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Sunspots, whether they are risk or uncertainty, cannot matter in the static Arrow-Debreu economy

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Abstract

It is well-known that in the static Arrow-Debreu economy with complete markets, extrinsic uncertainty cannot matter. This paper re-examines this result when agents preferences exhibit aversion to Knightian uncertainty. We then show that extrinsic uncertainty still cannot matter.

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1. Introduction

In the last three decades, many studies on macroeconomics have demonstrated effects of psychological factors such as "animal spirits," "market psychology," or "sunspots" on equilibrium prices and allocations. In these studies, there is a significant concept: *extrinsic uncertainty.*

Extrinsic uncertainty means the combination of nonfundamental shocks and extraneous beliefs in effects of the shocks. One of important studies on extrinsic uncertainty is Cass and Shell (1983), which established extrinsic uncertainty can matter in the overlapping generations economy with restricted participation but cannot matter in the static Arrow-Debreu (AD) economy with complete markets. An intuition, here, is that the latter result may be broken when we precisely distinguish, as Knight (1921), between risk, a situation that one can assign a unique probability measure to future events, and uncertainty, a situation that one cannot do so.

It is well-known that under Bayesian model of decision-making with the Savage (1954) axioms, Knightian distinction between risk and uncertainty is no longer meaningful. With Ellsberg (1961) as a turning point, however, we can now avail a number of *extensions* of the Bayesian model with the Savage axioms that admit a distinction between risk and uncertainty. The important one of such extensions is the Choquet Expected Utility (CEU) axiomatized by Schmeidler (1989). Furthermore, he showed that under the "uncertainty aversion" axiom, a decision-maker maximizes CEU with a convex nonadditive measure. CEU is, for example, applied to portfolio selection (Dow and Werlang, 1992), to asset pricing (Epstein and Wang, 1994), or to search (Nishimura and Ozaki, 2004), and accounts for several puzzles.

The purpose of this paper is to investigate whether the intuition mentioned above is true. For this purpose, we re-examine the result in Cass and Shell (1983, Proposition 3) under the assumption that agents' preferences exhibit aversion to Knightian uncertainty, that is, agents maximize their CEU with convex nonadditive measures.

The base of our model is the almost same with the reduced-form model in Cass and Shell (1983). However, we assume that agents maximize CEU with a common convex nonadditive measure. We then demonstrate that the intuition is false. That is, we show that, even when agents' preferences exhibit aversion to Knightian uncertainty, extrinsic uncertainty still cannot matter in the static AD economy with complete markets.

Our result indicates the fact as follows: In the static AD economy, extrinsic uncertainty, whether it is risk or uncertainty, cannot matter. It also indicates, mathematically, the robustness of Cass and Shell (1983, Proposition 3) with respect to agent's confidence: We can weaken the additivity of agents' beliefs to show that extrinsic uncertainty does not matter in the static AD economy.

The organization of this paper is as follows. Section 2 recall some definitions on capacities and Choquet integrals. In Section 3, we present the model. Section 4 presents the main result of this paper. Section 5 provides concluding remarks.

2. Nonadditive measures and Choquet integrals

We first recall some definitions about capacities and Choquet integrals.

Let $S = \{1, \ldots, k\}$ for some positive integer k and 2^S be the power set of S. Also let Δ_S be the set of probabilities on S.

A set function $\nu : 2^S \to [0, 1]$ a *nonadditive measure* if $\nu(\emptyset) = 0$, $\nu(S) = 1$, and $\nu(A) \leq \nu(B)$ for $A, B \in 2^S$ such that $A \subseteq B$. A nonadditive measure ν is *convex* if $\nu(A \cup B) + \nu(A \cap B) \geq \nu(A) + \nu(B)$ for all $A, B \in 2^S$. The *core* of a nonadditive measure ν is the set

$$\mathrm{core}(\nu) = \{\pi \in \Delta_S | \, \pi(A) \geq \nu(A) \quad \text{for all} \quad A \in 2^S\}.$$

It is well-known that if ν is convex, then $\mathrm{core}(\nu) \neq \emptyset$.

Let $B(S, \mathbb{R})$ be the space of bounded functions of S to $\mathbb{R}$. For each $X \in B(S, \mathbb{R})$, the Choquet integral of X with respect to a nonadditive measure ν is defined by

$$\mathbb{E}_\nu[X(s)] = \int_0^\infty \nu(\{s | X(s) \geq t\}) dt + \int_{-\infty}^0 [\nu(\{s | X(s) \geq t\}) - 1] dt,$$

where integrals on the right hand side are in the sense of Riemann integrals. In particular if X is such that $X(1) \leq \cdots \leq X(k)$, then

$$\mathbb{E}_\nu[X(s)] = \sum_{j=1}^{k-1} X(j)[\nu(\{j, \ldots, k\}) - \nu(\{j+1, \ldots, k\})] + X(k)\nu(\{k\}).$$

The following result is well-known.

Fact 1(Schmeidler, 1986). If ν is a convex nonadditive measure, then, for every $X \in B(S, \mathbb{R})$, $\mathbb{E}_\nu[X(s)] = \min_{\pi \in \mathrm{core}(\nu)} \mathbb{E}_\pi[X(s)]$.

At last, define $D_\nu(X) = \arg\min_{\pi \in \mathrm{core}(\nu)} \mathbb{E}_\pi[X(s)]$.

3. The Model

The base of our model is the almost same with the reduced-form model in Cass and Shell (1983). There are one standard commodities, H agents, $h = 1, \ldots, H$, and two possible states of nature, $s = \alpha, \beta$, that is, $S = \{\alpha, \beta\}$. As Cass and Shell (1983), we identify α with the state "sunspots" and β with the state "no sunspots."

Let $x_h(s) \geq 0$ denote consumption by agent h in state s and $x_h = (x_h(\alpha), x_h(\beta))$. Agent h is endowed with prospective goods $\omega_h \in \mathbb{R}^S_{++}$. Assume that $\omega_h(\alpha) = \omega_h(\beta)$, that is, endowments are not affected by sunspot activity. Let $\omega = \sum_h \omega_h$.

The preference of agent h is denoted by $v_h(x_h)$, which is defined over his/her prospective consumption plans. We assume, differently from Cass and Shell (1983), that agents maximize CEU with a common nonadditive measure ν, that is, they maximize $v_h(x_h) = E_\nu[u_h(x_h(s))]$, where $u_h : \mathbb{R}_+ \to \mathbb{R}$ is increasing, strictly concave, and continuously differentiable with $\lim_{x \downarrow 0} u'(x) = \infty$.

An allocation $(x_h)_h$ is *feasible* if $\sum_h x_h \leq \sum_h \omega_h$. A feasible allocation $(x_h)_h$ is a *Pareto efficient allocation* (PEA) if there is no feasible allocation $(x'_h)_h$ such that $v_h(x'_h) \geq v_h(x_h)$ for every $h = 1, \ldots, H$ and $v_i(x'_i) > v_i(x_i)$ for some $i = 1, \ldots, H$. A feasible allocation $(x_h)_h$ is a *rational expectations equilibrium allocation* (REEA) if there is a contingent-claims price vector $p = (p(\alpha), p(\beta))$ such that:

(i) for every x'_h, $v_h(x'_h) > v_h(x_h)$ implies $p \cdot x'_h > p \cdot \omega_h$; and

(ii) $\sum_h x_h(s) = \sum_h \omega_h(s)$ for each $s \in S$.

We say *extrinsic uncertainty matters* (or simply, sunspots matter) to an allocation if $x_h(\alpha) \neq x_h(\beta)$ for some h, and, otherwise, say *extrinsic uncertainty do not matter* (or simply, sunspots do not matter). An allocation $(x_h)_h$ is *comonotone* if, for all h, h' and all $s, s' \in S$, $(x_h(s) - x_h(s'))(x_{h'}(s) - x_{h'}(s')) \geq 0$.

4. Results

This section presents and proves our results. To prove the main result, we need several preparations.

Proposition 1 *If ν is convex, then an allocation $(x_h)_h$ is a PEA if and only if it is a PEA of an economy in which agents have von Neumann-Morgenstern (vNM) utility index u_h and identical probability over S. In particular, PEAs are comonotone.*

Proof. Similar to Chateauneuf et al. (2000, Proposition 3.1). ■

Proposition 2 *If ν is convex, then an allocation $(x_h)_h$ is a REEA if and only if there exists $\pi \in D_\nu(\omega)$ such that $(x_h)_h$ is a REEA of an economy in which agents have vNM utility index u_h and π as prior.*

Proof. Similar to Dana (2004, Theorem 3.3). ■

Proposition 3 *If ν is convex, then REEAs are PEAs and comonotone.*

Proof. It is well-known that REEAs of an vNM economy are PEAs of the economy. Hence, by Proposition 1 and 2, every REEA is a PEA and comonotone. ■

The main result is that when agents' preferences exhibit aversion to Knightian uncertainty, there is no REEA to which sunspots matter.

Proposition 4 *If ν is convex, then there exists no REEA to which sunspots matter.*

Proof. Suppose otherwise. That is, assume that there is some REEA $(x_h)_h$ with an equilibrium price vector p such that $x_i(\alpha) \neq x_i(\beta)$ for some $i = 1, \ldots, H$. We assume w.l.o.g. that $x_i(\alpha) < x_i(\beta)$.

Since $(x_h)_h$ is a REEA, it follows from Proposition 3 that $(x_h)_h$ is comonotone. Hence, for any h, $x_h(\alpha) \leq x_h(\beta)$.

Then, define the alternative allocation $(\bar{x}_h)_h = ((\bar{x}_h(\alpha), \bar{x}_h(\beta)))_h$ by $\bar{x}_h(t) = \mathbb{E}_\nu[x_h(s)]$ for $t \in S$. Since it follows from $x_h(\alpha) \leq x_h(\beta)$ and the definition of REEAs that, for each $s \in S$,

$$\begin{aligned}
\sum_h \bar{x}_h(s) &= \sum_h [(1 - \nu(\{\beta\}))x_h(\alpha) + \nu(\{\beta\})x_h(\beta)] \\
&= (1 - \nu(\{\beta\})) \sum_h x_h(\alpha) + \nu(\{\beta\}) \sum_h x_h(\beta) \\
&= (1 - \nu(\{\beta\})) \sum_h \omega_h(\alpha) + \nu(\{\beta\}) \sum_h \omega_h(\beta) \\
&= \sum_h \omega_h(s).
\end{aligned}$$

Hence $(\bar{x}_h)_h$ is feasible.

Note that $u_i(x_i(\alpha)) < u_i(x_i(\beta))$ because u_i is increasing and $x_i(\alpha) < x_i(\beta)$. It follows from $\bar{x}_i(\alpha) = \bar{x}_i(\beta)$ that

$$\begin{aligned}
v_i(\bar{x}_i) &= \mathbb{E}_\nu[u_i(\bar{x}_i(s))] \\
&= u_i(\mathbb{E}_\nu[x_i(s)]) \\
&= u_i((1-\nu(\{\beta\}))x_i(\alpha) + \nu(\{\beta\})x_i(\beta)) \\
&> (1-\nu(\{\beta\}))u_i(x_i(\alpha)) + \nu(\{\beta\})u_i(x_i(\beta)) \\
&= \mathbb{E}_\nu[u_i(x_i(s))] \\
&= v_i(x_i),
\end{aligned}$$

where the strict inequality results from the strict concavity of u_i. Again by Proposition 3, this, however, contradicts the fact that the REEA $(x_h)_h$ is a PEA. ■

It is obvious that if ν is a (usual) probability, then ν is convex. Hence we can get the following corollary.

Corollary 1 (Cass and Shell, 1983, Proposition 3) *If $\nu \in \Delta_S$, then there exists no REEA to which sunspots matter.*

5. Concluding Remarks

The paper shows that in the static AD economy with complete markets, extrinsic uncertainty, whether it is risk or uncertainty, cannot matter.

Gilboa and Schmeidler (1989) axiomatized the maxmin expected utility (MMEU), i.e., the minimum expected utility overthrough some set of probability measures. Schmeidler (1989) showed that CEU with a convex nonadditive measure is a special case of MMEU. The key point in the proof of the main result is, however, rather the comonotonicity of REEAs than properties of MMEU. Since a REEA is comonotone, we can regard an *arbitrary* agent as an expected-utility maximizer with the probability measure $\pi \in \Delta_S$ defined by $\pi(\{\alpha\}) = 1 - \nu(\{\beta\})$ and $\pi(\{\beta\}) = \nu(\{\beta\})$.

Note that there exists a situation, as Cass and Shell (1983) mentioned, that *intrinsic* uncertainty matters even when we consider Knightian uncertainty. Intrinsic uncertainty cannot matter in the case of no aggregate risk, although REE is indeterminate. It can, however, matter in the case of small aggregate risk (See, for example, Dana (2004, Sec.3)).

In the static model, whether or not sunspots can matter does not depend on Knightian distinction between risk and uncertainty. One of our further researches is to examine, in the dynamic model, whether or not the distinction has effects on results of Cass and Shell (1983).

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