Abstract

Potential output and the related concept of output gap play a central role in the macroeconomic policy interventions and evaluations. In particular, the output gap, defined as the difference between actual and potential output, conveys useful information on the cyclical position of a given economy. This paper proposes estimates of the Italian potential output based on a structural VAR model using data coming from business surveys. This kind of data, given their cyclical profile, are particularly useful for detrending purposes, as they allow to include information concerning the business cycle activity. The ability of the cyclical GDP component obtained with the SVAR decomposition to forecast inflation and to detect business cycle turning points over the expansion and recession phases is then performed.

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1 Introduction

Potential output and output gap are considered important indicators of the economic activity evolution. More in detail, the output gap, i.e. the difference between the actual output level and its potential, provides information concerning the cyclical position of the economy. In this sense it represents a benchmark to achieve non inflationary growth since if the output gap is positive (negative) the inflationary pressures raise (fall) and the policy makers are expected to tighten (ease) monetary policies. This indicator it is also used by central banks to fix interest rates according to the so-called Taylor rules (Taylor, 1993).

However, in spite of the attention received, the estimates of those aggregates are still surrounded by a huge amount of uncertainty (cfr. Orphanides and van Norden, 1999 and 2001). This is mainly due to the fact that the output decomposition into its trend and cyclical components are not unique depending on the method used.

In the literature different methods have been used to estimate potential GDP. The most known univariate statistical techniques are based on the use of univariate filters (i.e. Hodrick and Prescott, 1997 and Baxter and King, 1995). Other univariate approaches include unobserved components models (see for details, Harvey, 1985 and Clark, 1987) and the Beveridge and Nelson (1981) decomposition. In addition, multivariate decompositions based on those techniques (i.e. multivariate filters or multivariate unobserved components models) have also been developed. Recently, considerable attention has been focused on the use of VAR models. To this end St-Amant and van Norden (1997) use a VAR model with long run restrictions including output, inflation, unemployment and real interest rate to estimate the Canadian output gap. Similarly Claus (2003) employs a SVAR model with long run restrictions to estimate New Zealand output gap for the period 1970-99.

The aim of this paper is to estimate Italian potential output using a multivariate decomposition based on the use of a structural VAR model. Compared to other standard techniques, this kind of models show several advantages. Firstly, the use of a multivariate decomposition model allows to include information coming from more then one variable. In this sense, if compared to univariate decomposition methods, which only incorporate information coming from the decomposed variable, the multivariate method takes into account the external dynamics coming from other data. Secondly, as against other decomposition methods based on univariate filtering, the detrended series obtained with the SVAR methodology satisfies the Cogley and Nason (1995) critique, inasmuch the decomposition introduces no spurious cyclicality in the data. Furthermore, compared to other multivariate techniques (i.e. multivariate filters) the SVAR model allows for an economic interpretation of each variable’s shocks. Finally, given its ability to act as a prediction model, the SVAR can be applied for forecast purposes.

The paper is organized as follows. Section 2 introduces the SVAR model and the identifying restrictions. Section 3 reports the output gap estimates for Italy and includes an assessment of the ability of the estimated GDP cyclical components to detect turning points of the Italian cyclical chronology. Section 4 includes an evaluation of the output gap’s ability to forecast inflation. Section 5 concludes the work.

2 The model

To provide output gap estimates for Italy, we apply a SVAR model based on Blanchard and Quah (1989) identifying restrictions. The MA representation of the bivariate structural VAR model is given by:
where $\Delta y_t$ is the growth rate of output, $bs_t$ is a cyclical stationary variable coming from business tendency surveys, $v_{st}$ and $v_{dt}$ represent structural uncorrelated supply and demand shocks and $A(L)$ is a 2x2 dimension polynomial matrix in the lag operator $L$. Alternatively, the model can be written in a compact form:

$$x_t = k + A(L)v_t$$

where $x_t = [\Delta y_t, bs_t]$ represents the vector of endogenous variables and $v_t = [v_{st}, v_{dt}]$ is the vector of aggregate shocks. Moreover, the shocks are normalized in order to have unit variance ($E(v_t,v_t') = I$).

The identifying restrictions are provided by assuming that demand-side shocks (i.e. to the cyclical indicator) only have a short-run impact on output, whereas supply-side shocks (i.e. productivity shocks) can produce long-run effects on output. More in detail, the identification is ruled out, imposing long-run restrictions on the coefficients of the MA representation of the structural VAR model.

Since the structural shocks are not observed, to evaluate the effects on the economy we need to derive them from the estimated residuals of the reduced-form model. The standard matrix representation of the bivariate reduced VAR form is given by:

$$\begin{bmatrix} \Delta y_t \\ bs_t \end{bmatrix} = \begin{bmatrix} \Phi_{10} \\ \Phi_{20} \end{bmatrix} + \begin{bmatrix} \Phi_{11}(L) & \Phi_{12}(L) \\ \Phi_{21}(L) & \Phi_{22}(L) \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ bs_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{st} \\ \varepsilon_{dt} \end{bmatrix}$$

or in a more compact formula:

$$x_t = \Phi_0 + \Phi_1(L)x_{t-1} + \varepsilon_t$$

Where $\varepsilon_t = [\varepsilon_{st}, \varepsilon_{dt}]$ indicates the residual vector of the estimated model and $\Sigma_\varepsilon = E(\varepsilon_t,\varepsilon_t')$ indicates the variance and covariance residual matrix, which generally are not diagonal. If the process is invertible (the polynomial matrix $\Phi(L)$ has unit root out of the unit circle), its moving average representation is given by:

$$x_t = K + C(L)\varepsilon_t$$

where $K = (I - \Phi_1)^{-1}\Phi_0$ and $C(L) = (I - \Phi_1(L)L)^{-1}$

Under the hypothesis that innovations are a linear combination of structural shocks, by equating (2) and (5) we obtain:

$$K + A(L)v_t = K + C(L)\varepsilon_t$$

For $L=0$, since $C(0) = I$ we have:

$$A(0)v_t = \varepsilon_t$$

where $E(\varepsilon_t,\varepsilon_t') = A(0)E(v_t,v_t')A(0)' = \Sigma_\varepsilon$

The sigma matrix is given by:
\[\Sigma_{e} = \begin{bmatrix}
A_{11}(0)^2 + A_{12}(0)^2 & A_{11}(0)A_{21}(0) + A_{12}(0)A_{22}(0) \\
A_{11}(0)A_{21}(0) + A_{12}(0)A_{22}(0) & A_{21}(0)^2 + A_{22}(0)^2
\end{bmatrix}\]  

(8)

Structural shocks \( v_t \) are determined from equation (7):

\[v_t = A(0)^{-1} \varepsilon_t\]  

(9)

or in a matrix form:

\[
\begin{bmatrix}
\varepsilon_{yt} \\
\varepsilon_{gt}
\end{bmatrix} =
\begin{bmatrix}
A_{11}(0) & A_{12}(0) \\
A_{21}(0) & A_{22}(0)
\end{bmatrix}^{-1}
\begin{bmatrix}
\varepsilon_{yt} \\
\varepsilon_{gt}
\end{bmatrix}
\]  

(10)

To recover the structural form shocks, it is necessary to know the coefficients of the \( A(0) \) matrix. This latter expresses the contemporaneous effects of structural shocks on the variables considered. To identify the four coefficients of matrix \( A(0) \), the following restrictions are applied:

\[Var(\varepsilon_{yt}) = A_{11}(0)^2 + A_{12}(0)^2\]  

(11)

\[Var(\varepsilon_{gt}) = A_{21}(0)^2 + A_{22}(0)^2\]  

(12)

\[Cov(\varepsilon_{yt}, \varepsilon_{gr}) = A_{11}(0)A_{21}(0) + A_{12}(0)A_{22}(0)\]  

(13)

\[C_{11}(L)A_{12}(0) + C_{12}(L)A_{22}(0) = 0\]  

(14)

The first three restrictions stem from (8), the last restriction is obtained by assuming that cumulated demand shocks have no permanent effects on output.

For the GDP to be decomposed into cycle/trend components, the output gap \( \Delta y_t^{gap} \) is obtained by cumulating the demand shocks to output. Similarly, the potential output component \( \Delta y_t^{p} \) is determined by cumulating supply-side shocks. Starting from (2) and given that \( C(L)A(0) = A(L) \), we have:

\[x_t = K + A(L)v_t = K + C(L)A(0)v_t = K + \sum_{i=0}^{\infty} \Phi^{iL}A(0)v_{t-i}\]  

(15)

Considering only the first variable, we obtain:

\[\Delta y_t = K_1 + A_{11}(L)v_{st} + A_{12}(L)v_{gt}\]

\[= K_1 + A_{11}(0)v_{st} + A_{12}(0)v_{gt} + A_{11}(1)v_{st} + A_{12}(1)v_{gt} + A_{11}(2)v_{st} + A_{12}(2)v_{gt} + A_{11}(3)v_{st} + A_{12}(3)v_{gt} + \ldots\]

The potential GDP growth rate is given by:

\[\Delta y_t^{pot} = K_1 + A_{11}(L)v_{st} = K_1 + A_{11}(0)v_{st} + A_{11}(1)v_{st} + A_{11}(2)v_{st} + A_{11}(3)v_{st} + \ldots\]

(16)

the output gap is given by:

\[\Delta y_t^{gap} = A_{12}(L)v_{dt} = A_{12}(0)v_{dt} + A_{12}(1)v_{dt} + A_{12}(2)v_{dt} + A_{12}(3)v_{dt} + \ldots\]

(17)

By using this kind of decomposition is thus possible to obtain an estimate of potential growth and cyclical output component based on economic hypothesis of the structural shocks effects.
3 Empirical results

As a preliminary analysis, we estimated different bivariate models by using output and various business survey data indicators (i.e. degree of plant utilization, inventories, the production level and confidence climate index) coming from Italian Manufacturing Business Surveys. Such data, given their cyclical behaviours are particularly useful for detrending purposes, since allow to incorporate information on the cyclical economic activity.

The selection of survey data to be included in the model was based on the analysis of their contemporaneous correlations with the GDP cyclical component obtained with an Hodrick-Prescott filter.

Although we tried different specifications in what follows we show the results of the bivariate model including the degree of plant utilization. This variable is able capture the whole economy cyclical dynamics\(^1\) with a greater precision and to match business cycle evolution without introducing phase shifts. Output is defined as the Italian Gross Domestic Product (expressed in euros at constant 1995 prices, seasonally adjusted source ISTAT). The structural model specification, called SVAR, thus includes GDP in log differences and the degree of plant utilization.\(^2\) The confidence intervals of the estimates were calculated using bootstrap resampling and drawing random shocks from the estimated structural shocks.

![Figure 1 Output gap SVAR Model](image)

Figure 1 shows the estimated cyclical GDP component. The output gap determined through the SVAR specification appears to be positive from the second half of the Eighties till the Nineties and from 1994 to 1996. The end-of-sample cycle becomes more erratic. These findings reflect the stagnation experienced by the Italian manufacturing sector in the past five years. The results of 90 percent confidence intervals show that the uncertainty surrounding the output gap estimates appears to be significant.

\(^1\) Although the survey data refer to the manufacturing sector, they are able to thoroughly capture the whole economy dynamics (on this point see Hearn and Woitek, 2001 and Cesaroni, 2007).

\(^2\) The lag structure of the reduced form was selected by using the Schwartz and Akaike criteria. The results of the Portmanteau test for the residual autocorrelation do not allow to reject the null hypothesis of autocorrelation absence. The usual heteroscedasticity test indicates omoscedastic residuals.
To evaluate whether the estimated cyclical component obtained with the SVAR decomposition accurately indicates business cycle turning points, we make a comparison between the turning points obtained through the Italian cyclical chronology and the peaks and troughs identified through different output gap estimates. In particular, the output gap estimates obtained using the SVAR decomposition, a quadratic trend, the Hodrick-Prescott (1997) filter with a lambda parameter set to 1600 are compared. The sample period is 1985-05. The Italian cyclical chronology used here comes from Altissimo, Marchetti and Oneto (1999). This methodology detects turning points and cyclical phases on the basis of the coincident indicator absolute variation level and it is based on the classical business cycle definition by Burns and Mitchell (1946).

Figure 3 Cyclical chronology (recession periods: grey area/expansion periods: white area).

Figure 3 reports maximum and minimum turning points of the Italian cyclical chronology. Looking at the graph, we notice that all the output gap estimates are able to indicate quite precisely the business cycle turning points, even though each estimate differs from the other in the dynamics displayed into the expansion and recessions zones. Moreover, the results show that, although the quadratic trend and the Hodrick-Prescott evolutions (left scale) are relatively similar, the SVAR model estimates (right scale) differ from those methods, particularly starting from 2001. The output gap, which is negative from 2001 to 2005 when using univariate estimates, seems positive in the same period when adopting the VAR model estimates. The difference in the two output gap indicator dynamics of SVAR as against the univariate methods stems from the use of an external signal (i.e. coming from business survey data).

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3 Variables included in the coincident indicator are GDP, the industrial production index, imports of investment goods, the share of overtime hours, railway transport, machinery and equipment investments and the market services’ value added.
Figure 4 Comparison between output gap SVAR estimates and output gap estimates obtained with the OECD methodology (annual frequency).

To provide a further check of the reliability of the SVAR output gap measure, figure 4 reports a comparison between the output gap obtained with the bivariate SVAR decomposition (right scale) and the annual output gap estimates for Italy published in the OECD Economic Outlook Database (2006). The OECD estimates are obtained by using the production function approach according to European Commission guidelines. To compare the dynamics of OECD series, the SVAR data were collapsed to annual frequency. The results show that, although the two measures are based on rather different methodologies and assumptions, the dynamics appear to be quite similar with a correlation in the sample considered of roughly 0.7.

4 Forecasting inflation

In order to investigate whether the output gap obtained with the SVAR decomposition has any ability to forecast inflation, we estimated a forecasting equation for inflation growth using the output gap as regressor. We then report an out-of-sample exercise comparing the forecast accuracy at different horizons and with respect to a benchmark autoregressive model. The estimated equation was:

$$\pi_t = \alpha + \sum_{j=1}^{k} \beta_j GAP_{t-j} + \gamma \tau_{t-1} + \delta D_{t,m} + \varepsilon_t$$

where $\pi_t = \log P_t - \log P_{t-4}$ is the inflation rate, $GAP$ is the output gap estimated with the SVAR decomposition, $k$ are the number of GAP lags in the equation and $D_{t,m}$ is a dummy variable. The unknown coefficients were estimated by OLS. The estimates are obtained for the period 1985:3 - 1997:2. The one- and two-step ahead out-of-sample value forecasts are generated recursively. The

4 More in detail, the forecast of the first observation in the period 1997:3 was obtained with parameter estimates using data up to 1997:2. Subsequent forecasts were calculated by re-estimating each model with the new data point and then forecasting the next observation.
one quarter forecasts started in 1997:3 and end in 2004:4. The two quarter forecasts started in 1997:4 and ended in 2005:1. The forecasts were compared to an AR(2) model.

Table 3 RMSFE of the out of sample forecasts. Recursive estimation

<table>
<thead>
<tr>
<th>Models (recursive)</th>
<th>RMSFE(h=1)</th>
<th>RMSFE(h=2)</th>
</tr>
</thead>
<tbody>
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<td>SVAR_OG ( \pi(t)= \pi(t-1)+\text{og}(t-1)+\text{og}(t-4) )</td>
<td>0.0406</td>
<td>0.0689</td>
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<tr>
<td>Benchmark AR ( \pi(t)= \pi(t-1)+\pi(t-2) )</td>
<td>0.0038436</td>
<td>0.082433</td>
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<tr>
<td>Relative RMSFE</td>
<td>10.56301</td>
<td>0.83583</td>
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The results show that the forecast ability of the output gap model based on the structural VAR decomposition gives better results than the benchmark autoregressive model in a two quarter horizon. Conversely, the information content of the output gap seems not to yield value added in forecasting inflation at a shorter horizon (i.e. one quarter). This finding shows that the output gap tends to be helpful in forecasting inflation only when the forecast horizon is increased.

5 Conclusions

This paper investigates the effects of a decomposition of real GDP into its trend and cyclical components by using a multivariate decomposition. In particular, we focused on the possibility to obtain reliable estimates of potential output and output gap using structural VAR models including data from business surveys.

From an economic point of view those models provide an economic interpretation to the structural shocks. Furthermore, given that restrictions to shape the structure of each component are not required, the methodology does not impose an \textit{a priori} limitation to modelling trend and cycle dynamics in the data. In this sense, while most detrending methods assume a random walk process for the trend component, the VAR decomposition does not involve a similar assumption.

In our findings, the estimated output gap indicator is able to indicate quite precisely the turning points over the expansions and recessions periods of the Italian official chronology. The results also show that the output gap estimates based on the SVAR model seem to have some predictive power in forecasting inflation compared to the benchmark, and that they could be useful in forecasting inflation.
References

Appendix

Table 4 Portmanteau Test VAR model

<table>
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<tr>
<th>Lags</th>
<th>Q-Stat</th>
<th>Prob.</th>
<th>Adj Q-Stat</th>
<th>Prob.</th>
<th>df</th>
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H0: no residual autocorrelations up to lag h
Sample: 1985q1 2005q1
Included observations: 80
*The test is valid only for lags larger than the VAR lag order.
df is degrees of freedom for (approximate) chi-square distribution

Table 5 Lag selection criteria-VAR model

| Endogenous variables: delta y and degree of plants utilization |
| Exogenous variables: C |
| Sample: 1985q1 2005q1 |
| Number of observations included: 74 |

<table>
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<th>Lag</th>
<th>LogL</th>
<th>LR</th>
<th>FPE</th>
<th>AIC</th>
<th>SC</th>
<th>HQ</th>
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* lag order selection criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Criterio di informazione di Akaike
SC: Criterio di informazione di Schwartz
HQ: Criterio di informazione di Hannan-Quinn