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A Measure of Variability in Comovement for Economic Variables: a Time-Varying Coherence Function Approach

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Abstract

In this paper, we suggest a different dynamic measure of comovement which is unlike previous studies allowing to test instability in comovement between two non stationary economic time series. We use the frequency approach, which is based on evolutionary spectral analysis, to estimate the Time-Varying Coherence Function (TVCF). Then we test stability in both cross-spectra and TVCF by detecting endogenously various break points in each function. Applying this new methodology to the GDP growth rate of the US and UK, we get an interesting result about period of business cycle convergence and divergence for these economies.

1   Introduction

Not only does economy grow, but it fundamental structure also change across the time. Stressed by their opening to the international markets, these changes in several economies supported the increase in the frequency of shocks in such markets. In fact, economic series are more affected by shocks, therefore testing instability in mean, variance and covariance is needed at the first stage of statistical modelling. These shocks influence not only the series but also the nature and the degree of her relationship with other time series. For this reason, the presence of break dates in series can prejudice comovement estimation because of neglecting presence of structural change points. We can assume that this instability can affect the comovement between series.

The comovement estimation in economic series among different countries is assumed to be constant, but the presence of structural changes can seriously affect this assumption. To avoid such a situation, we must test the homogeneity of the series initially and, in the case of the presence of the breakpoint, split the initial sample into sub-periods. We then compare a degree of comovement between periods, an ad-hoc sub-samples are analysed by computing parametric and non-parametric statistics from which some conclusions are drawn. Our goal is to test this instability and to detect when comovements have changed. We therefore propose a new modelling strategy and measurement that avoid many of the shortcomings of previous studies. We introduce a different measure that allows comovement for increase or decrease in different types of cycles.

In previous studies, comovement between series is assumed time independent. Croux *et al.* (2001) propose new measures called cohesion and cross-cohesion to evaluate the issue of comovement of business cycles in Europe and the United States. The two statistics are based on the average weight of a coherence. Their proposed measures are appropriate for stationary processes only. Hughes Hallett & Richter (2002, 2004, 2006, 2008), however, use a 'short time Fourier transform' (STFT) to estimate power spectra and time-varying coherence. The STFT assumes local stationarity by applying a Kalman filter to the chosen AR(p) model. Hughes Hallett & Richter (2004) start from a time-varying spectrum for each country separately estimated using the Kalman filter. They then use these spectra to derive the time-varying coherence between national cycles. They include different long lag order of two growth rate GDP series. Hughes Hallett & Richter (2006) employ an AR model with dummy variables for outliers. This prior adjustment can affect spectrum and cross-spectrum estimation. They suggest that the UK has been diverging from the Euro zone at all frequencies except those at the long end of the spectrum, while increasing its coherence with the US at most frequencies. Using the evolutionary spectral approach we do not need any prior adjustment to the series in contrast with the Fourier approach which does need a prior adjustment. This latter methodology is inappropriate for non-stationary time series and choosing an excessive order of AR can bias coherence estimation. Zhao *et al.* (2005) estimate the Time-Varying Coherence Function (TVCF) based on the use of the Short-Time Fourier Transform (STFT). In their work, a TVCF was obtained by reformulating an ARMA model. Their TVCF values, however, can be greater than one. This fact happens because the linear Time-Varying Transfer Function (TVTF) is not suitable for capturing nonlinear characteristics of sudden breaks, especially in low frequencies. The spectral approach, however, adheres particularly to the description of the cycle's fluctuations and the cycle type. For this reason the spectral analysis is more appropriate than the Fourier analysis. Through our measure of comovement technique, we can not only decompose business cycle into their component cycles but also allow them to vary over time. Moreover, our methodology outperform previous literature through its capacity to estimate coherence between series without looking in advance to see if they are stationary or not and without any previous treatment of the series. Ombao & Van Bellegem (2008) develop a data adaptive procedure to estimate the time-evolutionary coherence but their method need time series with length of at least $T = 1024$. This fact presents a serious handicap for its use in economic time series.

In the present study, we use the evolutionary spectral analysis introduced by Priestley (1965-1996). We study the comovement between series and determine the structural change-points based on two tests: the TVCF test and the evolutionary cross-spectrum density test. Our measure of comovement can deter-

mine the nature of the dynamic correlation process for short run (high frequencies) and also for long run (low frequencies).

The advantage of the frequency approach is that it detects the variability in the synchronisation process in different frequencies. With this additional information we can know which cycles and periods are more synchronised than others. In this paper, we present an alternative empirical methodology to estimate TVCF based on spectral analysis, and to identify the break dates collected by low and high frequencies. If the break date is detected in low frequency, then the change is of long run, but if it is detected in high frequency it corresponds to a change of short run. The Time-Varying Coherence Function (TVCF) statistics can be easily calculated for sub-periods and can therefore provide insight into the evolution of synchronisation over time. Our study clearly shows the change in comovement between the two series. Moreover, we define endogenously periods of synchronisation or desynchronisation. This point has not been investigated before.

This paper is organised as follows. In Section 2, we introduce the theory of evolutionary spectral analysis, followed by a presentation of the coherence function in Section 3. In Section 4, we present the TVCF test and the evolutionary cross-spectrum density test, and perform an emprical study. Section 5 concludes.

2 Theory of Evolutionary Spectral Analysis

There are two distinct approaches to analysing time series: Spectral Approach (frequency approach) and Temporal Approach (dynamic approach). The advantage of the spectral approach is its simplicity in the visibility of the periodicity within the series. Thus it does not require an upstream treatment. Unlike in the temporal approach, the analysis of the time series is based on the assumption of the stationarity in covariance after the elimination of any trends. For this reason more researchers have been interested in time frequency analysis (Ahamada & Boutahar, 2002; Ahamada & Ben Aïssa, 2003; Ben Aïssa & Boutahar & Jouini, 2004, etc.).

The spectral approach presented by Priestley (1965-1996) became the reference for frequency analysis because of its respect to the properties of the ideal spectrum, namely unicity, positivity, the estimate from one realisation, etc. (Loynes, 1968). Many studies in the frequency domain show the ability of spectral analysis to identify the characteristics of non-stationary series.'Spectral analysis shows de- composes the variance of a sample of data across different frequencies' (see Prietley (1991), page 215). Based on the assumption of local stationarity, this literature has been extended to the Wigner-Ville distribution, namely the 'Short time Fourier transform' (STFT).

The frequency domain approach improves our analysis in six ways. First, it does not depend on any particular detrending technique. Second, there is no deletion of short or long cycles, so their importance relative to business cycle frequencies remains clear. Third, the coherence measure generalises simple correlation or concordance measures. Fourth, convergence or divergence periods are detected endogenously and their character is specified. Fifth, this approach can be applied to stationary or non-stationary processes. Finally, it performs our time-varying analysis at different frequencies simultaneously. Applying it, we are able to separate different dynamic components of the comovement; we obtain the short, medium and long run behaviour of the generating process of the comovement series.

2.1 Presentation of the Spectral Theory (Priestley, 1965-1996)

We denote $\{X_t\}$ the observable time series. The evolutionary aspect of the spectrum is related to the non-stationarity in this series which follows an oscillatory process.

$$X_t = \int_{-\pi}^{\pi} A_X(w,t)e^{iwt}dZ_X(w), \tag{1}$$

3

where for each w, the sequence $\{A_X(w,t)\}$, as function of t admits a maximum Fourier transform (in module) in zero with $\{Z_X(w)\}$ being an orthogonal process on $[-\pi,\pi]$, $E[dZ_X(w)]=0$, $E[|dZ_X(w)|^2]=d\mu_X(w)$ and $\mu_X(w)$ a measure. The evolutionary spectral density of $\{X_t\}$ is the function $S_X(w,t)$, with the respect to the family of sequences $\mathfrak{F} \equiv \{A_X(w,t)e^{iwt}\}$, and it's defined as follows:

$$S_X(w,t) = \frac{dH_X(w,t)}{dw}, -\pi \leq w \leq \pi, \tag{2}$$

where $dH_X(w,t) = |A_X(w,t)|^2 d\mu_X(w)$. The variance $\sigma^2_{X,t}$ of $\{X_t\}$ at time t depends on the evolutionary spectral density $S_X(w,t)$ through the following equation

$$\sigma^2_{X,t} = Var(X_t) = \int_{-\pi}^{\pi} S_X(w,t)dw, \tag{3}$$

2.2   Estimate of the Evolutionary Spectrum ($S_X(w,t)$)

Estimation of $S_X(w,t)$ is performed by use of two windows $\{g_u\}$ and $\{w_v\}$

$$\widehat{S_X}(w,t) = \sum_{v \in Z} w_v |U_{t-v}(w)|^2, \tag{4}$$

where $U_t(w) = \sum_{u \in Z} g_u X_{t-u} e^{-iw(t-u)}$. We choose $\{g_u\}$ and $\{w_r\}$ as follows:

$$g_u = \left\{ \begin{array}{ll} 1/(2\sqrt{h\pi}) & if \ |u| \leq h \\ 0 & if \ |u| > h \end{array} \right| \tag{5}$$

$$w_v = \left\{ \begin{array}{ll} 1/T' & if \ |v| \leq T'/2 \\ 0 & if \ |v| > T'/2 \end{array} \right| \tag{6}$$

Here $h=7$ and $T'=20$[1]. According to Priestley (1988) we have $E(\widehat{S_X}(w)) \approx S_X(w,t)$, $var(\widehat{S_X}(w))$ decreases when T' increases and $\forall(t_1,t_2), \forall(w_1,w_2)$,
$cov(\widehat{S_X}(w_1,t_1),\widehat{S_X}(w_2,t_2)) \approx 0$. If one of the two following conditions (j) and (jj) is satisfied[2].

$$(j) \ \ |t_1 - t_2| \geq T', \quad (jj) \ \ |w_1 \pm w_2| \geq \pi/h \tag{7}$$

Let $S_X^{iw} = log(S_X(w,t_i))$ and $\Lambda_X^{iw} = log(\widehat{S_X}(w,t_i))$. From Priestley (1988), we have:

$$\Lambda_X^{iw} \approx S_X^{iw} + e_X^{iw}, \tag{8}$$

where the sequence $\{e_X^{iw}\}$ is approximately normal, uncorrelated and identically distributed.

3   Presentation of the Coherence Function

Coherence can be interpreted as the squared linear correlation coefficient for each frequency of the spectra of two series. The time approach gives the instantaneous coherent peaks between two series and describes their patterns over time. For our case it is therefore crucial to know whether coherence has increased or not between the cycles of some economies so we can conclude if the economies are suitable to create a monetary union, present a high level of interdependency, etc.

In the frequency domain, we define the correlation between two components in frequency as the coherence ($\mathcal{K}$). Let us consider the case of bivariate stationary process $\{X_t;Y_t\}$. The two components are

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[1]This is the choice adopted by Artis *et al.* (1992), Ahamada *et al.* (2002).
[2]See Priestley (1969 and 1988) for more details

zero-mean stochastic processes. We denote $S_X(w)$ and $S_Y(w)$, $-\pi \leq w < \pi$, the spectral density functions and $S_{XY}(w)$ the cospectrum. Each of the two processes can be written as

$$X_t = \int_{-\pi}^{\pi} A_X(w)e^{iwt}dZ_X(w), \quad Y_t = \int_{-\pi}^{\pi} A_Y(w)e^{iwt}dZ_Y(w), \tag{9}$$

where $\{Z_k(w)\}$ $(for\ k = X, Y)$ is an orthogonal process,

$$E[dZ_X(w_1)\overline{dZ_Y(w_2)}] = \begin{cases} 0 & \text{if} \quad 0 \quad w_1 \neq w_2 \\ dw & \text{if} \quad w_1 = w_2 = w \end{cases}, \tag{10}$$

where $\overline{Z}$ denotes the complex conjugate of Z. The cospectrum of X and Y is given by $S_{XY}(w) = A_X(w)\overline{A_Y(w)}$. It is well-known that

$$S_{XY}(w) = C_{XY}(w) - iQ_{XY}(w), \tag{11}$$

where

$$C_{XY}(w) = \Re\{S_{XY}(w)\} \quad \text{and} \quad Q_{XY}(w) = -\Im\{S_{XY}(w)\}$$

are the Real Cospectrum (the gain) and the Quadrature Spectrum (the phase) respectively. $\Re$ and $\Im$ are the real and the imaginary parts of the cross-spectrum.

The coherence $\mathcal{K}_{XY}(w)$ at frequency w is given by

$$\mathcal{K}_{XY}^2(w) = \frac{C_{XY}^2(w) + Q_{XY}^2(w)}{S_X(w)S_Y(w)}, \tag{12}$$

Eq(11) verifies the inequality coherence, $C_{XY}^2(w) + Q_{XY}^2(w) \leq S_X(w)S_Y(w)$. Therefore $\mathcal{K}_{XY}^2(w)$ cannot be greater than one. $\mathcal{K}_{XY}^2(w)$ shows the degree of comovement between two series at the frequency w and it is analogous to the coefficient of the correlation between the two samples in the time domain. The coherence measure provides more detailed information than the conventional correlation and concordance measures through her capacity to decompose series into different cycle (frequencies).

3.1 A Time-Varying Coherence Function (TVCF)

In the time domain, dynamic correlation can give us some key responses to the comovement degree between series, but the choice of window can seriously affect the dynamic correlation pattern. Essaadi *et al.* (2009) shows that results of dynamic correlation had been significantly influenced by the width of the window. To overcome this limitation, we propose to measure the comovement variability by the frequency approach. Time-varying coherence function estimates not only a degree of comovement over time but also behaviour in each frequency. Through the TVCF, we can estimate the relationship between two economies, and their change over time in each frequency. Our goal is to locate the dates of changes in comovement between two series in time and frequency domains.

We introduce a new method to estimate time-varying coherence functions (TVCF) for economic series. The aim of this paper is to present a more illuminating interpretation of coherence and to develop a novel point-wise statically adaptive procedure for estimating the time-evolutionary coherence of non-stationary time series.

3.1.1 Estimate of Evolutionary Cross-Spectra ($S_{XY}(w, t)$)

Priestley & Tong (1973) extends the theory of evolutionary spectra to the case of bivariate non-stationary processes. Consider two oscillatory component processes, $(X_t; Y_t)$: we can write

$$X_t = \int_{-\pi}^{\pi} A_X(w,t)e^{iwt}dZ_X(w), \quad Y_t = \int_{-\pi}^{\pi} A_Y(w,t)e^{iwt}dZ_Y(w). \tag{13}$$

Where:

$$
\begin{aligned}
E[dZ_X(w_1)\overline{dZ_X(w_2)}] &= E[dZ_Y(w_1)\overline{dZ_Y(w_2)}] \\
&= E[dZ_X(w_1)\overline{dZ_Y(w_2)}] = 0 \quad \text{when } w_1 \neq w_2. \\
E[|dZ_X(w)|^2] &= d\mu_X(w), \ E[|dZ_Y(w)|^2] = d\mu_Y(w) \ \text{ and} \\
E[dZ_X(w)\overline{dZ_Y(w)}] &= d\mu_{XY}(w).
\end{aligned}
\tag{14}
$$

Let $\mathfrak{F}_X$ and $\mathfrak{F}_Y$ denote respectively the families of oscillatory functions $\{\phi_X(w,t) \equiv A_X(w,t)e^{iwt}\}$ and $\{\phi_Y(w,t) \equiv A_Y(w,t)e^{iwt}\}$. We define the evolutionary power cross-spectrum at t with respect to the families $\mathfrak{F}_X$ and $\mathfrak{F}_Y$, $dH_{XY}(w,t)$, by

$$dH_{XY}(w,t) = A_X(w,t)\overline{A_Y(w,t)}d\mu_{XY}(w). \tag{15}$$

We note that when we may choose $\mathfrak{F}_X \equiv \mathfrak{F}_Y$, equation (15) takes on the special form

$$dH_{XY}(w,t) = |A(w,t)|^2 d\mu_{XY}(w), \tag{16}$$

where $A(w,t) \equiv A_X(w,t) \equiv A_Y(w,t)$.

According to Priestley (1988), in the non-stationary case cross-spectrum is time-varying and is defined as $dH_{XY}(w,t)$. By virtue of the Cauchy-Schwarz inequality, we have

$$|dH_{XY}(w,t)|^2 \leq dH_X(w,t)dH_Y(w,t), \quad \text{for all } t \text{ and } w. \tag{17}$$

We can write

$$dH_{XY}(w,t) = S_{XY}(w,t)\,dw, \tag{18}$$

where $S_{XY}(w,t)$ is the evolutionary cross-spectrum density function.

$$S_{XY}(w_j,t) = C_{XY}(w_j,t) - iQ_{XY}(w_j,t), \tag{19}$$

where

$$
\begin{aligned}
C_{XY}(w_j,t) &= \Re\{S_{XY}(w_j,t)\} \ \text{and} \\
Q_{XY}(w_j,t) &= -\Im\{S_{XY}(w_j,t)\}
\end{aligned}
\tag{20}
$$

are the Real Time-Varying Cospectrum (the gain) and the Time-Varying Quadrature Spectrum (the phase) respectively. $\Re$ and $\Im$ are the real and the imaginary parts of the time-varying cross-spectrum.

Following Priestley (1965-1996), in an evolutionary spectral theory, we propose a time-varying cross-spectrum estimator using spectral density. Estimation of $S_{XY}(w,t)$ is obtained by use of the 'double window technique' $\{g_u\}$ and $\{w_v\}$ given in (5) and (6).

$$\widehat{S_{XY}}(w,t) = \sum_{v \in Z} w_v U_X(w,t-v)U_Y(w,t-v), \tag{21}$$

where $U_X(w,t) = \sum_{u \in Z} g_u X_{t-u}e^{-iw(t-u)}$ and $U_Y(w,t) = \sum_{u \in Z} g_u Y_{t-u}e^{-iw(t-u)}$.

Here $h = 7$ and $T' = 20$. According to Priestley (1988), we have $E(\widehat{S_{XY}}(w)) \approx S_{XY}(w,t)$, $var(\widehat{S_{XY}}(w))$ decreases when T' increases and $\forall(t_1,t_2), \forall(w_1,w_2)$, $cov(\widehat{S_{XY}}(w_1,t_1), \widehat{S_{XY}}(w_2,t_2)) \approx 0$, if one of the two conditions (j) and (jj) of (7) is satisfied.

3.1.2 Estimation of the Coherence of the Non-stationary Spectra

In this section, we estimate the TVCF by using evolutionary spectral density. First, let the observable bivariate time series be $(X_t; Y_t)$ which are not necessarily stationary. Their time-varying spectra are denoted by $S_X(w, t)$ and $S_Y(w, t)$ respectively. The time-varying cross-spectrum given by (19) therefore has a polar representation

$$S_{XY}(w_j, t) = A_{XY}(w_j, t) \, exp\{i\theta_{XY}(w_j, t)\}, \tag{22}$$

this function allows us to compute the time-varying cross-amplitude in the following way

$$
\begin{aligned}
A_{XY}(w_j, t) &= |S_{XY}(w_j, t)| \\
&= [C_{XY}^2(w_j, t) + Q_{XY}^2(w_j, t)]^{1/2}
\end{aligned}
\tag{23}
$$

and the time-varying phase spectrum

$$\theta_{XY}(w_j, t) = \arctan[\frac{-Q_{XY}(w_j, t)}{C_{XY}(w_j, t)}]. \tag{24}$$

The Time-Varying Magnitude Squared Coherence is given by

$$\mathcal{K}_{XY}^2(w_j, t) = \frac{A_{XY}^2(w_j, t)}{S_X(w_j, t) S_Y(w_j, t)}. \tag{25}$$

4 Empirical Study

The TVCF is a useful tool for studying problems of business cycle synchronisation; to investigate short-run and long-run dynamic properties of multiple time series. The importance of this time-varying effect is to conclude if we have common cycles between the analysed series and to determine periods and frequencies when they diverge or converge. Convergence of the business cycle can approve existence of an optimum currency area. We examine the structural change in comovement using the time-varying coherence function.

Chauvet & Potter (2001) argue that US business cycles cannot be assumed to be constant. Hence, the spectrum of US GDP growth rate would no longer be constant over time owing to the changing distribution of weights associated with each of the elementary cycles. Evolutionary spectral analysis allows us not only to decompose movements of output series into their component cycles but also takes those cycles as they vary over time in importance and characteristics.

4.1 Presentation of the Data

The empirical study are based on gross domestic product (GDP) growth rate series of two countries: the United-States (US) and the United Kingdom (UK). GDP growth rate is calculated by the first difference of the logarithm of the quarterly GDP, as follows:

$$X_t = \Delta(\log(GDP_t)) = \log(\frac{GDP_t}{GDP_{t-1}}). \tag{26}$$

The data are sampled over the period from 1960Q1 to 2006Q4 (yielding 187 observations). We obtain the data from *International Financial Statistics* data statistics, published by the IMF (GDP deflator (2000=100)). In this section on synchronisation nonlinearity, GDP is used as a measure of economic activity. We restrict our analysis to bilateral links in order to avoid multicollinearity between series. Note that for the evolutionary spectral estimation necessity, we lose ten observations at the beginning and ten at the end. Therefore we apply a different test to $T = T^* - 20$ [3].

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[3] For both spectral and cross-spectra density estimation we lose 10+10 observations.

Following the (j) and (jj) conditions we choose $\{t_i\}$ and $\{w_j\}$ as follows:

$$\{t_i = 18 + 20i\}_{i=1}^{I} \ \ where \ \ I = \left[\frac{T^*}{20}\right] \ \ and \ \ T^* \ \ the \ \ sample \ \ size, \tag{27}$$

$[x]$ denotes the integer part of x.

$$\{w_j = \frac{\pi}{20}(1 + 3(j-1))\}_{j=1}^{7}. \tag{28}$$

Following (jj) condition, we inspect instability in these frequencies: $\pi/20$, $4\pi/20$, $7\pi/20$, $10\pi/20$, $13\pi/20$, $16\pi/20$ and $19\pi/20$.

From the evolutionary spectral figure (2) we can conclude the change in importance of the low frequency cycles for the US and UK economies. This result is in line with that of Hughes Hallett and Richter (2006). The evolutionary spectrum seems to have similar forms but the importance of the low frequencies component in the UK is higher than that of the US. The two economies react approximately in the same way at the same time to the shocks but to different degrees. As figure 2 shows, US economy is more stable compared to the UK one. This difference seems to be the source of the difference in amplitude in the low frequencies component. Compared with the time-varying coherence series, the dynamic cross-spectrum series is more stable at all frequencies. This is caused by the change in each evolutionary spectrum series of the studied countries. For this reason, it's recommended to test stability in both coherence and cross spectra statistics. Our methodologies allow us to understand more if change in comovement was caused by a change in relation between series and/or to the change in their own variance.

4.2 Spectral and Cross-spectra Density Stability Test

The test of the structural changes of the spectral density amongst other things makes it possible to collect the instability of the coherence function. Using the wavelets and the theory of the local stationary process, Sachs *et al.* (2000) proposed a test to examine the stationarity of the auto-covariance function. Ahamada & Boutahar (2002) studied the stability of the spectral density around each time and each frequency by using the test of T_w. We follow them to propose a nonparametric test for the stationarity of covariance based on the stability of the evolutionary spectral density. We extend their work to a sequential test of the stability in evolutionary cross-spectrum density.

4.2.1 Presentation of the Tests CS_w

From the Cusum Test approach, we test the stability of the evolutionary cross-spectrum, by adopting the same strategy as Ahamada & Boutahar (2002) used in their test of the stability in evolutionary spectral density. For each of the two series of data $\{X_t\}_{t=1}^{T^*}$ and $\{Y_t\}_{t=1}^{T^*}$ we can associate a time frequency cross-spectrum density $S_{XY}(w,t)$. Let $\{t_i\}_{i=1}^{I}$ a set of size I representing a time scale in which all element respect the condition (j) of (7).

$$\begin{aligned} S_{XY}^{iw} &= \log(|S_{XY}(w,t_i)|) \\ &= \log(A_{XY}(w,t_i)) \\ &= \log([C_{XY}^2(w,t_i) + Q_{XY}^2(w,t_i)]^{1/2}). \end{aligned} \tag{29}$$

Let $\Lambda_{XY}^{iw} = \log(|\widehat{S_{XY}(w,t_i)}|)$, $\mu_w = \frac{1}{I}\sum_{i=1}^{I}\Lambda_{XY}^{iw}$, $\hat{\sigma}_w^2 = \frac{1}{I}\sum_{i=1}^{I}(\Lambda_{XY}^{iw} - \mu_w)^2$ and $\delta_r^w = \frac{1}{\hat{\sigma}_w\sqrt{I}}\sum_{i=1}^{r}(\Lambda_{XY}^{iw} - \mu_w)$ when $r = 1, ..., I$. According to Priestley (1988), we have:

$$\Lambda_{XY}^{iw} \approx S_{XY}^{iw} + e_{XY}^{iw}, \tag{30}$$

where the sequence $\{e_{XY}^{iw}\}$ is approximately normal, uncorrelated and identically distributed. We apply the Cusum test to the constancy of the evolutionary cross-spectrum density $S_{XY}(w,t)$.

Theorem 1. *Let $CS_w = max_{r=1,...,I}|\delta_r^w|$. Then, under the null hypothesis of stationarity of X_t and Y_t, the limiting distribution of CS_w is given by:*

$$F_1(a) = 1 - 2\sum_{k=1}^{\infty}(-1)^{k+1}exp(-2k^2a^2), \tag{31}$$

through the statistic of Kolmogorov-Smirnov we calculate the critical value C_α, ie. $Pr(CS_w > C_\alpha) = \alpha$.[4]

4.2.2   Procedure for Detecting Breakpoints in Evolutionary Cross-Spectra

For each w we calculate the statistic CS_w and we reject the stability of cross-spectral density at level α if $CS_w > C_\alpha$. Therefore, when this statistic reaches its maximum in r_{max}, this point is considered a potential structural change date. We split the sample into two subsamples, and we do the same work in each of the subsample. If $CS_w < C_\alpha$ in the sub-period we accept the stability of the density function in w. In each subsample i, $r_{i,max}$ is an estimator of breakpoint if the null hypothesis of stability is rejected. We can therefore detect more than one break date for each frequency w. The test presented above has two main advantages in detecting structural change in both time and frequency. It detects endogenously structural change dates and reports the occurrence of the dates of break.

4.3   Time-Varying Coherence Function Stability Test (Bai & Perron, 1998-2003b)

Some techniques have recently been developed to test multiple structural breaks. We adopt Bai & Perron's (1998, 2003b) test to detect a mean-shift in TVCF. Using GAUSS software, we obtain estimates by running the code created by Bai & Perron (1998, 2003b). The choice of this type of model is motivated by TVCF characteristics. The graphical pattern of this statistic seems to be affected only by breaks in mean. Employing Bai & Perron's test (1998) allows us to determine endogenously break dates when the change of coherence between two series is significant. In this part, we are interested in breakpoint in the coherence between two economies. We define breakpoint as a change in the underlying relationship of the two economies that occurs as a response to an exogenous event or a change in monetary policy.

4.3.1   The Model and Estimators

We consider the following mean-shift model with m breaks, $(T_1,...,T_m)$:

$$
\begin{aligned}
TVCF_{w_j,t} &= \mu_1 + u_t, \quad t = 1,...,T_1, \\
TVCF_{w_j,t} &= \mu_2 + u_t, \quad t = T_1+1,...,T_2, \\
&\vdots \\
TVCF_{w_j,t} &= \mu_{m+1} + u_t, \quad t = T_m+1,...,T,
\end{aligned}
\tag{32}
$$

for $i = 1,2,...,m+1$, $T_0 = 0$ and $T_{m+1} = T$, where T is the sample size[5]. $TVCF_{w_j,t}$ is the time-varying coherence function in the neighborhood of the w_j frequency. μ_i are the means, and u_t is the disturbance at time t. The breakpoints $(T_1,...,T_m)$ are explicitly treated as unknown. From the ordinary least-squares (OLS) principle Bai & Perron (1998) estimate the vector of the regressor coefficients μ_j $(1 \leq j \leq m+$

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[4]$C_{0.1=1.22}$, $C_{0.05=1.36}$ and $C_{0.01=1.63}$.
[5]$T = T^* - 20 = 187 - 20 = 167$.

1) by minimising the sum of squared residuals $\sum_{i=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} (TVCF_{w_j,t} - \mu_i)^2$. Let $\widehat{TVCF}_{w_j,t} \left(\{T_j\} \right)$ denote the resulting estimate. Substituting it in the objective function and denoting the resulting sum of squared residuals as $S_T(T_1, \ldots, T_m)$, we see that the estimated break dates $(\hat{T}_1, \ldots, \hat{T}_m)$ are such that

$$(\hat{T}_1, \ldots, \hat{T}_m) = \arg \min_{(T_1, \ldots, T_m)} S_T(T_1, \ldots, T_m), \tag{33}$$

where the minimisation is taken over all partitions $(T_1, \ldots, T_m)$ such that $T_i - T_{i-1} \geq O$, where O takes some value.[6]

4.3.2 The Test Statistic and the Model Selection Criteria

This test locates multiple breaks without imposing any prior expectations on the data. The procedure estimates unknown regression coefficients together with the breakpoints when T quarters are available. In order to determine the number of breakpoints, we use the Bayesian Information Criterion (BIC) as suggested by Yao (1988) and defined as follows:

$$BIC(m) = (T^{-1}S_T(\widehat{T}_1, \ldots, \widehat{T}_m)) + p^* T^{-1} \ln(T), \tag{34}$$

where $p^* = 2m + 1$ is the number of unknown parameters. The author shows that, for normal sequence of random variables with shifts in mean, the number of breaks can be consistently estimated.

Bai & Perron (1998) present some asymptotic critical values for the arbitrary small positive number (ε) and the maximum possible number of breaks (M): ($\varepsilon = 0.10$, $M = 8$), ($\varepsilon = 0.15$, $M = 5$), ($\varepsilon = 0.20$, $M = 3$) and ($\varepsilon = 0.25$, $M = 2$). For our empirical computation, we choose ($\varepsilon = 0.15$, $M = 5$) and we use Bai & Perron's (1998, 2003b) algorithm to obtain global minimisers of the squared residuals.

4.4 Results and comments

'Many observers have noted how the shape of economic cycles has varied over time in terms of amplitude, duration and slope: long expansions, short recessions; expanding cycle lengths; steeper expansions than recessions and so on'(Hughes Hallett & Richter, 2006). To capture these features we use an evolutionary spectral approach and we propose a TVCF statistic based on this approach to detect the dynamic relationship between economies. Our study accommodates the possibility of structural breaks in these relationship caused by external events or a change in monetary policy. It also allows us to decompose movements into component cycles and allow those cycles to vary in importance and characteristics over time. The aim of this study is to find whether the UK business cycle has changed in the same way, to become more like the US one, or not. In time domain, the observed cyclical behaviour of the real business cycle hides many economic influences on cycles of different lengths and amplitudes. The use of frequency domain allows us to distinguish between properties of each cycle and to detect the dynamic coherence in each cycle. Because the spectral approach is nonparametric with no explicit economic structure imposed, we have many possibilities to explain the spectral peak. We will exemplify break dates detected in the first three frequencies, which correspond to the long-run effect. These frequencies also have an economic basis in business cycle literature: $\frac{\pi}{20}$, $\frac{4\pi}{20}$ and $\frac{7\pi}{20}$; correspond to 40 quarters' (the Juglar fixed investment cycle (7-11 years)), ten quarters' (the Kitchin inventory cycle (3-5 years)) and five quarters' cycle length respectively.

Indeed, as regards international change in the nature and the amplitude of the disturbance in world economy, we can find other fundamental factors that contribute to model cycles in the past and should continue to play a role, especially monetary and budgetary policies, globalisation, common shocks, actions executed by policymakers, etc.

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[6]From Bai and Perron (2003b), O is an arbitrary value correspond to the minimum distance, greater than q and not depending on T

We summarise in Tables (1 *and* 2) the results of the dynamic in comovement between the US and UK economies by looking for the tendency in level of synchronisation. We also find a clear difference in the comovement for the long term and the short term. Significant de-synchronisation in the two cycles (40 quarters, ten quarters) is observed between the two last regimes in contrast with the other cycle where we have an appreciation and a high level of synchronisation.

($1960Q1 - 1969Q2$) During the Vietnam War in the late 1960s and early 1970s, the Johnson administration still applied the Keynesian policies like Kennedy. The two factors seriously worsened the balance of payments. The sentiment of 'mini-recession' in the US economy drives the de-synchronisation process, keeping a decrease in coherence for 0.474543 to 0.255965. Indeed, Artis *et al.* (1997) argue that an industrial production recession occurred in the UK around 1971.

($1980Q4$, $1982Q1$, $1984Q4$ *and* $1987Q1$): our argument in favour of these dates is the beginning of the globalisation period. A possible explanation for the latter dates is that, while the US liberalised its capital accounts in the 1970s, the UK did not remove all of the barriers on capital account transactions until the beginning of the 1980s. These dates concern an instability in the long, middle and short term because they appear in all frequencies. Loosely speaking, the effect of the financial integration was felt early on during the common shock period in the two countries, where the full impact of financial reforms occurred only during the globalisation period. Indeed, the federal reserve bank shifted to a less expansionary policy and adopted a new monetary policy based on inflation control and emphasised by interest rates. Inflation and unemployment decreased from 5.57% to 3.03% and from 7.27 to 5.67 respectively between the 1980s and the 1990s, in contrast with the average annual growth rate in real GDP, which stagnated at 3.02%. Curtis (2005) shows that federal reaction to variations in inflation rates and unemployment started in 1987Q1 and he concludes that the Fed's objective was some mix of inflation-rate stability and output stability. It also corresponds to the Carter-Reagan defence build-up of the 1980s. Indeed, in the 1980s, the UK governments enacted a series of economic reforms to establish a more market-oriented economy. In sharp contrast with the convergence of inequality between the UK and the United States, the rates of poverty measured in absolute terms diverged between the two countries.

$1991Q3$, $1992Q2$ *and* $1993Q2$: we can explain these dates by the first Gulf War in the early 1990s and essentially the European Monetary System Crisis between 1992 and 1993. These dates are characterised by a general desynchronisation in the business cycles in all the OCDE countries. The crisis of the European exchange-rate mechanism (ERM) ($1992 - 1993$) was a critical event in the post-Bretton Woods history of the international monetary system. This event represents a turning-point in the use of exchange rate tools in the design of disinflation policies. The integration of two national economies under very different systems and with a substantial gap in productivity resulted in the adoption of a controversial monetary-fiscal policy (Buiter, Corsetti & Pesenti, 1998). Increase in the German interest rate and public deficit adding to the speculative attack against the lira and later against the franc intensified the conflict on exchange-rate matters among European policymakers. 'Black Wednesday' in the UK announced the beginning of the end of this exchange rate arrangement. On the morning of 16 September 1992, the Bank of England raised the minimum lending rate from 10% to 12%. On the same day, it announced a new increase to 15% and the 'temporary' withdrawal of the pound from the ERM was announced. Later, Italy followed Britain out of the ERM. At the same time several European countries were the victims of speculative attacks against their money. The 1992-1993 ERM crisis created a macroeconomic disturbance not only in Europe but in the rest of the world.

1996Q3, 1998Q1: the UK economy continued to experience lower rates of productivity than its major competitors. In December 1998, the Government published a white paper outlining a variety of measures aimed at shifting the economy into a new era of success, based upon the idea of a 'knowledge-driven economy'. Britain had a productivity gap of 40% with the USA and 20% with France and Germany. In contrast with these differences in productivity common international shocks promoted synchronisation between the two economies. These dates correspond to a number of international events that seriously affected world economy: the East-Asian crisis in July 1997, the Russian cold (1998) and the Brazilian fever (1998). Through the spectral properties, we can say that these observed regime shifts concern the

middle term.

Now we are interested in break dates at the short term: 1971Q1, 1975Q4, 1977Q2. We think that these dates summarise the persistence of relatively high inflation between the sixties and the seventies since annual inflation went up gradually from 2% to approximately 10% at the end of the seventies. In the United States, as in Great Britain, this increasing inflation rhythm started to accelerate a long time before the explosion of the import prices. The early/mid-1970s were also crisis years in the UK, with accelerating inflation, rising unemployment, massive industrial unrest and the first oil price shock (Dow, 1998).

5 Conclusion

In this paper, we propose a measure of comovement that is able to detect not only periods of convergence or divergence but to locate them endogenously and in different frequency. These findings allow us to distinguish between the properties of the cycles in the long term and the short term. We show, by applying these methods to GDP growth rate, how economic business cycles change over time and how synchronisation between US and UK business cycles changed from 1960 to 2006. As expected, the degree of synchronisation between the US and the UK has changed over time in all cycles, and the breakpoint in coherence series corresponds to the change in monetary policy, especially to that of the US. Our new methodology detects appropriately the known recessions of the mid-1970s, the beginning of the 1980s and 1992 to 1993. We also infer a higher degree of business synchronisation between US and UK economies, especially in short cycles. After 1992, we observe a divergence in the long-run cycles caused by a change in UK monetary policy.

Appendices

A   Proof of theorem 1.

Under the null assumption of the stability of the frequency w, the evolutionary spectral density is independent of time, i.e., $S_X^{iw} = S_X$ and $S_Y^{iw} = S_Y$; of Piestley's relation (8), we have:

$$\Lambda_X^{iw} \approx S_X^{iw} + e_X^{iw},$$

$$\Lambda_Y^{iw} \approx S_Y^{iw} + e_Y^{iw}, \quad i = 1,...,I = [\frac{T}{20}].$$

We can suggest for this case a time-varying cross-spectrum which is independent of time, i.e., $S_{XY}^{iw} = S_{XY}$; so we have:

$$\Lambda_{XY}^{iw} \approx S_{XY}^{iw} + e_{XY}^{iw}, \quad i = 1,...,I = [\frac{T}{20}],$$

where the sequence e_{XY}^{iw} is approximately normal, uncorrelated and identically distributed. The estimator of S_{XY} is given by $\hat{S}_{XY} = \frac{1}{I} \sum_{i=1}^{I} \Lambda_{XY}^{iw} = \mu^j$ and the OLS residuals are $\widehat{e^{iw}} = \Lambda_{XY}^{iw} - \mu^j$. Thus $\delta_r^w = \frac{1}{\hat{\sigma}_w \sqrt{I}} \sum_{i=1}^{r} (\Lambda_{XY}^{iw} - \mu^j)$ represents the cumulative sum of the OLS residuals. Let $B^{(I)}(z) = \frac{1}{\hat{\sigma}_w \sqrt{I}} \sum_{i=1}^{[Iz]} \widehat{e^{iw}}$ for $0 \leq z \leq 1$. Since all the conditions of the theorem of Ploberger & Krämer (1992) are trivially satisfied, then the limit in distribution of $B^{(I)}(z)$ is a Brownian bridge standard $B(z)$. Therefore the limit in the distribution of $sup_{0 \leq z \leq 1} |B^{(I)}(z)|$ is $sup_{0 \leq z \leq 1} |B(z)|$. According to Billingsley (1968), however, we have:

$$P(sup_{0 \leq z \leq 1} |B(z)| > a) = 2 \sum_{k=1}^{\infty} (-1)^{k+1} exp(-2k^2 a^2),$$

12

The desired conclusion holds, since $CS_{t,w} = max_{r=1,...,t}|\delta_r^w| = sup_{0 \leq z \leq 1}|B^{(I)}(z)|$.

B   Figures

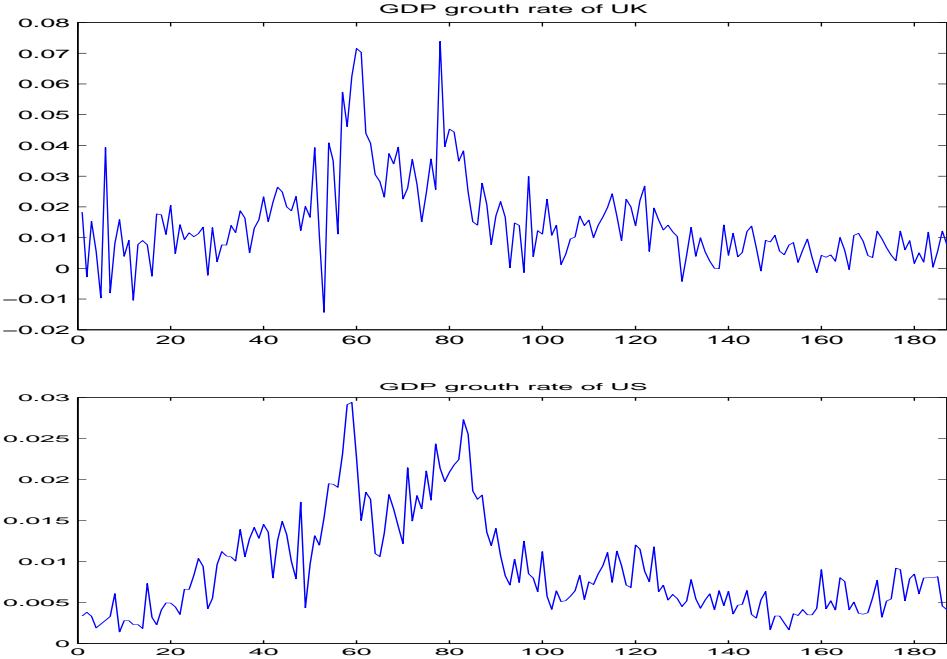Figure 1: GDP growth rate of UK and US

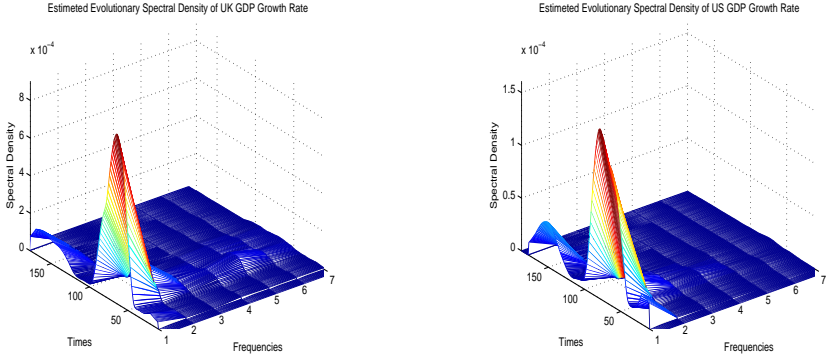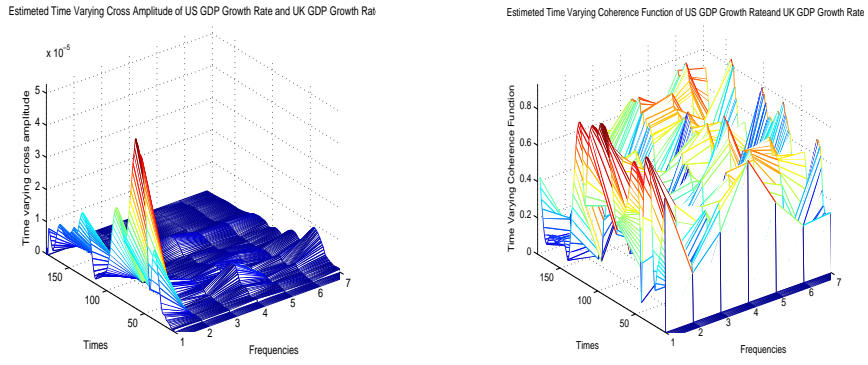Figure 2: Evolutionary Spectrum of UK and US

Figure 3: Time Varying Cross-spectrum and Time Varying Coherence Function

C   Tables

Table 1: Break Date Identification for the Evolutionary Cross-Spectrum of US and UK

Frequencies	estimators $\widehat{T}_1$	$\widehat{T}_2$
$\pi/20$	1982Q1	
$4\pi/20$	1982Q1	
$7\pi/20$	1982Q1	
$10\pi/20$	1982Q1	1991Q3
$13\pi/20$	1982Q1	1996Q3
$16\pi/20$	1982Q1	
$19\pi/20$	1982Q1	

14

Table 2: Break Date Identification for the TVCF of US and UK at Different Frequency

Estimators	$\hat{T}_1$	$\hat{T}_2$	$\hat{T}_3$	$\hat{T}_4$	
USA-UK					
$\pi/20$					
Break dates	1980Q4	1987Q1	1993Q2		
Coefficients	0.5860	0.8934	0.6993	0.1657	
Standard errors	0.1955	0.0195	0.2103	0.2872	
$4\pi/20$					
Break dates	1968Q4	1976Q4	1984Q1	1992Q1	
Coefficients	0.2487	0.4070	0.2821	0.4462	0.0874
Standard errors	0.1906	0.0538	0.0640	0.1167	0.0150
$7\pi/20$					
Break dates	1969Q2	1984Q4	1998Q1		
Coefficients	0.4745	0.2559	0.3249	0.4634	
Standard errors	0.0246	0.0660	0.1158	0.0275	
$10\pi/20$					
Break dates	1971Q1	1977Q2	1986Q1	1993Q4	
Coefficients	0.5472	0.1881	0.4630	0.3227	0.6571
Standard errors	0.1653	0.1324	0.0516	0.4579	0.0732
$13\pi/20$					
Break dates	1969Q2	1981Q1	1988Q2	1998Q1	
Coefficients	0.3809	0.5841	0.4367	0.6530	0.5049
Standard errors	0.0422	0.3011	0.2769	0.0269	0.1136
$16\pi/20$					
Break dates	1969Q2	1975Q4	1987Q3	1998Q1	
Coefficients	0.1710	0.5759	0.2677	0.4260	0.6911
Standard errors	0.0354	0.0397	0.0227	0.0962	0.0634
$19\pi/20$					
Break dates	1975Q4	1987Q1	1993Q2		
Coefficients	0.3880	0.6840	0.3498	0.4964	
Standard errors	0.2897	0.0794	0.1167	0.5229	

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