Abstract
Melnik et al. [Melnik, A., Shy, Oz, Stenbacka, R. Assessing market dominance. Journal of Economic Behavior and Organization 68, 63-72] have proposed a new statistic to assess market dominance. In this comment we expand their discussion of certain mathematical properties in their analysis and link their methodology to some previous approaches.
1. Introduction

In a recent paper Melnik et al. (2008) suggest a novel approach to the existence of dominance of the leading firm in an industry. The formula they put forward, they claim, can be easily applied in antitrust cases by competition and regulatory agencies without the requirement of estimating demand elasticities or marginal costs, the latter being notoriously difficult to obtain. Their approach is to be commended since they restore the prominence of the market share concept that has been given (in our view, unduly) short shrift by some recent research.

By relating the paper to previous papers by LaCour and Møllgaard (2002, 2003) and Dobbs and Richards (2005) to which Melnik et al. (2008) do not refer, the purpose of the present comment is firstly, to show the interconnection of these approaches and, secondly, to elaborate on this paper by checking the validity of the main findings and to expand on them.

The proposed single-market measure (“threshold”), which would enable regulators to draw inferences about dominance of the leading firm, takes the following form:

\[
s^D = \frac{1}{2} [1 - \gamma (s_1 - s_2)(1 - \sum_{i=3}^{N} s_i)] = \frac{1}{2} [1 - \gamma (s_1^2 - s_2^2)]
\]

(1)

where \( s_i \) are firm market shares indexed in an order satisfying \( s_1 \geq s_2 \geq \ldots \geq s_N \), and \( \gamma \) is an exogenous parameter interpreted “as an industry-specific assessment of the entry barriers relevant for the industry. Lower values of \( \gamma \) correspond to lower entry barriers, which would mean that potential (future) competition will limit the ability of firm 1 to exploit its market power more effectively” (ibid., p. 65).

We proceed by equating the share of the second largest firm (\( s_2 \)) to \( (1-\sum s_i-s_1) \). Thus, equation (1) is transformed into:

\[
s^D = \frac{1}{2} \left[1 - \gamma [2s_1(1-\sum s_i) - (1-\sum s_i)^2] \right]
\]

(2)

2. The parameter \( \gamma \)

Following Dobbs and Richards (2005) we now introduce the concept of “output restriction test” elasticity:

\[
\tilde{\varepsilon}_{ORT} = \frac{\Delta Q}{Q} \frac{\Delta q_1}{\Delta q_1} = \frac{\Delta Q}{\Delta q_1} s_1 = zs_1
\]

(3)

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1 The paper by Hellmer and Wärell (2009, p. 3239) describe the paper as “interesting and intriguing”.

2 On this, see the earlier discussion in Shepherd et al. (2000).

3 Hereafter, for simplicity purposes, we use the notation \( \sum s_i \) instead of \( \sum_{i=3}^{N} s_i \).
where \( Q \) is the industry’s output, \( q_1 \) the leading firm’s output, \( z = \Delta Q / \Delta q_1 \). Additionally, according to Dobbs and Richards (ibid., p. 573): “more usually, output restriction [by the leading firm] will give rise to some degree of reduction in total output, and hence to a positive ORT elasticity”\(^4\).

These authors claim that, under reasonable assumptions, this elasticity will lie between zero and one. Thus, expression (2) can be written as:

\[
S^D = \frac{1}{2} \left\{ 1 - \gamma \left[ 2 \epsilon^{ORT} \left( 1 - \sum s_i \right) - \left( 1 - \sum s_i \right)^2 \right] \right\}
\]  

(4)

and

\[
\frac{\partial S^D}{\partial \gamma} = - \frac{\epsilon^{ORT}}{z} \left( 1 - \sum s_i \right) + \frac{\left( 1 - \sum s_i \right)^2}{2}
\]  

(5)

For this partial derivative to become negative, as Melnik et al. (2008, pp. 66, 69) maintain, the following inequality should apply: \( 1 - 2\epsilon^{ORT} / z < \sum s_i < 1 \), where \( 0 < \sum s_i < 1 \) and as hypothesized \( \epsilon^{ORT} < z \) (since \( s_i = \epsilon^{ORT} / z \) and \( 0 < s_i < 1 \))

Moreover, Dobbs and Richards (op.cit., p. 574) maintain that, in the case of a homogenous good Cournot industry, “assuming locally linear demand and constant marginal costs, the ORT elasticity for any given firm, at the current Cournot equilibrium, is simply the market share of that firm divided by the total number of firms in the industry”: that is \( \epsilon^{CORT} = s_i / N \). In this case, expression (2) can be written as:

\[
S^D = \frac{1}{2} \left\{ 1 - \gamma \left[ 2 N \epsilon^{CORT} \left( 1 - \sum s_i \right) - \left( 1 - \sum s_i \right)^2 \right] \right\}
\]  

(6)

and

\[
\frac{\partial S^D}{\partial \gamma} = - N \epsilon^{CORT} \left( 1 - \sum s_i \right) + \frac{\left( 1 - \sum s_i \right)^2}{2}
\]  

(7)

For this partial derivative to become negative, as Melnik et al. (2008, pp. 66, 69) argue, the following inequality should hold: \( 1 - 2N \epsilon^{CORT} / z < \sum s_i \) which is equivalent to \( 1 - 2s_1 < \sum s_i < 1 - s_1 - s_2 \); and is

\(^4\) In the words of Azevedo and Walker (2002, p. 366): “Any firm can restrict its own output, but in most markets any unilateral output restriction would be replaced by an output expansion by other players in the market. This is not true of a dominant firm’s output restriction”. Also, “The greater the firm’s market share the less likely that other firms will be able to expand production to defeat the unilateral price increase” (McFalls, 1997).

\(^5\) We doubt the correctness of equating higher strictness of the dominance criterion with a lower value of \( S^D \) (Melnik et al. 2008, p. 65). On the contrary, we think that higher strictness is associated with a higher value of \( S^D \) and vice versa.
also valid since \( s_1 > s_2 \). Furthermore, as we proved previously in the case of the OPT elasticity and for the same reasons, the partial derivative cannot be positive. Therefore, the negative relationship between the parameter \( \gamma \) and the dominance threshold is established once again.

We now turn to the papers by LaCour and Møllgaard (2002, 2003). One of the factors considered in testing for dominant position is the own-price elasticity, \( e_{\text{own}} = -(\Delta q_1/\Delta p_1).(p_1/q_1) \) which we can transform into \( e_{\text{own}}^\prime = -(\Delta q_1/\Delta p_1).(p_1/s_1Q) \) and hence \( s_1 = y/e_{\text{own}}^\prime \) where \( y = -(\Delta q_1/\Delta p_1).(p_1/Q) \). From expression (2) we then get:

\[
D_i^{\text{ort}} = \sum_{s \in \text{set}} \frac{\frac{\partial \gamma}{\partial \varepsilon}}{1} \leq 0, \quad \text{where } 0 < \sum_{s} < 1, \gamma, z > 0 (10)
\]

\[
\frac{\partial \gamma}{\partial \varepsilon^\prime} = -
\]

\[
D_i^{\text{ort}} = \sum_{s \in \text{set}} \frac{\gamma \cdot y}{(e_{\text{own}}^\prime)^2} \leq 0, \quad \text{where } 0 < \sum_{s} < 1, y < 0 \text{ and } \gamma > 0 (12)
\]

With reference to equations (10) and (11) we can infer that both results are plausible since the higher the value of the elasticity the less the remaining firms can counteract the change (decrease) in quantity initiated by the leading firm, reflecting a state of weak competition. Thus the threshold required for dominance need not be high. As far as the own-price elasticity is concerned, the negative value of the partial derivative indicates that the higher (in absolute terms - or the lower in negative terms) the elasticity, the lower the dominance threshold. This is, again, plausible and accords to what LaCour and Møllgaard (2003, p.133) point out: “A very high negative value of \( e \) is a sign that the demand curve disciplines the firm strongly: customers rush away if the price is increased”.

4. The joint market share of all but the two largest firms

We come to the final variable under review, namely the joint market share of all but the two largest firms. According to Melnik et al. (2008, p. 65) “…the dominance threshold is lower… the higher is the joint market share of the two largest firms” or alternatively, the dominance threshold is lower the lower the joint market share of the remaining firms (\( \Sigma s_i \)).
From (4), (6) and (8) we get the three partial derivatives with respect to \( \Sigma s_i \):

\[
\frac{\partial D_i}{\partial \sum s_i} = \frac{\gamma e^{ORT}}{z} - \gamma (1 - \sum s_i), \quad \text{where } s_1 = \frac{e^{ORT}}{z}
\]  

(13)

\[
\frac{\partial D_i}{\partial \sum s_i} = \gamma e^{CORT} N - \gamma (1 - \sum s_i), \quad \text{where } s_1 = e^{CORT} N
\]  

(14)

and

\[
\frac{\partial D_i}{\partial \sum s_i} = \frac{\gamma y}{e^{own}} - \gamma (1 - \sum s_i), \quad \text{where } s_1 = \frac{y}{e^{own}}
\]  

(15)

These relations can be summarized in one expression:

\[
\frac{\partial D_i}{\partial \sum s_i} > 0 \quad \text{when } s_1 < 1 - \sum s_i
\]

But the positive partial derivative is not validated since \( s_1 > s_1 + s_2 \) cannot hold, thus the higher the joint share of the remaining firms (\( \Sigma s_i \)) the lower the dominance threshold. This is, however, the opposite of the Melnik et al. result. Thus, a word of caution and further elaboration are needed.

The partial derivatives presented above are valid only when \( s_1 \) is kept constant. This means that if the \( \Sigma s_i \) decreases (increases), as \( s_1 \) remains constant, the share of the second largest firm (\( s_2 \)) goes up (goes down) resulting in the decrease (increase) of (\( s_1 - s_2 \)) which in turn tends to raise (lower) the threshold (\( s^D \)). At the same time though, the increase (decrease) of (1 - \( \Sigma s_i \)) exerts a negative (positive) influence on the threshold (\( s^D \)) but not as strong as the positive (negative) contribution of the diminution (enlargement) of the market share difference of the two largest firms, hence the overall increase of the threshold (\( s^D \)). Consequently, Melnik et al. err in stating “…the dominance threshold is lower…the higher is the joint market share of the two largest firms…” (p.65) as the comparison of market E with either market C or market D of their Table 1 (p. 66) indicates. The diminution of the market share difference predominates, causing the increase of the dominance threshold (\( s^D \)) despite the increase of (\( s_1 + s_2 \)).

If however, the market share difference remains constant, which is not an option in our equations (4), (6) and (8), when \( \Sigma s_i \) varies, then the threshold increases (decreases) as \( \Sigma s_i \) goes up (goes down). Obviously, if all three shares, i.e. \( s_1, s_2 \) and \( \Sigma s_i \), vary concurrently, then it is not possible to attribute the new value of the dominance threshold to the change of a single variable.

The following table provides a self-explanatory numerical example of our previous arguments:

<table>
<thead>
<tr>
<th>Market</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_1 - s_2 )</th>
<th>( \Sigma s_i )</th>
<th>( s^D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.40</td>
<td>0.15</td>
<td>0.25</td>
<td>0.45</td>
<td>0.43</td>
</tr>
<tr>
<td>B</td>
<td>0.40</td>
<td>0.25</td>
<td>0.15</td>
<td>0.35</td>
<td>0.45</td>
</tr>
<tr>
<td>C</td>
<td>0.45</td>
<td>0.20</td>
<td>0.25</td>
<td>0.35</td>
<td>0.42</td>
</tr>
<tr>
<td>D</td>
<td>0.45</td>
<td>0.30</td>
<td>0.15</td>
<td>0.25</td>
<td>0.44</td>
</tr>
<tr>
<td>E</td>
<td>0.35</td>
<td>0.20</td>
<td>0.15</td>
<td>0.45</td>
<td>0.46</td>
</tr>
</tbody>
</table>
5. Conclusion
It has been argued, echoing the Chicago-school view, that “the presumption associating market power with high market shares is rebuttable” (Hay, 1992, p. 822). Along the same lines, “…the case law puts a great deal of stock in the size of market shares in inferences of market power - a general inference that modern economics tells us is not supported by either theory or empirical evidence” (Scheffman, 1992, p. 919). However, opponents of this view have maintained that “…market share is the leading fact, the most basic determinant of the degree of competition or monopoly” (Shepherd et al., 2001, p. 841). Neither view elevates “market share” into the absolute criterion or denigrates it to such an extent as to render it completely inoperable and useless. As Melnik et al. suggest and as the present comment has insisted other factors also come into play. Generally speaking, the fast-track formula seems to rest on solid grounds. But the use of supplementary variables, whose estimation is admittedly more demanding, would impact on the study of dominance in a more useful manner.

6 “Though market shares are recognized to be important, no serious scholar has claimed that they control market outcomes precisely. They are simply the best single indicator, as business experience has also fully recognized. Their evidence establishes a presumption about market power’s possible effect in raising price, which secondary conditions may modify” (Shepherd et al., 2000, p.852, fn. 26) and “The relationship between market share and market power is not exact or tight....But within any market, larger market shares give significantly more market power; and in almost every market, market shares above 30% provide substantial market power. That is one reason why firms struggle so relentlessly to gain more market share” (ibid., p. 860, fn. 42).
References