Specialization through Cross-licensing in a Multi-product Stackelberg Duopoly

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Abstract

We argue that cross-licensing is a device to establish specialization in a multi-product Stackelberg duopoly under process innovation. The optimum licensing contracts are royalty contracts. These are designed so as to implement the joint-profit maximization (monopoly) outcome as the unique Nash equilibrium of the competition game. The monopoly-First-Best optimum is attained: each firm produces solely the good for which it has a technological advantage, firms' joint profits attain the First Best optimum. We study the implications of limitations to contract enforceability and find that this may reduce the attained degree of specialization, but social welfare may increase.

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Introduction

Patent licensing is a main channel through which dissemination of innovations takes place.

The literature on licensing has identified several incentives to license process innovation or products to competitors or to others. Licensing may increase profits through the replacement of inefficient production techniques, by increasing demand, by facilitating collusion, by eliminating R&D expenditures or by deterring entry of stronger competitors.

Antitrust law historically has viewed certain types of licensing contracts, cross-licensing or pooling agreements, with suspicion because these mechanisms are potentially capable of promoting collusion in the product market (Gallini, 2002; Gilbert, 2004).

Cross-licensing is simply an agreement between two companies that grant each the right to practise the other’s patents. Through cross-licensing each firm is enabled to innovate more quickly and more effectively without fear that the other will hold up by asserting a patent that it has unintentionally infringed.

In a patent pool, an entire group of patents is licensed in a package, either by one of the patent holders or by a new entity established for this purpose, usually anyone willing to pay the associated royalties.

Empirical evidence shows that innovations in hardware, software or biotechnology often build on a number of innovations developed and patented by a set of different producers. In many cases there are multiple pathways to be investigated, for instance in bio-medical research. Cross-licensing agreements and patent pools are therefore an economic necessity.

The literature on cross-licensing has stressed that it facilitates collusion. Firms can achieve a collusive outcome by adopting proper punishment schemes. In fact Shapiro (1985 p.26) states that: "two rivals (with or without innovations) alternately could design a cross-licensing agreement whereby each would pay the other a royalty per unit of output, ostensibly for the right to use the other’s technology. By imposing a “tax” on each other (or by writing pricing or territorial restrictions into the cross-licensing contract), the firms could again achieve the fully collusive outcome. A cross-licensing contract may be required to achieve the fully collusive outcome if the firms produce different products or are otherwise heterogeneous”.

Eswaran (1994) assumes that the firms license their technologies to each other but tacitly agree not to produce from the acquired technology as long as the contracting firm does not defect. In an infinitely repeated game it is shown that collusion can be sustained from tacitly restricted level of production by credibly introducing the threat of increased rivalry in the market for each firm’s product. Lin (1996) is close to Eswaran’s contribution as fixed fee licensing makes firms’ costs
symmetric and increases the licensee’s scope for retaliation.

Fershtman and Kamien (1992) deals with cross licensing of complementary technologies, that may be independently developed by different firms. Relevant to this note is the problem the firms face about how to design a cross licensing agreement such that the resultant non-cooperative game, yields equilibrium profits identical to the cooperative outcome.

K.Kultti et al. (2006) shows that cross-licensing is a device to establish multi-market contact (Bernheim and Whinston, 1990) and is likely to raise anti-trust concerns only in so far multi-market contact does and that cross-licensing is irrelevant for sustaining tacit collusion.

This note studies product specialization in a duopoly where each firm produces two imperfect-substitute goods. In the case of Stackelberg competition, we show that under process innovation, specialization is the equilibrium attained under optimal cross-licensing arrangements. The optimum licensing contracts are royalty-contracts. Royalties are set so as to implement the joint-profit maximization (monopoly) outcome as the unique Nash equilibrium of the competition game. The monopoly-First-Best optimum is attained: i) each firm produces solely the good for which it has a technological advantage; ii) the quantities of goods which are produced are the monopoly levels; iii) firms’ joint profits attain the First Best optimum.

In case of limitations to contract enforceability, it is not convenient for firm i to specialize in good i and both firms produce both goods. Therefore the licensing contract is not a Nash equilibrium and renegotiation proof.

These results compliments the Cournot case discussed in Filippini (2006) in three extensions: checking out the robustness of the findings in the Stackelberg framework that fits better the assumption of process innovation in one good, the good where each firm is a leader; firms behave cooperatively to maximize their joint profits along the lines of the widely used taxonomies of R&D cooperation (Kamien et al., 1992), and finally contract enforceability is not assumed.

The plan of the paper is as follows. Section 2 describes the basic framework in Stackelberg competition, where the two firms diversify their production and considers the introduction of the process innovation that may lead to product specialization. Section 3 discusses the cross-licensing case and the product specialization which results from that. Section 4 analyzes the case where contract enforceability is not assumed.

Two Stackelberg firms diversifying their production

Let’s consider an industry composed by two symmetric firms, and two imperfect substitute goods, good 1, good 2. Each firm can produce both goods, and \( q_{ij} \) is the quantity of good \( i \) produced by firm \( j \).

Demands are derived from the maximization problem of a representative consumer (as shown by Singh and Vives, 1984), endowed with a utility function separable and linear in the numeraire good, \( m \), given by:
\[ u(q_1, q_2) = a(q_1 + q_2) - q_1^2/2 - q_2^2/2 - \theta q_1 q_2 + m, \]

where: \( q_1 = q_{11} + q_{12}; \) \( q_2 = q_{21} + q_{22}, \) and \( \theta \) an element of \((0, 1]\) represents the degree of product differentiation.

Therefore inverse demand functions are linear:

\[
p_1 = a - (q_{11} + q_{12}) - \theta(q_{21} + q_{22})
\]

\[
p_2 = a - \theta(q_{11} + q_{12}) - (q_{21} + q_{22}),
\]

where \( p_i \) is the price of good \( i, i = 1, 2. \)

Firm cost functions are linear and symmetric: each firm produces both good \( i, \) and suppose that Firm 1 discovers and patents a cost reducing technology, respectively, for good 1, and Firm 2 for good 2, that lead to zero cost in production. The other is produced at the constant marginal cost, \( c. \) We assume \( c < a \) in order to avoid a corner solution.

The profits functions are (the subscript \( P \) denotes process innovation):

\[
\Pi_{1P} = p_1 q_{11} + p_2 q_{21} - cq_{21}
\]

\[
\Pi_{2P} = p_1 q_{12} + p_2 q_{22} - cq_{12}.
\]

Let consider Stackelberg quantity competition (Boyer and Moreaux, 1987), under the demand and cost assumptions above-mentioned. Firms 1 and 2 are, respectively, leader for good 1 and 2 and choose their (individual) profit-maximizing outputs. Specifically, firm 1:

\[
\text{Max} \{q_{11} p_1 + b_2[q_{21}(p_2 - c)] \}
\]

\[
s.t.
\]

firm 2’s best response for good 1, \( b_2; \)

\[
q_{11} \geq 0, q_{21} \geq 0,
\]

firm 2:

\[
\text{Max} \{b_1[q_{12}(p_1 - c)] + q_{22}p_2 \}
\]

\[
s.t.
\]

firm 1’s best response for good 2, \( b_1, \)

\[
q_{12} \geq 0, q_{22} \geq 0.
\]

As profit functions are concave in output, \( FOC \) are necessary and sufficient for a maximum.
Solving system (3.a) – (3.b) leads to:

1) if \( c < d' = \frac{a(1 - \theta)(2 - 2\theta - \theta^2)}{(6 + 4\theta - \theta^3)} \), and \( \theta < \sqrt{3} - 1 \)

then there is limited specialization:
Equilibrium outputs are strictly positive, and are given by:

\[
q_{11} = q_{22} = \frac{a(1-\theta)(4+2\theta+\theta^2)+c(4+6\theta-\theta^2)}{(1-\theta)(1+\theta)(8+\theta^2)}
\]

Prices and profits (the subscript \( LS \) denotes limited specialization and Stackelberg quantities) are:

\[
p_i = \frac{(a + c)(2 + \theta^2)}{(8 + \theta^2)}
\]

\[
\Pi_{i,LS} = \frac{6(a-c)^2(2+\theta^2)^2+2ac(8+8\theta+10\theta^2+\theta^3)}{(1+\theta)(1+\theta)(8+\theta^2)^2} + \frac{c^2(4+6\theta-\theta^2)^2}{(1-\theta)(1+\theta)(8+\theta^2)}
\]

2) if:

\[
c \geq d' = \frac{a(1 - \theta)(2 - 2\theta - \theta^2)}{(6 + 4\theta - \theta^3)}
\]

then there is full specialization:

Equilibrium outputs, prices and profits (the subscript \( FM \) denotes full specialization and monopoly quantities) are equal to system:

\[
q_{12}^* = q_{21}^* = 0
\]

\[
q_{11}^* = q_{22}^* = \frac{a}{2+\theta}
\]

\[
p_i^* = \frac{a}{2+\theta}
\]

\[
\Pi_{i,FM} = \left(\frac{a}{2+\theta}\right)^2
\]
This leads to:

**Proposition**: The Nash-Stackelberg equilibrium entails:

- **i)** full specialization in case of drastic innovation, i.e. condition (5) holds.
- **ii)** limited specialization in case of non-drastic innovation, i.e. condition (4) holds.

Clearly, in the case of non-drastic innovation, firms would be better off if they could commit to joint profit maximization.

However, the only credible commitments are those that are incentive compatible, and \( q_{12}^* = q_{21}^* = 0 \), are not. Indeed, the unique Nash-Stackelberg equilibrium has \( q_{ij} > 0 \) (by ii) of Proposition 1).

We show below that there exists a cross-licensing scheme that implements the collusive outcome: the unique Nash-Stackelberg equilibrium entails full specialization, and firm profits attain the First Best optimum level.

### Cross-licensing in a Stackelberg framework

In the cross-licensing game, at the first stage (i.e. the licensing stage), firms act cooperatively: firms 1 and 2 choose \((r_i, F_i)\) that maximize joint profits. In the second stage, firms again engage in quantity Stackelberg competition.

Specifically, for any given royalty rates, equilibrium outputs, prices and profits are:

\[
q_{11} = \frac{a(1-\theta)(4+2\theta^2)+3\theta^2 r_1+2\theta r_2(2+\theta^2)}{(1-\theta)(1+\theta)(8+\theta^2)}
\]
\[
q_{12} = \frac{a(1-\theta)(2-2\theta-\theta^2)-2(2+\theta^2)r_1-3\theta^3 r_2}{(1-\theta)(1+\theta)(8+\theta^2)}
\]
\[
q_{21} = \frac{a(1-\theta)(2-2\theta-\theta^2)-2(2+\theta^2)r_2-3\theta^3 r_1}{(1-\theta)(1+\theta)(8+\theta^2)}
\]
\[
q_{22} = \frac{a(1-\theta)(4+2\theta^2)+3\theta^2 r_2+2\theta r_1(2+\theta^2)}{(1-\theta)(1+\theta)(8+\theta^2)}
\]
\[
p_1 = \frac{a(2+\theta^2)+r_1(4-\theta^2)}{(8+\theta^2)}
\]
\[
p_2 = \frac{a(2+\theta^2)+r_2(4-\theta^2)}{(8+\theta^2)}
\]
At the first stage, each firm $i$ chooses $(r_i,F_i)$ in order to maximize joint profits subject to output non-negativity constraints. That is:

$$\text{Max}_{r_i,F_i} \{ \Pi_{1SLic}(r_1,r_2) + \Pi_{2SLic}(r_1,r_2) \}$$

s.t.

$$q_{11} \geq 0, q_{21} \geq 0, q_{12} \geq 0, q_{22} \geq 0,$$
$$r_i,F_i \geq 0$$

In the unique Nash equilibrium, licensing contracts are pure royalty contracts:

$$r_i = \frac{a}{2}$$

**Proposition**: The optimum licensing contracts are the royalty-contracts defined by [9]. These implement the monopoly-First-Best optimum in the Stackelberg case.

We now compare social welfare between cross-licensing and the process innovation status quo. The social welfare functions are:

I) in the no-licensing case ($W_{LS}(q_1,q_2) = u(q_1,q_2) - cq_{12} - cq_{21}$) is by [4.a]:

$$W_{LS} = \frac{12a^2(5+\theta^2)(1-\theta)-4ac(1-\theta)^2(\theta^2+29+10)+c^2(92+680+8\theta^2-4\theta^3-\theta^4-\theta^5)}{(1-\theta)(1+\theta)(8+\theta^2)^2}$$

II) in the cross-licensing case ($W_{Lic}(q_1^*,q_2^*) = u(q_1^*,q_2^*)$) is by [6]:

$$W_{Lic} = a^2(3+\theta)/(2+\theta)^2.$$  

We have:

**Proposition**: Social welfare in the cross-licensing case is always lower than with no-licensing, for all $\theta$ and $c \in (0,c = d')$.

Proof:

First note that $W_{LS}$ attains its minimum value when the cost, $c$, equals $c_{min} = \frac{2a(1-\theta)^2(10+20+\theta^2)}{92+680+8\theta^2-4\theta^3-\theta^4-\theta^5}$, where $W_{Lic} < W_{LS}(c_{min})$.

Further note that $W_{Lic} < W_{LS}(c = 0) \text{ and } \lim W_{LS}(c \to d') \rightarrow W_{Lic}$.

This result is in contrast with that attained in the Cournot framework (Filippini, 2006). The economic intuition for Proposition 3 is in that cross-licensing implements monopoly output, and therefore induces an output decrease with respect to Stackelberg competition. However, it also gives rise to efficiency gains, because
production is obtained with the most efficient technology. Social welfare improves when the efficiency gain overcomes the output loss, and vice versa.

**Stackelberg Cross-Licensing: limitations to contract enforceability**

Contract offers are made by one firm, the other either rejects the offer or accepts it. If the latter rejects it then it will necessarily use the old technology, if it accepts it then royalty-payment obligations are due independently of the technology used and therefore its profit-maximizing choice is necessarily to adopt the new (cost-reducing) technology.

This section derives the implications that would result from limitations to contract enforceability, and specifically from the possibility that the Firm $i$, after having signed the contract, reneges on its royalty-contractual obligations on the grounds that it does not use the new technology. Therefore in equilibrium Firm $i$ produces both goods. This possibility implies that Firm $i$ fulfills its contractual obligations if and only if it finds it optimal to specialize. Figure 1 shows all combinations of firms’ profits when each firm produces either one good or two goods.

**Figure 1 - Profits for Firm 1 and 2**

The focus of this section is to derive sufficient condition so that both Firms are monopolists of a substitute good.

The game played by the two firms is a non-cooperative two-stage game. At the first stage, each firm $i$ chooses $(r_i, F_i)$ in order to maximize joint profits subject to its own and its rival’s individual-rationality constraints, and to output non-negativity constraints. In the second stage firms engage in Stackelberg competition as described in Section 2.

This adds to the licensor’s optimization problem the constraint that the profits of Firm $i$ in the licensing case, $\Pi_{i\text{Lic}}(r_1, r_2, \ldots)$, must exceed the profits of Firm $i$ that reneges its obligations and still produces goods $j(\neq i)$ at cost $c$, $\Pi_{i\text{Lic}}(r_i, c, \ldots)$.

That is:
In the unique Nash equilibrium, licensing contracts are:
\[
\begin{align*}
Max & \quad \langle \Pi_{1\text{SLic}}(r_1, r_2, \ldots) + \Pi_{2\text{SLic}}(r_1, r_2, \ldots) \rangle \\
\text{s.t.} & \quad \Pi_{1\text{SLic}}(r_1, r_2, \ldots) \geq \Pi_{1\text{SLic}}(r_1, c, \ldots) \\
& \quad \Pi_{2\text{SLic}}(r_1, r_2, \ldots) \geq \Pi_{2\text{SLic}}(c, r_2, \ldots) \\
& \quad q_{11} \geq 0, q_{21} \geq 0, q_{12} \geq 0, q_{22} \geq 0, r_i, F_i \geq 0
\end{align*}
\]

In the unique Nash equilibrium, licensing contracts are:
\[
r_i > c, F = 0.
\]

The optimal licensing contract is a royalty and it exceeds the cost reduction (see also Filippini, 2005). Equilibrium outputs increase and social welfare may too.

This leads to:

**Proposition** : If contract enforceability is not assumed, in cross licensing it is not convenient for firm \(i\) to specialize in good \(i\) and both firms produce both goods. Therefore the licensing contract is not a Nash equilibrium and renegotiation proof. But social welfare may increase.

From the antitrust perspective the implications of Propositions 3 and 4 are clear. Antitrust laws discourage collusion not for its own sake but because collusion harms consumers. The above Propositions shows that cross licensing may increase welfare.

**Conclusion**

We have studied product specialization in a Stackelberg framework where each firm produces two imperfect-substitute goods. We have shown that under process innovation, specialization is the equilibrium attained under cross-licensing arrangements. The optimum licensing contracts are royalty contracts. These are designed so as to implement the joint-profit maximization (monopoly) outcome as the unique Nash equilibrium of the competition game. The monopoly-First-Best optimum is attained: Each firm produces solely the good for which it has a technological advantage, firms’ joint profits attain the First Best optimum.

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**References**


