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A note on information of firms and collusion

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Abstract

We study the effect of more information of firms about consumers' preferences on collusion sustainability within a differentiated Hotelling duopoly. We show that the increase of information may increase or decrease collusion sustainability, depending on the type of information involved (shared information or unshared information), on the characteristics of the information distribution, and on the product differentiation degree.

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1. Introduction

In a series of recent articles (Schultz, 2004, 2005, 2009), Schultz assumes spatially differentiated firms and considers the case where only a fraction of consumers at each location is informed about firms' prices. Schultz (2005) shows that an increase of the fraction of informed consumers makes collusion harder to sustain if firms are sufficiently differentiated, while it has no effect if firms are homogenous. The purpose of this article is to analyse the effect on collusion sustainability of a different type of information. While Schultz (2004, 2005, and 2009) considers the information that consumers have about firms' prices, we consider the inverse relation, i.e. the information that *firms* have about *consumers' preferences*. This seems particularly relevant nowadays, since the ability of firms to store and use consumers' specific information has greatly increased due to the Internet.

Following Schultz (2004, 2005), we adopt a symmetric Hotelling duopoly framework. Each firm knows the preferences of a subset of consumers. The two subsets, assumed of equal size, do not necessarily fully overlap. This yields four types of consumers: consumers which are known by both firms; consumers which are unknown by both firms, consumers which are known by firm *A* only and consumers which are known by firm *B* only. The fraction of consumers which are known by both firms amounts to what we call "shared" information, while the sum of the fractions of consumers known by one firm only amounts to what we call "unshared" information.

We find that when shared information expands at the expense of the set of unknown consumers, collusion unambiguously becomes more difficult to sustain. Instead, if shared information expands at the expense of unshared information, collusion sustainability decreases when firms are sufficiently homogenous, but if firms are sufficiently differentiated, collusion sustainability decreases (increases) if and only if the set of unknown consumers is sufficiently large (small). Finally, if unshared information expands at the expense of the set of unknown consumers, collusion sustainability decreases when firms are sufficiently differentiated, while it increases when firms are similar enough. For intermediate levels of differentiation, more unshared information decreases (increases) collusion sustainability if and only if shared information is sufficiently small (large).

2. The model

Assume a segment of length 1 where consumers are uniformly distributed. Denote by $x \in [0,1]$ the location of each consumer. Each point in the segment represents a variety of a certain good. Consumers buy one or zero unit of the good. There are two firms, *A* and *B*, with zero marginal and fixed costs, and located respectively at $a \in [0,1/2]$ and $1-a$: the higher is a the lower is product differentiation.[1] Denote by p_x^J the price set by firm $J = A, B$ on consumer *x*. The utility of consumer *x* when buys from firm *A* (*B*) is: $u_x^A = v - p_x^A - t(x-a)^2$ ($u_x^B = v - p_x^B - t(x-1+a)^2$). We assume $v \geq 13t/4$: this guarantees that the market is covered and that the cheating firm serves all consumers.

Based on the information that firms have about consumers' preferences (locations), four different types of consumers can be identified: a fraction γ of consumers is known by both firms; a fraction φ of consumers is known by no firm; a fraction η_A (η_B) of consumers is known by firm *A* (*B*), but not by firm *B* (*A*). We assume: $\eta_A = \eta_B = \eta$. Clearly, $\gamma + \varphi + 2\eta = 1$. Parameter γ identifies the amount of shared information, while 2η identifies the amount of unshared information.

Suppose that firms interact repeatedly in an infinite horizon. In supporting collusion, we assume the *grim trigger* strategy (Friedman, 1971).[2] Denote by Π^C, Π^D and Π^N, respectively the one-shot

[1] Here we differentiate from Schultz (2004, 2005), which uses *t* as a measure of differentiation. However, the adoption of *a* to measure differentiation is equally common in this type of models: see Hackner (1995) and the references therein.

[2] Even if it is not optimal, the *grim trigger* strategy "*is one of very realistic punishment strategies because of its simplicity*", as argued by Matsumura and Matsushima (2005, p 263). The most part of the papers studying collusion

collusive, deviation and Nash profits. The market discount factor, δ, is exogenous and common. Collusion is sustainable if and only if $\delta \geq \delta^* \equiv (\Pi^D - \Pi^C)/(\Pi^D - \Pi^N)$: the higher is δ^* the smaller is the set of market discount factors supporting collusion. In the rest of the analysis, only perfect collusion (joint profits maximization) is considered.[3] We are interested in the effect of a variation of shared and unshared information on collusion sustainability.

Consider consumers φ. Firms do not know consumers' preferences and must set a uniform price. This case has been analysed by Hackner (1995). We simply refer to his paper for relevant payoffs:

$$\Pi_\varphi^N = \frac{t(1-2a)}{2};\quad \Pi_\varphi^C = \begin{cases} [v - t(1/2 - a)^2]/2 & if \quad a \leq 1/4 \\ (v - ta^2)/2 & if \quad a \geq 1/4 \end{cases};\quad \Pi_\varphi^D = \begin{cases} v - t(5/4 - 3a + a^2) & if \quad a \leq 1/4 \\ v - t(1-a)^2 & if \quad a \geq 1/4 \end{cases}$$

Consider consumers γ. Both firms know exactly consumers' location. Therefore, they can perfectly target the price on the consumers' location. This case has been discussed by Gupta and Venkatu (2002) and Colombo (2010). We report the relevant payoffs:

$$\Pi_\gamma^N = t(1-2a)/4; \qquad \Pi_\gamma^C = [v - t(a^2 - a/2 + 1/12)^2]/2; \qquad \Pi_\gamma^D = v - t(1/3 + a^2 - a)$$

Consider consumers η_A. As shown by Eber (1997), Nash profits of firm *A* when only firm *A* sets personalized prices are:

$$\Pi_{\eta_A}^N = 9t(1-2a)/16$$

Consider collusion. If firm *B* sets the uniform price $\tilde{p}_B$, the highest personalized prices firm *A* can set are the solution of $u_x^A(\tilde{p}_A) = u_x^B(\tilde{p}_B)$, that is: $\tilde{p}_A = \tilde{p}_B + t(x-1+a)^2 - t(x-a)^2$. The threshold consumer is $\tilde{x} = \tilde{p}_B/2t(1-2a) + 1/2$: consumers located at $x \in [0, \tilde{x}]$ buy from firm *A*, while the other consumers buy from firm *B*: it follows that firm *B* has a positive demand only if $\tilde{p}_B < t(1-2a)$. The joint profits on η_A can be written directly as a function of $\tilde{p}_B$. That is: $\Pi_T \equiv \Pi_{\eta_A}^A + \Pi_{\eta_A}^B = \int_0^{\tilde{x}} [\tilde{p}_B + t(x-1+a)^2 - t(x-a)^2]dx + \tilde{p}_B(1-\tilde{x})$. As $\partial \Pi_T / \partial \tilde{p}_B = 1 - \tilde{p}_B/2t(1-2a)$ is strictly positive when $\tilde{p}_B < t(1-2a)$, joint profits are maximized when firm *B* has zero demand. Provided that firm *A* serves the whole market, the highest price it can set is obtained leaving consumers with zero surplus, that is: $p_{\eta_A}^C = v - t(a-x)^2$. Collusive profits follow:

$$\Pi_{\eta_A}^C = \int_0^1 p_{\eta_A}^C dx = v - t(1/3 + a^2 - a)$$

When deviates, firm *A* cannot obtain more than monopolistic profits: the deviation profits coincide with the collusive profits. Consider now consumers η_B. When firm *A* sets a uniform price while firm *B* sets personalized prices, firm *A* obtains the following profits (Eber, 1997):

sustainability within the Hotelling framework adopt the *grim trigger* strategy. See for example, Friedman and Thisse (1993), Lambertini et al. (2002), Matsumura and Matsushima (2005) and Schultz (2005).

[3] This limitation is due to the fact that finding the highest sustainable couple of price schedules when firms are asymmetric in the knowledge of consumers' preferences (subsets η_A and η_B) is not possible analytically. For a similar approach, see Deneckere (1983).

$\Pi^N_{\eta_B} = t(1-2a)/8$

Under collusion, all consumers are served by firm *B* at price $p^C_{\eta_B} = v - t(1-a-x)^2$. Collusive profits of firm *A* are zero. Suppose firm *A* deviates. It sets the highest uniform price which allows serving all consumers given $p^C_{\eta_B}$. Since the farthest consumer is located at 1, the deviation price is the solution of $u^A_1(p^D_{\eta_B}) = u^B_1(p^C_{\eta_B})$, that is: $p^D_{\eta_B} = v - t(1-a)^2$, which yields:

$\Pi^D_{\eta_B} = v - t(1-a)^2$

The critical discount factor can be obtained inserting the profits functions into $\delta *$ and using the fact that $\eta_A = \eta_B = \eta$:

$$\delta * = \frac{\varphi(\Pi^D_\varphi - \Pi^C_\varphi) + \gamma(\Pi^D_\gamma - \Pi^C_\gamma) + \eta_A(\Pi^D_{\eta_A} - \Pi^C_{\eta_A}) + \eta_B(\Pi^D_{\eta_B} - \Pi^C_{\eta_B})}{\varphi(\Pi^D_\varphi - \Pi^N_\varphi) + \gamma(\Pi^D_\gamma - \Pi^N_\gamma) + \eta_A(\Pi^D_{\eta_A} - \Pi^N_{\eta_A}) + \eta_B(\Pi^D_{\eta_B} - \Pi^N_{\eta_B})} =$$

$$= \begin{cases} \dfrac{2[12v(2\eta+\gamma+\varphi) - t[24\eta(1-a)^2 + \gamma(7-18a+12a^2) + 3\varphi(9-20a+4a^2)]]}{48v(2\eta+\gamma+\varphi) - t[\eta(97-210a+96a^2) + 4[\gamma(7-18a+12a^2) + 3\varphi(7-16a+4a^2)]]} & \text{if} \quad a \le \dfrac{1}{4} \\ \dfrac{2[12v(2\eta+\gamma+\varphi) - t[24\eta(1-a)^2 + \gamma(7-18a+12a^2) + 12\varphi(2-4a+a^2)]]}{48v(2\eta+\gamma+\varphi) - t[\eta(97-210a+96a^2) + 4[\gamma(7-18a+12a^2) + 6\varphi(3-6a+2a^2)]]} & \text{if} \quad a \ge \dfrac{1}{4} \end{cases}$$

Suppose that shared information increases at the expense of unshared information, while the set of unknown consumers does not change. Substituting $\eta = (1-\varphi-\gamma)/2$ into $\delta *$ and taking the derivative with respect to γ, we get:

<u>*Proposition 1.*</u> Given φ, $\partial\delta *\!/\partial\gamma \ge 0$ if and only if:

- $a \ge 1/18, \forall \varphi$, or
- $a \le 1/18 \cap \varphi \ge \overline{\varphi}(a)$, with $\partial\overline{\varphi}(a)/\partial a \le 0$.[4]

Therefore, in this case more information about consumers' preferences decreases (increases) collusion sustainability when products are sufficiently homogenous and/or the set of unknown consumers is sufficiently large (small).

Suppose instead that shared information increases at the expense of the set of unknown consumers, while unshared information is constant. Substituting $\varphi = 1 - 2\eta - \gamma$ into $\delta *$ and taking the derivative with respect to γ, we get:

<u>*Proposition 2.*</u> Given η, $\partial\delta *\!/\partial\gamma \ge 0 \quad \forall \eta, \gamma, a$.[5]

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[4] The expression of the derivative is: $\dfrac{\partial\delta *}{\partial\gamma} = \dfrac{4t[12v(18a-1) + t(7-216a^3+260\varphi-18a(8+65\varphi)+12a^2(28+111\varphi))]}{[96v-t[97+96a^2-41\varphi+6a(-35+11\gamma-29\varphi)+71\varphi]]^2}$ when $a \le 1/4$ and $\dfrac{\partial\delta *}{\partial\gamma} = \dfrac{4t[12v(18a-1) + t(7-216a^3+257\varphi-48a(3+23\varphi)+12a^2(28+93\varphi))]}{[96v-t[97+96a^2-41\varphi+6a(-35+11\gamma-13\varphi)+47\varphi]]^2}$ when $a \ge 1/4$. The expression of the threshold is $\overline{\varphi}(a) = \dfrac{(1-18a)[12v-t(7-18a+12a^2)]}{2t(130-585a+666a^2)}$.

In this case, more information about consumers' preferences always decreases collusion sustainability.

Finally, suppose that unshared information increases at the expense of the set of unknown consumers, while shared information is fixed. Still substituting $\varphi = 1 - 2\eta - \gamma$ into δ^*, but now taking the derivative with respect to η, we observe:

<u>*Proposition 3.*</u> Given γ, $\partial\delta^*/\partial\eta \geq 0$ if and only if:

- $a \leq 13/42, \forall\gamma$, or
- $a \in (13/42, 44/100] \cap \gamma \leq \bar{\gamma}(a)$, with $\partial\bar{\gamma}(a)/\partial a \leq 0$.[6]

Therefore, in this case more information about consumers' preferences makes collusion less sustainable when firms are sufficiently differentiated, while makes collusion more sustainable when firms are sufficiently similar. For intermediate levels of *a*, higher market transparency decreases (increases) collusion sustainability if and only if shared information is sufficiently small (large).

The intuition is the following. When parameter i increases at the expense of parameter j taking parameter z constant, where $i = \gamma, \eta$, $j = \varphi, \eta$ and $z = \gamma, \varphi, \eta$, we observe two effects: both the temptation to deviate ($\Pi^D - \Pi^C$) and the strength of punishment ($\Pi^D - \Pi^N$) increase, the reason being that the possibility to set personalized prices makes deviation more profitable (the deviating firms can better target the deviation prices) but at the same time makes punishment harsher (competition with discriminatory prices is fiercer than competition with uniform prices, as shown, among the others, by Thisse and Vives, 1988). The strength of these two effects depends on the product differentiation degree, *a*, and on the consumers' set which has not been affected by variation of *i*, *z*.[7] When shared information expands at the expense of the set of unknown consumers (Proposition 2), the first effect always dominates, thus determining a higher critical discount factor. Instead, when unshared information expands at the expense of the set of unknown consumers (Proposition 3), the temptation to deviate increases more than the strength of punishment when product differentiation is sufficiently high, while the reverse occurs when product differentiation is sufficiently low. For intermediate values of product differentiation, the first effect dominates only when parameter γ is low enough. At the opposite, when shared information expands at the expense of unshared information (Proposition 1) and when product differentiation is high, the temptation to deviate increases more than the strength of punishment only when parameter φ is sufficiently high: for low values of product differentiation instead, the first effect always dominates.

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[5] The expression of the derivative is: $\frac{\partial\delta^*}{\partial\gamma} = \frac{4t[144v(1-2a) + t[288a^3 - 18a^2(32+37\eta) - 2(42+65\eta) + a(384+585\eta)]]}{[48v - t[84 + 48a^2 - 71\eta + 6a(-32+29\eta+20\gamma) - 56\gamma]]^2}$ when $a \leq 1/4$ and $\frac{\partial\delta^*}{\partial\gamma} = \frac{2t[288v(1-2a) + t[-168 + 576a^3 - 257\eta + 48a(16+23\eta) - 36a^2(32+31\eta)]]}{[48v - t[72 + 48a^2 - 47\eta + 6a(-24+13\eta+12\gamma) - 44\gamma]]^2}$.

[6] The complete expression is $\frac{\partial\delta^*}{\partial\eta} = \frac{t[12v(49-23\gamma+6a(5\gamma-19)) + t(-603+421\gamma-72a^3(5\gamma-19)+12a^2(179\gamma-331)-6a(299\gamma-475))]}{[24v(1+3\eta+\gamma) - t[42+55\eta-6a(16+19\eta-4\gamma)-14\gamma+24a^2(1+3\eta+\gamma)]]^2}$ when $a \leq 1/4$ and $\frac{\partial\delta^*}{\partial\eta} = \frac{2t[6v(49-23\gamma+6a(5\gamma-19)) - t(300+209\gamma+36a^3(5\gamma-19)+a^2(1878-966\gamma)+48a(18\gamma-29))]}{[24v(1+3\eta+\gamma) - t[36+61\eta-6a(12+23\eta)-8\gamma+24a^2(1+3\eta+\gamma)]]^2}$ when $a \geq 1/4$.

The expression of the threshold is $\bar{\gamma}(a) = \frac{6[v(49-114a) - t(50-232a+313a^2-114a^3)]}{6v(23-30a) - t(209-864a+966a^2-180a^3)}$.

[7] Since δ^* is a fraction, parameter *z*, even if constant, does not disappear in the derivative of δ^* with respect to *i*, thus contributing to the sign of the derivative.

3. Conclusions

Using a symmetric Hotelling duopoly, we studied the effect of more information of firms about consumers' preferences on the sustainability of a collusive agreement. We showed that an increase of information may increase or decrease collusion sustainability, depending on which type of information increases (shared information or unshared information), on the characteristics of the information distribution, and on the product differentiation degree.

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