -2882.22 |
| | 1.88 | 11.31 | 34.44 | |
| Cap-10 | (0.07) | (2.26) | (1.41) | -2705.16 |

**Notes on Table 2**

1. See notes on Table 1.
2. The following unobserved component or state space model with non-normality (stable model) is employed to estimate the results shown in this Table:

$$r_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim c_t z_t, \qquad z_t \sim iid\ S_\alpha(0,1) \qquad (1a)$$

2. The model is estimated using value weighted market excess returns (Value-w), and ten size-based portfolio returns where the smallest size-based portfolio is denoted by Cap-1 and the largest size-based portfolio is termed as Cap-10. The variables estimated for each of the series are the characteristic exponent $\alpha$, mean excess returns μ, and scale ratio $c$ that are shown respectively in columns 2 to 4, and finally the log likelihood estimates from the model are shown in column 5.

3. All estimates are rounded off to the second decimal place. Hessian-based standard errors for the parameter estimates are reported in parentheses.

**Table 3: Model 3 Estimates: Gaussian Models with Predictable Components**

| Portfolio | $\mu$ | $c$ | $c_\eta$ | $\phi$ | *Log Likelihood* | $LR(\phi = c_\eta = 0)$ |
|---|---|---|---|---|---|---|
| | 11.08 | 38.19 | 0.00 | -0.31 | | 0.40 |
| Value-w | (2.41) | (1.20) | (0.25) | (1.58) | -2725.73 | (0.79) |
| | 23.37 | 68.28 | 0.00 | 0.93 | | 1.74 |
| Cap-1 | (4.43) | (2.15) | (0.01) | (0.07) | -3018.55 | (0.27) |
| | 17.35 | 175.37 | 0.33 | 0.26 | | 33.94 |
| Cap-2 | (4.85) | (681.62) | (1.27) | (0.04) | -2929.98 | (0.00) |
| | 14.81 | 89.42 | 0.61 | 0.23 | | 27.28 |
| Cap-3 | (4.46) | (512.77) | (3.48) | (0.04) | -2903.15 | (0.00) |
| | 14.51 | 61.12 | 0.86 | 0.22 | | 25.68 |
| Cap-4 | (4.25) | (1016.49) | (14.23) | (0.04) | -2884.15 | (0.00) |
| | 13.74 | 51.98 | 0.01 | 0.14 | | 0.02 |
| Cap-5 | (3.27) | (1.64) | (0.15) | (1.89) | -2882.10 | (0.81) |
| | 14.17 | 51.03 | 0.01 | 0.91 | | 0.94 |
| Cap-6 | (3.29) | (1.59) | (0.08) | (0.13) | -2871.79 | (0.43) |
| | 13.59 | 49.59 | 0.00 | 0.91 | | 1.12 |
| Cap-7 | (2.77) | (1.56) | (0.15) | (0.09) | -2857.42 | (0.38) |
| | 12.99 | 46.65 | 0.01 | -0.01 | | 0.22 |
| Cap-8 | (2.94) | (1.47) | (0.28) | (0.00) | -2826.49 | (0.72) |
| | 12.54 | 43.98 | 0.01 | 0.063 | | 0.20 |
| Cap-9 | (2.78) | (1.38) | (0.29) | (0.00) | -2796.78 | (0.79) |
| | 10.65 | 36.74 | 0.16 | 0.35 | | 0.48 |
| Cap-10 | (2.39) | (1.66) | (0.09) | (0.00) | -2713.37 | (0.59) |

**Notes on Table 3**

1. The following model with predictable component is employed to estimate the results shown in this Table:

$$r_t = x_t + \varepsilon_t, \quad \varepsilon_t \sim \sqrt{2}\, c z_{1t}, \quad z_{1t} \sim iid \;\; N(0,1) \qquad (3a)$$

$$(x_t - \mu) = \phi(x_{t-1} - \mu) + \eta_t, \qquad \eta_t \sim \sqrt{2}\;\, c_\eta c_t z_{2t}, \qquad z_{1t} \sim iid \;\; N(0,1) \qquad (3b)$$

2. All estimates are rounded off to the second decimal place.
3. The model is estimated using value weighted market excess returns (Value-w), and ten size-based portfolio returns where the smallest size-based portfolio is denoted by Cap-1 and the largest size-based portfolio is termed as Cap-10. The variables estimated for each of the series are the characteristic exponent $\alpha$, mean excess returns μ, scale ratio $c$, and AR coefficient $\phi$ that are shown respectively in columns 2 to 5, and finally the log likelihood estimates from the model are shown in column 6.
4. The LR test statistics ($LR(\phi = c_\eta = 0)$) for testing the null of "no predictable component" in Gaussian settings for all the series are shown in the column 7 of this Table.
5. The LR test statistics for this test are calculated from log likelihood estimates from model 3 and its restricted version at Gaussian settings (model 4) that restricts predictable component ($\phi$) in it.
6. Under this null, the distribution of the LR test statistic is non-standard, therefore, the *p*-values generated by estimating Gaussian versions of Models 1 and 2 with data simulated from the estimated Gaussian Model 2 are reported in parentheses.

**Table 4: Model 4 Estimates: Gaussian Models without Predictable Components**

| Portfolio | $\mu$ | $c$ | *Log Likelihood* |
|---|---|---|---|
| | 10.79 | 38.21 | |
| Value-w | (2.41) | (1.20) | -2725.93 |
| | 22.15 | 68.40 | |
| Cap-1 | (4.31) | (2.15) | - 3019.42 |
| | 17.46 | 59.23 | |
| Cap-2 | (3.73) | (1.87) | -2946.95 |
| | 14.94 | 55.79 | |
| Cap-3 | (3.52) | (1.76) | -2916.79 |
| | 14.53 | 53.65 | |
| Cap-4 | (3.28) | (1.69) | -2896.99 |
| | 13.74 | 51.97 | |
| Cap-5 | (3.28) | (1.64) | -2881.11 |
| | 13.59 | 51.08 | |
| Cap-6 | (3.21) | (1.61) | -2872.26 |
| | 12.96 | 49.65 | |
| Cap-7 | (1.56) | (1.56) | -2857.98 |
| | 12.92 | 46.66 | |
| Cap-8 | (2.94) | (1.47) | -2826.60 |
| | 12.47 | 43.99 | |
| Cap-9 | (2.77) | (1.39) | -2796.88 |
| | 10.51 | 37.29 | |
| Cap-10 | (2.35) | (1.17) | -2713.61 |

**Notes on Table 4**

1. The following Gaussian model without predictable component is employed to estimate the results shown in this Table:

$$r_t = \mu + \varepsilon_t\,,\ \varepsilon_t \sim \sqrt{2}\ cz_t\,,\ z_t \sim iid\ N(0,1) \qquad (4)$$

2. The model is estimated using value weighted market excess returns (Value-w), and ten size-based portfolio returns where the smallest size-based portfolio is denoted by Cap-1 and the largest size-based portfolio is termed as Cap-10. The variables estimated for each of the series are mean excess returns μ, and scale ratio $c$, that are shown respectively in columns 2 and 3 ,and finally the log likelihood estimates from the model are shown in column 4 .
3. All estimates are rounded off to the second decimal place.