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Atemporal non-expected utility preferences, dynamic consistency and consequentialism

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Abstract

This note studies conditions which allow to maintain a non-expected utility representation (Max-min expected utility and Choquet expected utility), dynamic consistency and consequentialism in an atemporal and purely subjective framework. By contrast with a dynamic set-up, where consistency can be reached with non-expected utility models, we show that both Maxmin expected utility and Choquet expected utility degenerate into an expected utility representation.

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1 Introduction

Non-Expected Utility (NEU) models of choice under uncertainty have generated a growing interest over the last decades among decision theorists. In this paper, we focus on the two most popular approaches, namely the Choquet Expected Utility (CEU) model and the Max-min Expected Utility (MEU) model. In situations of *Knightian uncertainty* or *ambiguity* (i.e. where probability distributions on the outcomes are not given), these models allow to describe Ellsberg-type preferences by taking into account ambiguity and ambiguity attitudes. This is done by assuming that the decision maker's beliefs are not necessarily represented by a single additive prior. But various economic situations involve not only ambiguity, but also sequential resolution of the uncertainty. In these situations, it can be suitable, especially in economic applications, to impose the axioms of *dynamic consistency* and *consequentialism*. Whereas these assumptions are automatically satisfied by the classical Expected Utility (EU) model from Savage (1954), they have to be explicitly stated when ambiguity do matter.

Several works (Hammond 1988, Segal 1990, Karni and Schmeidler 1991, Volij 1994) have shown that consequentialism, dynamic consistency and reduction of compound lotteries together imply the independence axiom in risky situations. Under knightian uncertainty, in a set-up *à la Savage*, Ghirardato (2002) proved that the EU model can be obtained by keeping Savage postulates, except the sure-thing principle, replaced by dynamic consistency and consequentialism.

In a dynamic set-up, where the domain of events is restricted to the algebra delivered by a filtration, Sarin and Wakker (1998), Epstein and Schneider (2003), Eichberger, Grant and Kelsey (2005) and Dominiak and Lefort (2010) have shown that CEU and MEU models are not necessarily reduced to the EU model under dynamic consistency and consequentialism.

Nevertheless, in a static set-up, it has not been proved that the EU model can be obtained by adding up dynamic consistency and consequentialism to a NEU representation. The aim of this note is to fill this gap. We show that given these two axioms in an atemporal and purely subjective framework, both CEU and MEU models degenerate into expected utility.

The remainder of the paper is organized as follows. In section 2, we present our set-up and axioms. Sections 3 and 4 report results for CEU and MEU models. Section 5 concludes.

2 Set-up and axioms

We assume that uncertainty is described by a finite *state space* noted S[1] such that $|S| = n \geq 3$. A state in S is represented by s. Subsets of S are called events. Hence $\Sigma = 2^S$ is a σ-algebra. For all E in Σ, the event $S \backslash E$ is denoted E^c. X is an *outcome space* and it is assumed to be an interval from $\mathbb{R}$. We denote by $\mathscr{A} \subseteq X^n = \{f : S \to X\}$ the set of *acts*, that are measurable functions taking only finite values. Throughout we assume that the following asssumption on the structure of X holds:

Assumption 1 (Non-triviality) *There are x_* and x^* in X such that $x^* > x_*$.*

A decision maker (DM) is represented by a class of preference relations $\{\succcurlyeq_E\}_{E\in\Sigma}$ on $\mathscr{A}$. $\succcurlyeq_S$ ($\succcurlyeq$ henceforth) is defined ex-ante, i.e. when no information is given to the DM. For all E in Σ, $\succcurlyeq_E$ compares acts conditionally to E, i.e. if the DM is informed that only $s \in E$ can obtain. We write $f =_E g$ if $f(s) = g(s)$ for all s in E, and $f_E g$ refers to the compound act such that $f_E g =_E f$ and

[1] Assuming that S is finite does not involve any essential loss of generality. Indeed, our results can be extended to the infinite case by imposing some measurability assumptions on S.

$f_E g =_{E^c} g$. $\succ_E$ and $\sim_E$ are defined in the usual way. The class of preference relations $\{\succcurlyeq_E\}_{E\in\Sigma}$ can satisfy several axioms. We are mainly concerned with the two followings:

Axiom 1 (Consequentialism) *For all E in Σ and f, g in $\mathscr{A}$, $f =_E g$ implies $f \sim_E g$.*

It means that the counterfactuals outcomes are not relevant to the DM (for an extensive discussion, see Machina 1989), so that each conditional preference is only dependent on the information received.

Axiom 2 (Dynamic consistency) *For all E in Σ and f, g in $\mathscr{A}$ such that $f =_{E^c} g$, $f \succcurlyeq g$ if and only if $f \succcurlyeq_E g$.*

This says that the DM's preferences does not reverse when new information arrives. From a normative point of view, dynamic consistency is relevant for many reasons. For instance, this avoids money pump argument and allows the information (in the sense of Wakker (1988)) to have a non-negative value.

Note that our set-up implicitly assumes a reduction of compound acts axiom. Indeed, given an event E, we do not distinguish the act $f = (x_1, \ldots, x_n)$ and the compound act $f_E f$.

3 Choquet Expected Utility

An important class of NEU models is the CEU one. In this model, the beliefs are represented by a Choquet capacity, i.e. a set function $\nu : \Sigma \to [0;1]$ such that $\nu(\emptyset) = 0$, $\nu(S) = 1$ and $\forall A, B \in \Sigma, A \subseteq B \Rightarrow \nu(A) \leq \nu(B)$. Wakker (1989) axiomatizes the Choquet Expected utility representation with a finite state space in a Savage framework.

Definition 1 (CEU) *The preference relation $\succcurlyeq$ is represented by a* Choquet Expected Utility *functional if there exist a unique capacity ν and a continuous and strictly increasing function $u : X \to \mathbb{R}$, unique up to a positive affine transformation, s.t. the value of any act $f = (x_1 \,\text{on}\, D_1, \ldots, x_n \,\text{on}\, D_n)$, with $x_1 \geq \ldots \geq x_n$, is given by:*

$$\int_S u(f)d\nu = u(x_1)\nu(D_1) + \sum_{i=2}^{n} u(x_i)(\nu(\cup_{j=1}^{i} D_j) - \nu(\cup_{j=1}^{i-1} D_j))$$

Moreover, given an event E, the conditional CEU of f, noted $\int_S u(f)d\nu_E$, uses the conditional set function for $\nu(.)$ given E, denoted by $\nu_E(.)$, and the utility function $u(.)$.

Theorem 1 *Let $\{\succcurlyeq_E\}_{E\in\Sigma}$ be a class of preference relations $\mathscr{A}$. Then the following statements are equivalent:*

(i) $\{\succcurlyeq_E\}_{E\in\Sigma}$ satisfy (CEU) and axioms 1-2;

(ii) There exist a probability measure $p : \Sigma \to [0;1]$ s.t. $\succcurlyeq$ is represented by $\int_S u(.)dp$. Moreover, for all E in Σ, $\succcurlyeq_E$ is also represented by an expected utility form $\int_E u(f)dp_E$, where $p_E(.)$ is the conditional probability given by $p_E(.) = \frac{p(.\cap E)}{p(E)}$.

Proof Part A $(ii) \Rightarrow (i)$. This part is straightforward.

Part B $(i) \Rightarrow (ii)$. Let $f = (x_1 \,\text{on}\, D_1, \ldots, x_n \,\text{on}\, D_n)$, $x_1 \geq \ldots \geq x_n$, and $g = (x'_1 \text{ on } D_1, \ldots, x'_l \,\text{on}\, D_l, x_{l+1} \text{ on } D_{l+1}, \ldots, x_n \,\text{on}\, D_n)$, $x'_1 \geq \ldots \geq x'_l \geq x_{l+1} \geq \ldots \geq x_n$, be two acts s.t. $f \sim g$. The utility $u(.)$ keeps these rank-orderings because it is strictly increasing. We define any event E_l by $E_l = (\cup_{i=1}^{l} D_i)$ and $E_l^c = (\cup_{i=l+1}^{n} D_i)$. Because $\succcurlyeq$ satisfies CEU, we have:

$$\int_S u(f) d\nu = \int_S u(g) d\nu \tag{1}$$

By definition of the Choquet integral of utility, equation (1) holds if and only if:

$$\begin{aligned} &u(x_1)\nu(D_1) + \sum_{i=2}^{n} u(x_i)(\nu(\cup_{j=1}^{i} D_j) - \nu(\cup_{j=1}^{i-1} D_j)) \\ &= u(x'_1)\nu(D_1) + \sum_{i=2}^{l} u(x'_i)(\nu(\cup_{j=1}^{i} D_j) - \nu(\cup_{j=1}^{i-1} D_j)) + \sum_{i=l+1}^{n} u(x_i)(\nu(\cup_{j=1}^{i} D_j) - \nu(\cup_{j=1}^{i-1} D_j)) \end{aligned}$$

Note that $u(f)$ and $u(g)$ are comonotonic real-valued functions, i.e. that verify $[u(f(s)) - u(f(s'))][u(g(s)) - u(g(s'))] \geq 0$ for all s and s' in S. It is well known that the capacity is additive on these acts (see Dellacherie 1971). Let us define a decision weight p as $\forall i = 2, \ldots, n, p_i = \nu(\cup_{j=1}^{i} D_j) - \nu(\cup_{j=1}^{i-1} D_j)$ and $p_1 = \nu(D_1)$. By construction, we have $\forall l \leq n, \sum_{i=1}^{l} p_i = \nu(\cup_{i=1}^{l} D_i)$. Then equation (1) holds if and only if:

$$\int_S u(f) dp = \int_S u(g) dp \tag{2}$$

that implies:

$$\sum_{i=1}^{l} u(x_i) p_i = \sum_{i=1}^{l} u(x'_i) p_i \tag{3}$$

By dynamic consistency and consequentialism, equation (1) gives:

$$\int_{E_l} u(f) d\nu_{E_l} = \int_{E_l} u(g) d\nu_{E_l} \tag{4}$$

Now consider a pair of acts $f' = (x_1, \ldots, x_l, x'_{l+1}, \ldots, x'_n)$ and $g' = (x'_1, \ldots, x'_n)$, with $x_1 \geq \ldots \geq x_{l-1} \geq x'_{l+1} \geq x_l \geq x'_{l+2} \geq \ldots \geq x'_n$ and $x'_1 \geq \ldots \geq x'_n$. By consequentialism, equation (4) holds if and only if:

$$\int_{E_l} u(f') d\nu_{E_l} = \int_{E_l} u(g') d\nu_{E_l} \tag{5}$$

and then dynamic consistency implies:

$$\int_S u(f') d\nu = \int_S u(g') d\nu \tag{6}$$

Because $u(g')$ is comonotonic with $u(f)$ and $u(g)$, we have:

$$\int_S u(g') d\nu = \int_S u(g') dp \tag{7}$$

Equations (1) and (6) together imply:

$$\int_S u(g)d\nu + \int_S u(f')d\nu = \int_S u(f)d\nu + \int_S u(g')d\nu \tag{8}$$

and adding up (2) and (7) with (6) gives:

$$\int_S u(g)dp + \int_S u(f')d\nu = \int_S u(f)dp + \int_S u(g')dp \tag{9}$$

Equation (9) can be rewritten as:

$$\sum_{i=1}^{n} u(x_i')p_i + \sum_{i=1}^{l} u(x_i)p_i$$
$$= \sum_{i=1}^{l} u(x_i')p_i + \sum_{i=1}^{l-1} u(x_i)p_i + u(x_{l+1}')(\nu(\cup_{j=1,j\neq l}^{l+1} D_j) - \nu(\cup_{j=1}^{l-1} D_j)) + u(x_l)(\nu(\cup_{j=1}^{l+1} D_j)$$
$$- \nu(\cup_{j=1,j\neq l}^{l+1} D_j)) + \sum_{i=l+2}^{n} u(x_i')p_i$$

hence:

$$\forall l \leq n, \nu(\cup_{j=1}^{l+1} D_j) - \nu(\cup_{j=1,j\neq l}^{l+1} D_j) = p_l \tag{10}$$

Because equation (10) holds for all $l \leq n$ and for all D in Σ, we have $\nu = p$ on Σ. This implies that for all disjoints A and B we have $\nu(A) + \nu(B) = \nu(A \cup B)$, hence $\nu(.)$ is finitely additive. Therefore, the value of any act f is given by $\int_S u(f)dp$. Moreover, the same implication holds on each conditional preference: for all D and E in Σ, $p(E)\nu_E(D) = p(D \cap E) \Rightarrow \nu_E(D) = \frac{p(D\cap E)}{p(E)} = p_E(D)$ hence $\succcurlyeq_E$ is represented by $\int_E u(.)dp_E$. □

4 Max-min Expected Utility

In this section, we suppose that the DM considers a non-empty, compact and convex set $\mathscr{P}$ of finitely additive probability measures, and maximizes expected utility with respect to the lower probability. MEU over Savage acts has been axiomatized by Casadesus-Masanell et al. (2000). We define the MEU representation:

Definition 2 (MEU) *The preference relation $\succcurlyeq$ is represented by a* Max-min Expected Utility *functional if there exist a non-empty, compact and convex $\mathscr{P}$ of finitely additive probability measure on Σ and a continuouys and strictly increasing function $u : X \to \mathbb{R}$, unique to a positive affine transformation, s.t. the value of any act $f = (x_1, \ldots, x_n)$ is given by:*

$$\min_{p\in\mathscr{P}} \int_S u(f)dp = \sum_{i=1}^{n} u(x_i)p_i$$

where $p \in \arg\min_{p\in\mathscr{P}} \int_S u(f)dp$.

Moreover, given an event E, the conditional MEU of f, noted $\min\limits_{p_E \in \mathscr{P}_E} \int\limits_S u(f) dp_E$, uses the conditional set of probabilities $\mathscr{P}_E$ and the utility function $u(.)$.

Theorem 2 *Let $\{\succsim_E\}_{E\in\Sigma}$ be a class of preference relations $\mathscr{A}$. Then the following statements are equivalent:*

(i) $\{\succsim_E\}_{E\in\Sigma}$ satisfy (MEU) and axioms 1-2;

(ii) There exist a probability measure $p : \Sigma \to [0,1]$ s.t. $\succsim$ is represented by $\int\limits_S u(.)dp$. Moreover, for all E in Σ, $\succsim_E$ is also represented by an expected utility form $\int\limits_E u(f) dp_E$, where $p_E(.)$ is the conditional probability given by $p_E(.) = \frac{p(.\cap E)}{p(E)}$.

Proof Part A $(i) \Rightarrow (ii)$. This implication is straightforward.

Part B $(ii) \Rightarrow (i)$. First consider events $E = (\cup_{i=1}^{l})$ and $E^c = (\cup_{i=l+1}^{n} D_i)$ and any act $f = (x_1 \,\text{on}\, D_1, \ldots, x_n \,\text{on}\, D_n)$, then let us fix any real numbers $x = u^{-1}(\min\limits_{p_E \in \mathscr{P}_E} \int\limits_S u(f) dp_E)$ and $y = u^{-1}(\min\limits_{p_{E^c} \in \mathscr{P}_{E^c}} \int\limits_S u(f) dp_{E^c})$ and define acts $g = x_E y$ and $h = f_E y$. By definition, $f =_E f_E y$. Consequentialism implies:

$$\min_{p_E \in \mathscr{P}_E} \int\limits_S u(f) dp_E = \min_{p_E \in \mathscr{P}_E} \int\limits_S u(h) dp_E = u(x) \tag{11}$$

and dynamic consistency gives:

$$\min_{p \in \mathscr{P}} \int\limits_S u(h) dp = \min_{p \in \mathscr{P}} \int\limits_S u(g) dp \tag{12}$$

By definition, $\min\limits_{p_{E^c} \in \mathscr{P}_{E^c}} \int\limits_S u(h) dp_{E^c} = u(y)$, hence

$$\min_{p_{E^c} \in \mathscr{P}_{E^c}} \int\limits_S u(h) dp_{E^c} = \min_{p_{E^c} \in \mathscr{P}_{E^c}} \int\limits_S u(f) dp_{E^c} \tag{13}$$

and a last application of dynamic consistency gives:

$$\min_{p \in \mathscr{P}} \int\limits_S u(h) dp = \min_{p \in \mathscr{P}} \int\limits_S u(f) dp \tag{14}$$

Together with equation (12), equation(14) gives:

$$\min_{p \in \mathscr{P}} \int\limits_S u(f) dp = \min_{p \in \mathscr{P}} \int\limits_S u(g) dp \tag{15}$$

equivalent to

$$\min_{p \in \mathscr{P}} \int\limits_S u(f) dp = \min_{p \in \mathscr{P}} \int\limits_S \left(\min_{p_E \in \mathscr{P}_E} \int\limits_S u(f) dp_E, \min_{p_{E^c} \in \mathscr{P}_{E^c}} \int\limits_S u(f) dp_{E^c} \right) dp \tag{16}$$

If

$$m \in \arg\min_{p\in\mathscr{P}} \int_S \left(\min_{p_E\in\mathscr{P}_E} \int_S u(f)dp_E, \min_{p_{E^c}\in\mathscr{P}_{E^c}} \int_S u(f)dp_{E^c} \right) dp \tag{17}$$

then:

$$\min_{p\in\mathscr{P}} \int_S u(f)dp = m(E)\left(\sum_{i=1}^{l} \frac{p_i}{\sum_{i=1}^{l} p_i} u(x_i)\right) + m(E^c)\left(\sum_{j=l+1}^{n} \frac{p'_j}{\sum_{j=l+1}^{n} p'_j} u(x_j)\right)$$

where $\forall i = 1,\ldots,l, \forall j = l+1,\ldots,n, \frac{p_i}{\sum_{i=1}^{l} p_i} = p_E(D_i)$ and $\frac{p'_j}{\sum_{j=l+1}^{n} p'_j} = p'_{E^c}(D_j)$ represent the conditional measures on E and E^c, so that $p_E \in \arg\min_{p_E\in\mathscr{P}_E} \int_S u(f)dp_E$ and $p'_{E^c} \in \arg\min_{p_{E^c}\in\mathscr{P}_{E^c}} \int_S u(f)dp_{E^c}$. We define the probability measure $\pi(.)$ as: $\forall i = 1,\ldots,l, \pi_i = p_i \frac{\sum_{i=1}^{l} m_i}{\sum_{i=1}^{l} p_i}$ and $\forall j = l+1,\ldots,n, \pi_j = p'_j \frac{\sum_{j=l+1}^{n} m_j}{\sum_{j=l+1}^{n} p'_j}$, with $\sum_{i=1}^{l} m_i = m(E)$ and $\sum_{j=l+1}^{n} m_i = m(E^c)$. Therefore, under dynamic consistency and consequentialism, we have[2]:

$$\pi \in \arg\min_{p\in\mathscr{P}} \int_S u(f)dp \tag{18}$$

Repeating this operation with events $E' = (\cup_{i=1,i\neq l}^{l+1} D_i)$ and $E'^c = (\cup_{j=l,j\neq l+1}^{n} D_j)$ yields:

$$\min_{p\in\mathscr{P}} \int_S u(f)dp = \min_{p\in\mathscr{P}} \int_S \left(\min_{p_{E'}\in\mathscr{P}_{E'}} \int_S u(f)dp_{E'}, \min_{p_{E'^c}\in\mathscr{P}_{E'^c}} \int_S u(f)dp_{E'^c} \right) dp \tag{19}$$

and defining m' as:

$$m' \in \arg\min_{p\in\mathscr{P}} \int_S \left(\min_{p_{E'}\in\mathscr{P}_{E'}} \int_S u(f)dp_{E'}, \min_{p_{E'^c}\in\mathscr{P}_{E'^c}} \int_S u(f)dp_{E'^c} \right) dp \tag{20}$$

gives:

$$\begin{aligned}\min_{p\in\mathscr{P}} \int_S u(f)dp &= \left(\sum_{i=1}^{l+1} m'_i - m'_l\right)\left(\sum_{i=1}^{l+1} \frac{q_i}{\sum_{i=1}^{l+1} q_i - q_l} u(x_i)\right) - \left(\sum_{i=1}^{l+1} m'_i - m'_l\right) \frac{q_l}{\sum_{i=1}^{l+1} q_i - q_l} u(x_l) \\ &+ \left(\sum_{j=l}^{n} m'_j - m'_{l+1}\right)\left(\sum_{j=l}^{n} \frac{q'_j}{\sum_{j=l}^{n} q'_j - q'_{l+1}} u(x_j)\right) - \left(\sum_{j=l}^{n} m'_j - m'_{l+1}\right) \frac{q'_{l+1}}{\sum_{j=l}^{n} q'_j - q'_{l+1}} u(x_{l+1})\end{aligned}$$

where $\forall i = 1,\ldots,l-1,l+1, \forall j = l,l+2,\ldots,n, \frac{q_i}{\sum_{i=1}^{l+1} q_i - q_l} = q_{E'}(D_i)$ and $\frac{q'_j}{\sum_{j=l}^{n} q'_j - q'_{l+1}} = q'_{E'^c}(D_j)$ represent the conditional measures on E' and E'^c, respectively. We define the probability measure π' as $\forall i = 1,\ldots,l-1,l+1, \pi'_i = q_i \frac{\sum_{i=1}^{l+1} m'_i - m'_l}{\sum_{i=1}^{l+1} q_i - q_l}$ and $\forall j = l,l+2,\ldots,n, \pi'_j = q'_j \frac{\sum_{j=l}^{n} m'_j - m'_{l+1}}{\sum_{j=l}^{n} q'_j - q'_{l+1}}$. Hence dynamic consistency and consequentialism together imply:

$$\pi' \in \arg\min_{p\in\mathscr{P}} \int_S u(f)dp \tag{21}$$

[2] Note that the set $\mathscr{P}$ is rectangular in the sense of Epstein and Schneider (2003).

Because equations (18) and (21) hold for all $l \leq n$, and thus for all π and π' in $\mathscr{P}$, we have $\forall \pi, \pi' \in \mathscr{P}, \pi = \pi'$ on Σ. Therefore, $\succcurlyeq$ is represented by $\int_S u(.)dp$, where $p(.)$ is a unique probability measure. Obviously, if $\mathscr{P} = \{p\}$, then $\mathscr{P}_E = \{p_E\}$ for all E in Σ so that the update from $p(.)$ is unique. Therefore, each conditional preference $\succcurlyeq_E$ can be represented by an expected utility form $\int_S u(.)dp_E$, as claimed. □

5 Conclusion

Our results suggest that some assumptions must be released to simultaneously preserve dynamic consistency and non-additive beliefs.

One way consists to assume that the DM is faced to a given and fixed dynamic choice problem. In this case, the domain of events is restricted to a filtration and MEU and CEU preferences do not degenerate into EU. Notably, Epstein and Schneider (2003) give a necessary and sufficient condition to recursivity of the MEU representation: the set of priors must be rectangular.

However, in an atemporal set-up, the only way to preserve dynamic consistency and non-additive beliefs is to relax consequentialism. Concerning MEU preferences, Hanany and Klibanoff (2007) drop consequentialism by assuming that past choices influence conditional choices. The sets of conditional measures are restricted in order to avoid dynamically inconsistent conditional probabilities. Concerning CEU preferences, Kast, Lapied and Toquebeuf (2008) relate conditioning and comonotony (or antimonotony) of information with the valued random variable. The DM minimizes the role of information (pessimism). She uses the Bayes updating rule when information is comonotonic with the valued act and Dempster-Shafer updating rule when information is antimonotonic with it. As a consequence, counterfactuals outcomes do matter and hence consequentialism does not hold.

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