Economics Bulletin

Volume 30, Issue 2

Does the 'Golden Rule of Public Finance' imply a lower long-run growth rate? A clarification

Alfred Greiner
Bielefeld University, 33501 Bielefeld, Germany

Abstract

In a recent paper Minea and Villieu (2009) assert that the 'golden rule of public finance' implies a lower long-run growth rate than the balanced-budget rule. Their contribution is misleading because it is not the 'golden rule of public finance' that generates their result but rather the fact that public debt grows at the same rate as capital and GDP in the long-run in their paper. In this note we demonstrate that the 'golden rule of public finance' yields the same long-run growth rate as the balanced-budget rule provided that public debt asymptotically grows at a smaller rate than capital and GDP.

Citation: Alfred Greiner, (2010) "Does the 'Golden Rule of Public Finance' imply a lower long-run growth rate? A clarification", *Economics Bulletin*, Vol. 30 no.2 pp. 975-982.
Submitted: Mar 31 2010. **Published:** April 01, 2010.

1. Introduction

Most industrialized countries in the world are confronted with high government debt. Therefore, the question of how governments should cope with high public debt to GDP ratios in the future should be a central concern for policy makers.

In a recent paper Minea and Villieu (2009) presented a contribution that addresses this problem in a growth setting. They analyze an endogenous growth model with productive public capital and public debt and show that a balanced-budget rule (BBR) implies a higher long-run growth rate compared to a situation where the government runs permanent deficits to finance public investment in a productive public capital stock. However, they only consider the situation where public debt grows at the same rate as all other economic variables in the long-run and the situation where public debt is constant while all other variables grow at a constant rate. Therefore, they erroneously conclude that the 'golden rule of public finance' (GRPF) leads to a lower long-run growth rate than the balanced-budget rule. This needs clarification because it is misleading.

The fallacy in the paper by Minea and Villieu (2009) is that the GRPF does not mean that public debt rises at the same rate as capital and GDP in the long-run. The GRPF simply states that the government may run deficits in order to finance productive public investment. But it does not say anything about how large deficits should be and about the rate at which public debt may grow in the long-run.

Assume that a government follows the GRPF and runs permanent deficits but sets the primary surplus such that public debt asymptotically grows at a smaller rate than capital and GDP. Then, the ratio of public debt to capital and the ratio of public debt to GDP converge to zero in the long-run, just as for the BBR. Consequently, the GRPF and the BBR yield the same long-run balanced growth rate and are identical asymptotically. Thus, it is not the GRPF that generates the result in the paper by Minea and Villieu (2009) but rather the fact that public debt grows at the same rate as capital and GDP in their model.

In the next section we present a formal model in order to rigorously derive our result.

2. The model

The model we consider is basically the same as the one in Minea and Villieu (2009) where a closed economy is considered with a producer-consumer representative household and with a government. First, we describe the household sector.

2.1 The household

The household maximizes the discounted stream of utility arising from per-capita consumption, $c(t)$, over an infinite time horizon subject to its budget constraint. Labour is assumed to be constant and set equal to one so that all variables give per-capita quantities.[1] The maximization problem can be written as

$$\max_c \int_0^\infty e^{-\beta t} S\left(c^{1-1/S} - 1\right)/(S-1), dt, \tag{1}$$

subject to

$$\dot{b} + \dot{k} = (1-\tau)\,y + r\,b - c - \delta^k\,k. \tag{2}$$

The parameter β is the subjective discount rate and S gives the constant inter-temporal elasticity of consumption. For $S = 1$ the utility function is given by the natural logarithm, $\ln c$, and the dot over a variable stands for the derivative with respect to time, d/dt.

Macroeconomic production y is given as in Futagami et al. (1993) by

$$y = k^{1-\alpha} g^{\alpha}, \tag{3}$$

with k private capital, g public capital and $(1-\alpha) \in (0,1)$ is the elasticity of production with respect to private capital. The parameter $\tau \in (0,1)$ in the budget constraint is a flat rate tax on output and $\delta^k > 0$ gives the depreciation rate of private capital. Government bonds are denoted by b and the return to government bonds is r that equals the marginal product of private capital in equilibrium.

Solving the optimization problem of the household leads to the Keynes-Ramsey rule given by

$$\frac{\dot{c}}{c} = S\left((1-\tau)(1-\alpha)(g/k)^{\alpha} - \beta - \delta^k\right). \tag{4}$$

Further, the transversality condition $\lim_{t\to\infty} e^{-\beta t}(k+b)c^{-1/S} = 0$ must hold.

2.2 The government

The government in our economy receives tax revenues and revenues from issuing government bonds it then uses for public investment, i_p, for public consumption, c_p, and for interest payments on outstanding debt, rb. As concerns public consumption we assume that this variable is set by the government such that it is a certain fraction of output c_r so that $c_p = c_r\,y$ holds, with $c_r < \tau < 1$. Further, public consumption does neither yield utility nor raise productivity but is only a waste of resources. The period budget constraint of the government, then, is obtained as

$$\dot{b} = rb - (\tau - c_r)y + i_p = rb - ps, \tag{5}$$

with ps giving the primary surplus of the government. In addition, the government is not allowed to play a Ponzi game, i.e. it must obey the inter-temporal budget constraint given by

$$b(0) = \int_0^\infty e^{-\int_0^\mu r(\nu)d\nu} ps(\mu)d\mu \leftrightarrow \lim_{t\to\infty} e^{-\int_0^t r(\mu)d\mu} b(t) = 0. \tag{6}$$

[1]From now on we omit the time argument t if no ambiguity arises.

As regards the primary surplus, ps, we assume that the government sets the primary surplus relative to GDP such that it is a positive linear function of the public debt to GDP ratio. There are two justifications for this assumption. First, this government behaviour guarantees that the inter-temporal budget constraint is fulfilled.[2] The economic intuition behind it is that a positive reaction of the primary surplus to rising public debt, relative GDP respectively, makes the time path of the debt to GDP ratio a mean-reverting process so that this ratio remains bounded.

The second justification for this assumption is that there is strong empirical evidence that governments indeed set the primary surplus according to that rule. For example, Bohn (1998) found a positive and statistically significant response of the primary surplus to rising debt ratios for the USA. Greiner et al. (2007) have demonstrated that there is statistical significance for this rule to hold true for countries of the EURO area. Therefore, integrating this assumption into a theoretical model seems to be justified.

Thus, the primary surplus relative to GDP can be written as

$$\frac{ps}{y} = \rho \, \frac{b}{y}, \tag{7}$$

where $\rho \in \mathbb{R}_{++}$ is constant. The parameter ρ determines how strong the primary surplus reacts to changes in public debt and must be strictly positive so that sustainability of public finances is given.

It should be pointed out that this rule regarding the primary surplus makes public investment a 'semi-endogenous' variable, for a fixed tax rate τ and for a fixed ratio of public consumption to GDP c_r. Public investment i_p, then, is given by[3]

$$i_p = (\tau - c_r)y - \rho b. \tag{8}$$

We call it 'semi-endogenous' because, on the one hand, it is endogenous since it is determined by public debt and by GDP. On the other hand, the government has some discretionary scope since it can set ρ, provided $\rho > 0$ holds.

Using equation (7) the period budget constraint of the government can be written as

$$\dot{b} = r\, b - \rho\, b. \tag{9}$$

Public investment, finally, raises the stock of public capital according to the following differential equation,

$$\dot{g} = i_p - \delta^g \, g = (\tau - c_r)y - \rho b - \delta^g g, \tag{10}$$

where $\delta^g > 0$ is the depreciation rate and where we used equation (8).

[2]A formal proof of that statement can be found in Bohn (1995) for discrete time and in Greiner (2008, 2009) for example for continuous time. We do not repeat it here.

[3]Note that $c_r < \tau$ must hold for i_p to be positive.

2.3 Analysis of the model

The economy-wide resource constraint is obtained by combining the budget constraint of the household, equation (2), with the period budget constraint of the government, given by equation (10). It is easily seen that this leads to

$$\dot{k} = y - c - \delta^k k - \tau y + \rho b, \tag{11}$$

where public investment plus public consumption is $i_p + c_p = \tau y - \rho b$ and with output y given by (3) and r equal to $r = \partial y / \partial k = (1 - \alpha)(g/k)^\alpha$. Thus, in equilibrium the economy is completely described by the equations (4), (9), (10) and (11).

Before we analyze our economy we give a formal definition of a balanced growth path (BGP).

Definition 1 *A balanced growth path (BGP) is a path such that the economy is in equilibrium and such that consumption, private capital and public capital grow at the same strictly positive constant growth rate, i.e.* $\dot{c}/c = \dot{k}/k = \dot{g}/g = \gamma$, $\gamma > 0$, $\gamma = constant$, *and either*
$\dot{b} = 0$ *(BBR) or*
$\dot{b}/b = \gamma_b$, *with* $0 < \gamma_b < \gamma$, $\gamma_b = constant$ *(GRPF).*

Definition 1 shows that on the BGP consumption, c, private capital, k, and public capital, g, grow at the same strictly positive growth rate as usual in endogenous growth models.[4] Public debt, however, is either constant implying that the government runs a balanced budget, i.e. it follows the BBR, or public debt grows but at a smaller rate than all other economic variables. In the latter case, the government runs permanent deficits which are due to public investment so that the government follows the GRPF in this case.[5]

To model the BBR we set the parameter ρ in (7) equal to the marginal product of private capital, i.e. $\rho = r$. It is immediately seen that this gives the BBR where $\dot{b} = 0$ holds. Public deficits are equal to zero in this case so that public investment is given by $i_p = \tau y - c_r y - rb$. To model the GRPF we set ρ such that $\rho < r$ holds. Equation (7), then, shows that the government runs deficits in this case so that $\dot{b} > 0$ holds.

Now, assume that we have two economies with identical initial conditions with respect to public debt, b_0, and with respect to GDP, y_0, and that have the same fiscal parameters, τ, c_r, where one obeys the BBR and the other follows the GRPF. Setting $\rho = r - \epsilon/b$, with $\epsilon > 0$, equation (8) gives $i_p = \tau y - c_r y - rb + \epsilon$ under the GRPF demonstrating that public investment, compared to the BBR, is larger by the amount ϵ that is deficit

[4]Of course GDP, y, grows at the same rate on the BGP as capital and consumption.

[5]We do not consider the scenario where public debt asymptotically grows at the same rate as all other variables. That case has been extensively studied in Greiner (2007).

financed and that causes public deficits.[6] Further, $\rho > r(1-(1-\tau)S) + S(\beta+\delta^k)$ must hold so that $\dot{b}/b < \dot{c}/c$ is fulfilled. This means that ρ must not become too small because otherwise on the BGP the growth rate of public debt would exceed that of consumption and of private and public capital. For example, in case of a logarithmic utility function, i.e. for $S = 1$, the GRPF is obtained when ρ is set such that $\tau\, r + \beta + \delta^k < \rho < r$ holds which is equivalent to $r - \gamma < \rho < r$.

In order to analyze our model we define the new variables $x = g/k$, $z = b/k$ and $v = c/k$. Differentiating these variables with respect to time gives

$$\dot{x} = x\left((\tau - c_r)x^{\alpha-1} - \rho z/x - \delta^g - (1-\tau)x^\alpha + v + \delta^k - \rho z\right),\ x_0 > 0 \tag{12}$$

$$\dot{z} = z\left((1-\alpha)x^\alpha - \rho x^\alpha/z - (1-\tau)x^\alpha + v + \delta^k - \rho z\right),\ z_0 > 0 \tag{13}$$

$$\dot{v} = v\left(S(1-\tau)(1-\alpha)x^\alpha - S(\beta+\delta^k) - (1-\tau)x^\alpha + v + \delta^k - \rho z\right),\ v_0 > 0. \tag{14}$$

A rest point of (12)-(14), that is variables[7] $x^\star$, $v^\star$, $z^\star$ such that $\dot{x} = \dot{z} = \dot{v} = 0$ holds, gives a BGP for our economy where the BBR and the GRPF are modelled by setting ρ to appropriate values as demonstrated above.

Proposition 1 gives the result as concerns existence and stability of a BGP under the BBR and under the GRPF and shows that these two rules imply the same long-run growth rate.

Proposition 1 *Assume that the depreciation rate of public capital is sufficiently small. Then, there exists a unique saddle point stable BGP for the BBR and for the GRPF. The long-run growth rate under the BBR is equal to that under the GRPF.*

Proof: See appendix.

Proposition 1 shows that both under the BBR and under the GRPF the economy is characterized by a unique saddle point stable growth path. Further, the two rules yield the same long-run growth rate. The latter is due to the fact that under both rules the ratio of public debt to private capital asymptotically converges to zero. Under the BBR public debt is constant while private capital monotonically rises so that the ratio tends to zero. Under the GRPF public debt grows in the long-run but at a rate that is lower than the growth rate of capital so that the ratio of public capital to GDP converges to zero, too.

[6] That holds at least initially at $t = 0$ when the initial conditions with respect to y and b are identical under the BBR and the GRPF.

[7] The * denotes BGP values and we neglect the case $x^\star = v^\star = 0$.

3. Conclusion

In this contribution we have demonstrated that the balanced budget rule and the 'golden rule of public finance' yield the same long-run growth rate if public debt grows at a lower rate than capital and GDP asymptotically. Hence, in a growing economy the decisive aspect about public finances is not whether the government runs deficits to finance public investment but the question of whether public debt grows less than capital and GDP in the long-run.

Appendix

Proof of proposition 1

To prove this proposition for the BBR, we set $\rho = (1-\tau)(1-\alpha)x^{\alpha}$ and $z^{\star} = 0$. $z^{\star}$ can be set to zero because a constant level of public debt and a monotonously rising private capital stock imply that the ratio of public debt to capital converges to zero.

Then, setting $\dot{x} = 0$ and solving this equation with respect to v gives v as a function of x and of the parameters. Substituting this function for v in $\dot{v}/v$ gives $q(x, \cdot) = S(1-\tau)(1-\alpha)x^{\alpha} - (\tau - c_r)x^{\alpha-1} - S(\beta + \delta^k) + \delta^g$, where $z^{\star} = 0$ was used. Recalling that $\tau > c_r$ holds, it is easily seen that $\lim_{x\to 0} q(x, \cdot) = -\infty$, $\lim_{x\to\infty} q(x, \cdot) = +\infty$ and $\partial q(\cdot)/\partial x > 0$. Thus, existence of a unique $x^{\star}$ is shown. This gives a unique value for i_p/g on the BGP leading to positive growth if the depreciation rate δ^g is not too large, which is immediately seen from (10).

To show saddle point stability, we compute the Jacobian matrix evaluated at the rest point of (12)-(14). The Jacobian is given by

$$\mathrm{J} = \begin{bmatrix} \partial\dot{x}/\partial x & \partial\dot{x}/\partial z & \partial\dot{x}/\partial v \\ 0 & \partial\dot{z}/\partial z & 0 \\ \partial\dot{v}/\partial x & \partial\dot{v}/\partial z & \partial\dot{v}/\partial v \end{bmatrix}.$$

One eigenvalue of this matrix is $\lambda_1 = \partial\dot{z}/\partial z = -\dot{k}/k = -\gamma$. Thus, we know that one eigenvalue, λ_1, is negative. Further, it is easily shown that $(\partial\dot{x}/\partial x)(\partial\dot{v}/\partial v) - (\partial\dot{x}/\partial v)(\partial\dot{v}/\partial x) = vx\left[(\tau - c_r)(\alpha-1)x^{\alpha-2} - S(1-\tau)(1-\alpha)\alpha x^{\alpha-1}\right] < 0$ holds, so that complex conjugate eigenvalues are excluded. The determinant of J is given by $\det \mathrm{J} = \partial\dot{z}/\partial z[(\partial\dot{x}/\partial x)(\partial\dot{v}/\partial v) - (\partial\dot{x}/\partial v)(\partial\dot{v}/\partial x)] > 0$. Since the product of the eigenvalues equals the determinant, $\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = \det \mathrm{J} > 0$, and because of $\lambda_1 < 0$, we know that two eigenvalues are negative and one is positive.

For the GRPF we again set $z^{\star} = 0$. Note that we can set $z^{\star} = 0$ because in our definition of the GRPF the level of public debt grows at the rate γ_b and private capital grows at the rate γ on the BGP, with $\gamma_b < \gamma$. Thus, the time path of the ratio of public debt to capital along the BGP is given by $z(t) = z(0)e^{(\gamma_b - \gamma)t}$ which converges to zero for $t \to \infty$, with $z(0)$ the initial debt to capital ratio.

Then, we proceed analogously to the BBR so that a solution of $q(\cdot) = 0$ with respect to x again gives the value $x^\star$ on the BGP. The function $q(\cdot)$ is the same as under the BBR, i.e. $q(x, \cdot) = S(1-\tau)(1-\alpha)x^\alpha - (\tau - c_r)x^{\alpha-1} - S(\beta + \delta^k) + \delta^g$, so that the GRPF yields the same $x^\star$ as the BBR and, thus, the same value for i_p/g giving identical long-run growth rates.

The Jacobian matrix is the same as for the GRPF because of $z^\star = 0$, except for $\partial \dot{z}/\partial z$. $\partial \dot{z}/\partial z$ now is given by $\partial \dot{z}/\partial z = \lambda_1 = \dot{b}/b - \dot{k}/k < 0$, because of $\dot{b}/b < \dot{k}/k$ on the BGP. In particular, the determinant is again positive implying that two eigenvalues are negative and one is positive. □

References

Bohn, H. (1995) "The sustainability of budget deficits in a stochastic economy" *Journal of Money, Credit and Banking*, **27,** 257-271.

Bohn, H. (1998) "The behaviour of U.S. public debt and deficits" *Quarterly Journal of Economics* **113,** 949-963.

Futagami, K., Morita Y. and A. Shibata (1993) "Dynamic analysis of an endogenous growth model with public capital" *Scandinavian Journal of Economics* **95,** 607-625.

Greiner, A., Köller U. and W. Semmler (2007) "Debt sustainability in the European Monetary Union: Theory and empirical evidence for selected countries" *Oxford Economic Papers* **59,** 194-218.

Greiner, A. (2007) "An endogenous growth model with public capital and sustainable government debt" *Japanese Economic Review* **58,** 345-361.

Greiner, A. (2008) "Does it pay to have a balanced government budget?" *Journal of Institutional and Theoretical Economics* **164,** 460-476.

Greiner, A. (2009) "Economic growth, public debt and welfare: Comparing three budgetary rules" available at SSRN: http://ssrn.com/abstract=1354035, forthcoming in *German Economic Review.*

Minea, A. and P. Villieu (2009) "Borrowing to finance public investment? The 'golden rule of public finance' reconsidered in a growth setting" *Fiscal Studies* **30,** 103-133.